126. Note on the Teaching of "Solution of Triangles" in Trigonometry

Author(s): G. H. Bryan

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In the series $a, b, a+b, 2a+b, 3a+2b, 4a+3b$, etc.,
in which U_n, U_{n+1}, U_{n+2} is an A.P. if n is even, and a G.A. if n is odd,

$$M = \pm(2a^2 - b^2), \text{ and } U_n = 2U_{n-2} + U_{n-4}.$$

In the series composed of two G.A. placed alternately

$$a, x, b, y, a+b, x+y, a+2b, x+2y, \text{ etc.},$$

M = alternately $\pm(ay - bx)$ and $\pm\{a+bx - by\}$; thus $M = \pm 1$, in

$$2, 1, 3, 2, 5, 3, 8, 5, 13, 8, \text{ etc.}$$

M is, in turn, zero and $\pm$ (a term of A.)² in the following series in which u_1, u_4, u_7 , etc., is A.; and also u_2, u_5, u_8 , etc.; and u_3, u_6, u_9 , etc.; viz.:

$$1, 2, 3, 2, 3, 5, 3, 5, 8, 5, 8, 13, \text{ etc.}$$

It is seen from

$$1, r, r^2, r+r^2, (1+r)^2, (1+3r+2r^2), \text{ etc.},$$

that it is possible to have a series of integral functions in which u_n, u_{n+1}, u_{n+2} is a G.P. if n is odd, and a G.A. if n is even. W. H. LAVERTY.

125. [K. 20. d.] Approximation to $\tan A$.

Up to 10° the ordinary formula, $\tan A^\circ = .0175A$, gives close enough results for many purposes; but from 10° to 25° or so it is better to use

$$\tan A = \frac{A}{60} + \frac{A}{6000} + \left(\frac{A}{120}\right)^2.$$

Three examples are given.

$A = 10^\circ.8$	$16^\circ.7$	$24^\circ.6$
$60 \overline{) 10.8}$	$60 \overline{) 16.7}$	$60 \overline{) 24.6}$
$\cdot 1800$	$\cdot 2783$	$\cdot 4100$
$\quad 18$	$\quad 28$	$\quad 41$
$\quad \underline{81}$	$\quad \underline{196}$	$\quad \underline{420}$
$\tan A = \cdot 1899$	$\cdot 3007$	$\cdot 4561$
From tables, $\cdot 1908$	$\cdot 3000$	$\cdot 4578$

are seen to be the more accurate values.

The error is usually less than that in the last example, and as far as I have tried, the calculation gives the true tangent of some angles within a very few minutes of the given one.

An approximate reverse formula is

$$A = 61 \tan A - 16 \tan^2 A,$$

which may be applied to any angle up to 45° with fair exactness, the maximum error in the value of A so found being about a quarter degree.

I have been told that the latter has already appeared in some book on Yachting. C. E. M'VICKER.

126. [V. a; K. 20. e.] Note on the Teaching of "Solution of Triangles" in Trigonometry.

The methods commonly described in text-books for the logarithmic solution of oblique-angled triangles do not permit of the subject being adequately studied until a knowledge has been acquired of the properties of multiple and submultiple angles, and the so-called semi-sum and semi-difference formulae of Trigonometry. The object of the present note is to show how a beginner may proceed directly to the logarithmic solution of oblique-angled triangles with no knowledge beyond what is involved in the definitions of the trigonometric functions, the use of logarithmic tables, together with a preliminary drilling in solution of right-angled triangles.

The cases where a side and two angles or two sides and an angle opposite one are given present no difficulty, as they are solved by the sine rule which can be easily proved. The remaining cases are those in which two sides and the included angle or three sides are given.

For the first of these cases let CA , CB be the two given sides, BCA being the included angle, and suppose CB is the greater of the two sides (so that we do not have to consider the possibility of B being obtuse).

Drop AM perpendicular on BC .
Then we have

if C be acute	if C be obtuse
$MA = b \sin C$,	$MA = b \sin (180^\circ - C)$,
$CM = b \cos C$,	$CM = b \cos (180^\circ - C)$,
$BM = a - CM$.	$BM = a + CM$.

$$\tan B = \frac{AM}{BM},$$

and

$$c = \frac{AM}{\sin B} \quad \text{or} \quad c = \frac{BM}{\cos B}.$$

Finally,

$$A = 180^\circ - B - C.$$

For the second case let a be the greatest of the three sides (so that we do not have to consider the possibility of either B or C being obtuse), and suppose $b > c$.

