ON THE PERIOD AND LOGARITHMIC DECREMENT
OF A CONTINUOUSLY VIBRATING WIRE.

BY JOSEPH O. THOMPSON.

THE experiments described in the following article originated from a desire to study more minutely the phenomena to which Lord Kelvin¹ called attention in the following words :

“ Experimental exercises performed by students in the physical laboratory of the University of Glasgow, during the session 1864–65, brought to light some very remarkable and interesting results, proving a loss of energy in elastic vibrators (sometimes as much as two or three per cent. of energy lost in the course of a single vibration in one direction) incomparably greater than anything that could be due to imperfections in their elasticity, and showing also a very remarkable fatigue of elasticity, according to which a wire which has been kept vibrating for several hours or days through a certain range came to rest much quicker when left to itself than when set in vibration after it had been at rest for several days and then immediately left to itself. Thus it was found that the rates of subsidence of the vibrations of the several wires experimented on were generally much less rapid on the Monday mornings, when they had been at rest since the previous Friday, than on other days of the week, or than after several series of experiments that had been made on a Monday.”

It is well to quote also the following statement of the object of an experiment :

“ Two equal and similar wires, with equal and similar vibrators, one of them kept as continually as possible in a state of vibration, from day to day ; the other kept at rest, except when vibrated in an experiment once a day (to test the effect of continued vibration on the viscosity of a metal).”

¹ Math. and Phys. Papers, Vol. III., p. 22.

One of the results of this experiment is given in the note of May 27 :¹ "No. 1 has been kept at rest from May 17, while No. 2 has been kept oscillating more or less every day till yesterday, May 26, when both were oscillated, with the following results :

	Time per Vibration.
No. 1 subsided from 20 to 10 after 100 vibrations in 242 sec.	2.42
No. 2 subsided from 20 to 10 after 44 or 45 vibrations.	2.495 "

Thus it seems that Lord Kelvin attributed the increased period and the increased logarithmic decrement of No. 2 to the fact that it had been kept in such constant vibration. I also had no doubt that both the above-mentioned quantities were functions of the time the wire had been kept in vibration, and I accordingly went to work to investigate the nature of those functions.

The variations in period of vibration, since they so accurately give a measure of the variations in the rigidity modulus of the wire, were first studied, and since the modulus of elasticity is such a complicated function, it was evidently desirable to keep constant the quantities that are ordinarily variable. The photometer-room in the basement of the Physical Laboratory of Amherst College furnished a reasonably constant temperature, the temperature change rarely exceeding half a degree in twenty-four hours. To keep the pendulum in constant vibration day and night, and, what is vastly more important, to keep the amplitude of vibration constant, the following arrangement was used : I took for the disc of the torsion pendulum a brass plate 24 cm. in diameter and weighing $10\frac{1}{2}$ lbs. Two iron rods $\frac{1}{4}$ " in diameter were screwed into diametrically opposite holes near the edge of the disc ; these rods rose a little way, and then were bent into arcs parallel to the edge of the disc, as indicated in the accompanying diagram. Above the disc and not connected with it were two solenoids wound on lead pipes bent into arcs similar to the iron rods just mentioned. The iron rods served as cores of the solenoids. Hanging from the bottom of the disc near the edge was a light strip of brass carrying on its lower end a platinum tip and also a glass tip. When the disc swung one way the platinum tip made a brief contact with a platinum plate below; this

¹ Vol. III., p. 26.

sent an electric current up the brass strip, across the disc to the center, down the wire to a mercury cup, through the solenoids. The iron cores were sucked in, and thus a small impulse was given to the vibrating disc at the moment it was passing its equilibrium

Fig. 1.

position. When the disc swung the other way the insulating glass tip made contact with the platinum plate and no current passed. Thus the plate could be kept vibrating any number of hours or days, and with an amplitude nearly constant. By proper regulation of the

current any arc of vibration from 10° to 250° could be maintained for any length of time. An excellent feature of this device is that it automatically varies the impulse according to the needs of the disc. Suppose for instance we wished to maintain a constant arc of vibration of 100° and the current was regulated accordingly, and suppose some temporary disturbance caused the arc to fall to 90° . The time of electrical contact and consequently the impulse would thereby be increased and the pendulum would soon regain its former amplitude. In most of my experiments half an ampère sufficed to maintain a total arc of vibration of 180° , and the time of electrical contact was about $\frac{1}{40}$ the entire period of vibration. Hence the drain on the storage battery was exceedingly slight.