Drop AM perpendicular on BC as before.

Then, by I. 47,

$$CM^2 - BM^2 = CA^2 - BA^2 = b^2 - c^2,$$

and

$$CM + BM = a.$$

Hence

$$\log (CM - BM) = \log (b + c) + \log (b - c) - \log a.$$

This finds CM and BM , and then the angles are given by the formulae

$$\cos B = \frac{BM}{c}, \quad \cos C = \frac{CM}{b}.$$

The following examples will show how simply and easily triangles can be solved by these methods with a book of four-figure tables.

Ex. 1. $a = 684$, $b = 504$, $C = 94^\circ 16'$.

Here C is obtuse, and $CM = b \cos 85^\circ 44'$.

		Log.	Antilog.	
	b	2.7024		(1)
	$\cos (180^\circ - C)$	8.8717		(2)
	$\sin (180^\circ - C)$	9.9988		(3)
From (1), (2),	CM	1.5741	37.51	(4)
From (1), (3),	MA	2.7012	502.4	(5)
	BM	2.8582	721.51	(6)
From (5), (6),	$\tan B$	9.8430	$B = 34^\circ 52'$	(7)
	$\sin B$	9.7577		(8)
From (5), (8),	AB	2.9441	$c = 879.2$	(9)

Finally,

$$A = 180^\circ - 94^\circ 16' - 34^\circ 52' = 50^\circ 52'.$$

Ex. 2. $a=45\cdot2$, $b=32\cdot9$, $c=15\cdot4$.

	No.	Log.
b	32·9	
c	15·4	
$b+c$	48·3	1·6839
$b-c$	17·5	1·2430
b^2-c^2		2·9269
$CM+BM=a$	45·2	1·6551
$CM-BM$	18·70	1·2718
$2CM$	63·90	
$2BM$	26·50	

	No.	Log.		No.	Log.
CM	31·95	1·5045	BM	13·25	1·1222
$CA(=b)$	32·9	1·5172	$BA(=c)$	15·4	1·1875
$\cos C$		9·9873	$\cos B$		9·9347
$\therefore C=13^\circ 47'$			$B=30^\circ 38'$		

Finally, $A=180^\circ - 13^\circ 47' - 30^\circ 38' = 135^\circ 35'$.

The solutions of these two examples by conventional methods would be as follows :

Ex. 1.			Ex. 2.		
	No.	Log.		No.	Log.
a	684		a	45·2	
b	504		b	32·9	
$a-b$	180	2·2553	c	15·4	
$a+b$	1188	3·0748		2)93·5	
		1·1805	s	46·75	1·6693
$\cot \frac{1}{2}C$	$\cot 47^\circ 8'$	9·9676	$s-a$	1·55	0·1903
$\tan \frac{1}{2}(A-B)$		9·1481	$s-b$	13·85	1·1415
$\frac{1}{2}(A-B) =$	$8^\circ 0'$		$s-c$	31·35	1·4962
$\frac{1}{2}(A+B) =$	$42^\circ 52'$		$(s-a) \div s$		2·5305
$A =$	$50^\circ 52'$		$(s-c) \div (s-b)$		·3557
$B =$	$34^\circ 52'$		$\tan^2 \frac{1}{2}B$		2·8862
$a+b$		3·0748	$\tan^2 \frac{1}{2}C$		2·1748
$\sin \frac{1}{2}C$		9·8651	$\tan \frac{1}{2}B$	$\tan 15^\circ 30'$	9·4437
$\cos \frac{1}{2}(A-B)$		12·9399	$\tan \frac{1}{2}C$	$\tan 6^\circ 58'$	9·0874
		9·9958	B	$31^\circ 0'$	
c	879·2	2·9441	C	$13^\circ 56'$	

Finally, $A=180^\circ - 31^\circ 0' - 13^\circ 56' = 135^\circ 4'$.

A comparison of the alternative methods shows that there is no appreciable saving of labour in the use of the elaborate formulae commonly taught, and that by the use of the simpler methods here suggested the subject may be taught to beginners at an earlier stage than has been hitherto customary.

G. H. BRYAN.

REVIEWS.

A Course of Modern Analysis. By E. T. WHITTAKER, M.A. Pp. xvi 378. (Cambridge University Press. 1902.)

This work is sure of a favourable reception because it gives in a moderate compass an attractive account of some of the most valuable and interesting results of recent analysis. The first part deals mainly with infinite series, especially