Since copper wires were used in Lord Kelvin's experiments I was especially careful in studying the behavior of that metal. Experiments were also made with wires of brass, aluminium and silver. No matter what metal was used, no matter whether the arc of vibration was as small as 20° or as high as 200° , no matter whether the wire was long or short, thick or thin (provided of course the breaking strength of the wire was at least twice the weight of the disc) the result was uniformly the same, namely, *that when temperature and amplitude of vibration remained constant, the period of vibration was a constant quantity*. In one experiment with a copper wire the period obtained after fifty hours of continuous vibration through an average arc of 180° was found to be just what it was in the first two minutes. Gauss' method of timing was used and the degree of accuracy aimed at and attained was about $\frac{1}{20000}$.

The following table shows the results of May 16th, a day on which the temperature at the beginning of the experiment in the

Time.		Temperature.	Period.
h.	min.	deg. cent.	sec.
9	20	17.0	21.564
9	58	17.2	21.568
11	30	17.4	21.588
11	49	17.4	21.588
1	57	17.0	21.575
3	09	17.0	21.571
4	12	17.0	21.565

morning was the same as at the end of the experiment in the afternoon. The wire was of copper, 37.7 cm. long and .081 cm. in diameter. The arc was 180° .

The figures for the period indicate, though they are not accurate enough to prove, what I suspect to be the truth, namely, that the period changes with the temperature, not instantly, but in a somewhat sluggish manner.

THE LOGARITHMIC DECREMENT.

Having obtained the above evidence that the continuance of the vibrations had no effect on the period it seemed best to make a still more careful investigation of the effect of this continued vibration on the logarithmic decrement. Since I doubted whether the ordinary method of measuring the decrement would be sufficiently accurate to give any conclusive results, and since the small amplitudes ordinarily employed seemed unlikely to produce any measurable variation in the decrement, I made the following modification of the ordinary method of measuring the logarithmic decrement.

Two silvered mirrors were mounted 8 mm. apart and were carefully adjusted so that they were parallel within $1'$. These two mirrors were placed on the vibrator, one on each side of the wire, and instead of being parallel to the telescope scale were made perpendicular to it at its central point. Then when the vibrator was turned 90° from its position of equilibrium readings could be made near the center of the scale; and, just as it is unnecessary to change tangents to arcs when the amplitude does not exceed 2 or 3 degrees, so in our case it is unnecessary to make any correction when the amplitude varies only a few degrees from 90° . Ordinarily in getting the decrement I allowed the total arc of vibration to fall from about 185° to 175° , and then the electrical driving device would be applied to bring the amplitude up to its original value. It must be remembered that in the case of all decrements mentioned later the mean arc was about 180° .

To show exactly how the method was applied I give the following figures, referring to a set of observations made on a copper wire, the one already mentioned above:

The distance from mirror to scale was 98.66 cm.; the circum-

ference of the circle of this radius is 619.9 cm., but I have called it 620 cm. in the computation.

Scale readings.	Differences.	Double arcs.	Logarithms.
cm.		cm.	
42.60			
54.70	+12.1	632.1	2.800786
47.80	+ 6.9	626.9	2.797198
49.55	+ 1.75	621.75	2.793616
52.88	— 3.33	616.67	2.790053
44.50	— 8.38	611.62	2.786481
57.90	—13.40	606.60	2.782902

1st log. — 4th log. = .010733

2d “ — 5th “ = .010717

3d “ — 6th “ = .010714

Mean .010721

.010721 ÷ 3 = .003574 = Log. Dec.

A careful inspection of the above figures will show the exceptional accuracy of the method. The period of the pendulum was 21.51 sec., and by making the vibrations slower, greater accuracy in getting the scale-readings could, of course, be secured. When the arc was so great, over 180°, a quick eye was needed in order to be sure of the fractions of a mm.; but if I had made the period greater there would have been less likelihood of detecting variations either in period or in decrement.

The following measurements will be interesting on account of the fact that the wire was kept vibrating through an average arc of 180°

Time.	Temperature.	Log. Dec.
July 5.	°	
2.27 P. M.	23	.00364
2.33	23.1	370
2.39	“	365
2.49	23.2	365
3.09	“	363
3.53	“	367
4.37	23.3	367
5.33	“	365
9.21	21.9	358
July 6.		
8.53 A. M.	20.6	367

for 50 consecutive hours. The wire was the copper wire already mentioned, and had a breaking strength of 32 lbs. The disc weighed 10.5 lbs. Since the experiments of Schmidt¹ show that the elastic after-effect has a very marked influence on the logarithmic decrement no measurements were made until the wire had been stretched over a week by its load.

Thus after 18 hours of continuous vibration the decrement was only a trifle higher than the average for the first afternoon. The second decrement, .00370, owes its large value to the fact that just before it was measured the vibrating disc was accidentally given an amplitude about two degrees larger than it had at the start. The small value, .00358, I cannot with certainty account for.

Since it seemed clear that nothing of value would be gained by trying to keep the amplitude constant, I varied the experiment by increasing the arc to about 250° , noting the increased decrement and the gradual recovery.

Time.	Temperature.	Log. Dec.
July 6.		
9.10 A.M.	20.9	.00372
9.16	21.1	370
11.45	21.4	365
2.27 P.M.	21.3	358

Then the largest possible arc permitted by the apparatus, 250° , was given and the three decrements measured in the next 15 minutes were .00367, .00367 and .00371. On my return at 9 o'clock in the evening the arc was for some reason reduced to about 120° and the decrement, as was to be expected, was lower than any hitherto found, .00346. It became .000363 after the disc had been given its greatest possible amplitude.

The next day, July 7th, at 9:40 A. M., the arc was again down to about 120° and the decrement was found to be .00343. Temperature was $20^\circ.8$. At two in the afternoon the temperature was $21^\circ.4$ and the decrement .00357. After the amplitude was given its greatest possible value the decrement was found to be .00367.

Between these two last named values of the decrement lies the

¹Schmidt, Annalen der Phys. n. F., Bd. II.

value obtained 48 hours before at the very beginning of the experiment, namely, .00364. Considering the foregoing evidence and the fact that the smallest decrement, .00343, means an arc ratio 1.0079 and the largest decrement, .00372, a ratio 1.0086, and considering the further fact that the former decrement was obtained after the arc had for some time been 120° while the arc was 250° shortly before the largest decrement was measured, it seems probable to me that for constant temperature and constant amplitude the logarithmic decrement is a constant.

Finally this experiment gives corroborative evidence as to the constancy of the period. The very first measurement of the period made in the early part of the afternoon of July 5th gave as a result 21.496 sec. at the temperature $23^\circ.2$. The very last measurement made July 7th after the disc had been vibrating for over two days through an average arc of 180° was identically the same, 21.496 sec. The temperature was 22° . The reason why this value of the period differs from the values given above is that those values were obtained as the pendulum was electrically driven, while this lower value was obtained when the pendulum was swinging absolutely free, being kept in vibration by an occasional tap of the pencil. This tapping method was the more convenient while I was giving my attention chiefly to the logarithmic decrement. The above figures indicate that the electrical impulse was given not at the instant the disc was passing its equilibrium position but a little later.

EXPERIMENTS WITH INCREASED AMPLITUDES.

After having reached the conclusion that under conditions of constant temperature and constant amplitude both the period and decrement are constant, it seemed very desirable to find out if possible how Lord Kelvin could have obtained results which apparently differ so much from the results I have mentioned.

I accordingly removed the solenoids so that the arc of vibration might be increased up to any desired amount, and then I increased the arc from its former value, 180° , to about 360° . With this large arc the period was 21.66 sec. and twenty minutes later, when the arc averaged about 200° , the period was exactly the same, 21.66 sec.

The decrement, as was anticipated, showed a large increase and a surprisingly rapid recovery, as is indicated in the following table :

Time.	Temperature.	Log. Dec.
July 7.		
3.58 P. M.	22.1	.00506
4.02	"	486
4.09	"	470
4.38	22.2	445

These decrements, as in all the preceding cases, were measured while the arc was decreasing from about 185° to 175° .

At 4.45 the period for an arc of 180° was 21.52 sec., thus showing a prompt recovery.

At 4.53 after over 50 hours of continuous vibration the disc was brought to rest.

The following day, after some experiments with the same wire, the decrement had recovered to .00391, and the period showed an entire recovery, being 21.496 sec. at $22^{\circ}.7$.

Finally the disc was given an arc of about $360^{\circ} + 180^{\circ}$, and after the arc had died down to 180° , the period was 21.68 sec. and the decrement .00672, by far the largest value yet found. *Thus was demonstrated the fact that by suitably modifying the antecedent amplitude of vibration any decrement within wide limits and any period within narrow limits might be obtained.*

Hence such results as those noticed by Lord Kelvin may be produced by giving the vibrating disc a very large amplitude. It seems to me not improbable that at times very large amplitudes were given to his disc, for his words "as continually as possible" imply that no device was used in order to keep up the vibrations. And if all the impulses had to be given by hand, and by students' hands, some of those impulses were probably pretty large, assuming of course that there are points of resemblance between American and Scottish students.

EXPERIMENTS WITH METALS OTHER THAN COPPER.

Since the experiments cited at the beginning of this article were made with copper wire I naturally gave more attention to that metal than to any other. It was very desirable, however, that corroborative experiments be made with other metals and accordingly a care-

ful study was made of the behavior of wires of aluminium, silver and brass. The results obtained with aluminium were entirely similar to those obtained with copper. The aluminium wire was 96 cm. long, .108 cm. in diameter, and at the beginning of the experiments, July 11th, had a period of 20.270 sec. and a decrement of .00670 at $19^{\circ}.6$. After over 10 hours of continuous vibration, the temperature being 21° , the period was 20.285 sec. and the decrement .00724. This large decrement is due to the fact that 45 minutes before this I had given the disc an arc of vibration of 360° . The decrement thereby produced was .00853, and the decrements measured in the next 45 minutes (.00764, .00739, .00724) indicated that in an hour more the decrement would have returned nearly to its original value. These large decrements mean of course a much more rapid dissipation of energy than what took place in the copper wire. The current used in keeping the aluminium wire in vibration was therefore considerably greater, being about .75 ampère, whereas in the case of the copper wire a current of .5 ampère sufficed.

The silver wire corroborated the testimony of the other wires even though it had to bear a load unreasonably heavy. The breaking strength was 18 pounds, and the disc weighed 10.5 pounds. No thicker wire was readily obtainable and accordingly experiments were made with this, although it was anticipated that the inevitable stretching of the wire would render the results of little value. The decrement was so large that in order to get four consecutive readings on the telescope scale I had to use a period of vibration considerably larger than any hitherto used.

On the morning of July 13th, the period was 30.65 sec. at $19^{\circ}.2$, the next day in the morning the period was 30.73 sec. at $19^{\circ}.6$, and in the afternoon, after 32 hours of continuous vibration through an arc of about 180° , the period was 30.75 sec. at $19^{\circ}.85$. Just how much of this increase was due to the rise in temperature I do not know; but at any rate, considering the length of the wire was only 29 cm. and the average arc was 180° , and the very important fact that the load was considerably more than half the breaking strength, this maltreated wire did remarkably well in maintaining a constant period.

Numerous data from which the logarithmic decrement might be

computed were taken ; but since a careful inspection showed that they modified in no way the results already mentioned no decrements were computed.

The obvious reason for measuring all periods and decrements only when the arc of vibration was between definite limits, say 185° and 175° , was that by so doing the resistance of the air did not need to be determined, being the same in all measurements.

In conclusion I will repeat that the above experiments seem to me to prove that in any vibrating wire that is not unduly loaded, when temperature and amplitude of vibration are constant, both the period and logarithmic decrement are constant quantities.

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