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have examined this interesting circumstance (see "Changes of Property of Amalgams by Repeated Fusion," *Phil. Mag.* September 1890, p. 228).

The mixtures of metals which produced the most sudden increases of current in these experiments were those which exhibited the most sudden liquefaction. An explanation I venture to suggest of the increase of voltaic energy during liquefaction is, that the molecules of the liquefied alloy are usually in a state of more active motion than those of the solid substance, partly because the liquid has absorbed more potential heat; and that the suddenness of the increase of the current at the period of liquefaction is due to absorption of heat being promoted, and chemical and voltaic action facilitated by the sudden occurrence of convection currents in the liquid alloy.

As energy is indestructible, the effect of each unit of heat absorbed by the alloy upon its total amount of energy of molecular motion must be the same whether the heat is absorbed as specific heat and raises its temperature, or as latent heat which, without altering its temperature, changes its physical state by fusion, or its other properties, as in the case of iron at its "critical point."

The results of the experiments show that changes of voltaic energy may be used in the above manner to detect physical and chemical changes in metallic alloys.

V. *A Kinetic Theory of Solids, with an Experimental Introduction.* By WILLIAM SUTHERLAND*.

THE present work started in an experimental inquiry having a twofold object,—to try to ascertain whether there is a general law connecting the elastic properties of metals with temperature, as is the case with gases, and to discover it if possible, and, failing that, to obtain empirical means of finding the values of the elastic constants of the metals at absolute zero, at which temperature they would be specially interesting, as the molecules would be at rest.

The first result was that a general law unfolded itself with singular ease, namely that the law of variation of the rigidity of the metals with temperature is the same for all: if n is the rigidity at absolute temperature θ , and N that at absolute zero, while T is the melting-point, then

$$\frac{n}{N} = 1 - \left(\frac{\theta}{T} \right)^2,$$

a law almost as simple as that of perfect gases.

* Communicated by the Author.

It next appeared in a similar manner that the law of variation of Young's modulus of elasticity with temperature is nearly the same for all metals, and nearly, though not quite, as simple as that for rigidity.

As these results amounted to an experimental proof that rigidity (and elasticity generally) is essentially a kinetic phenomenon I was encouraged to proceed with a kinetic theory of solids, which had been in view when the experimental inquiry was undertaken.

The present paper will be divided into two parts :—
Experimental Introduction and Theory.

EXPERIMENTAL INTRODUCTION.

This will be divided into two parts, Rigidity and Young's Modulus, in which the matter will be taken in the following order :—first, an account of my own experiments on the effect of temperature on elasticity ; second, a compilation of all available data on the same subject ; third, a compilation of all the available data as to the absolute values of Rigidity and Young's Modulus at about 15° C. ; and fourth, establishment of the general law connecting these elastic constants with temperature, and determination of their values at absolute zero.

Rigidity.

1. *Experiments.*—The variation of rigidity with temperature has been worked at by Napiersky, Kohlrausch, Pisati, and Tomlinson ; but these experimenters have almost confined their attention to the nobler metals with high melting-points. As I expected that in the case of the metals with low melting-points the law of variation would be easier to discover, it was necessary to make a study of these metals.

I used the torsional method of measuring rigidity both in its static and kinetic form, and found the two forms to give agreeing results ; but as the static proved less accurate, I will not waste space by giving its results here. The kinetic method was worked in the usual manner. The metal to be experimented on was taken in the form of a wire, with a cylindrical vibrator of thin sheet iron soldered to one end, and a strip of thin iron to the other by means of which it could be clamped. As chamber in which to produce variations of temperature, a wooden box about 65 centim. by 20 by 27 was taken and fixed with its long edge vertical. The top and bottom had a hole bored at each middle point to let the wire pass through freely. The iron strip was clamped to the top of the box, and the vibrator either swung just outside or inside

the box according to the length of the wire. It was convenient to have the vibrator swinging outside when it was necessary to largely increase the moment of inertia of the vibrator. To heat the chamber I used as a preliminary expedient a gas-flame burning inside it at one of the bottom corners with vent-holes so distributed as to prevent the concentration of a stream of hot air on any one point of the wire. The mean temperature of the chamber could be regulated by the size of the flame, and was measured by a thermometer the bulb of which was almost in contact with the middle point of the wire. Although this was intended to give only rough preliminary measurements, it worked so well and was so simple and handy that I retained it throughout. It was tested repeatedly in the following manner:—a gas-flame was turned on sufficient to maintain a constant temperature of say 70°C. , as recorded by the thermometer, and the period of vibration of the wire under experiment was observed. The flame was then increased till a temperature of 105°C. was reached, when the flame was turned off, and the vent-holes closed; the temperature of the chamber then fell slowly and the period of vibration of the wire was again observed while the temperature fell from 71° to 69° ; the difference in the two values of the period never amounted to more than what would correspond to an uncertainty of 2° in the reading of the temperature, and in most cases was actually inappreciable. This was a degree of accuracy quite sufficient for the purpose in view.

In determining the period of vibration of wires with such molecular viscosity as those of lead, tin, and zinc, there is this difficulty, that when vibration is started it tends to die away after a very few swings, so that accuracy in the measurement of the time of a single swing is not possible without some method of periodically reinforcing the swing. I found the simplest and best plan of doing this to be the application of a slight impulsive couple with the hands to the vibrator at the middle of every n th swing. With a little practice it was soon possible to make a large number of measurements of the period of a lead wire not differing amongst themselves by more than 1 in 300. Each determination of a period of vibration was made by observing with the seconds hand of a watch the time of between 100 and 300 swings.

The rigidity is calculated from the experimental data by the formula

$$n = \frac{2\pi^3 I l^3 \rho^2}{m^2 (t/2)^2},$$

n being the rigidity, l the length of the wire vibrated, m its mass, ρ its density, I the moment of inertia of the vibrator, and t the time of a complete vibration. Using C.G.S. units in the measurements, the rigidity is given in dynes per square centim., but as the gramme weight is the unit of force most employed by elasticians it will be used here.

When the vibrator swung outside the chamber, a small length of the wire was not heated directly but only by conduction; to allow for this the small length was assumed to be at the temperature of the external air. Let n_1 be the rigidity at the initial temperature θ_1 of the surrounding air, n_2 that at any higher temperature θ_2 , and λ the length of the small part of wire outside of the chamber, α the coefficient of linear expansion of the wire, t_1 and t_2 the periods observed at the two temperatures, then the ratio n_2/n_1 is given by the formula

$$\frac{n_2}{n_1} = \frac{l-\lambda}{l} \frac{1}{1+3\alpha(\theta_2-\theta_1)} \div \left[\left(\frac{t_2}{t_1} \right)^2 - \frac{l-\lambda}{l \{ 1 + \alpha(\theta_2-\theta_1) \}} \right].$$

Lead.—Commercial wire.

l .	λ .	m .	ρ .	I .
77.05 cm.	4 cm.	28.4 gm.	11.4	586500 gm. cm. ²
Temperature C. θ		21°	60°	102°
Half-period $t/2$		5.51 sec.	5.71	6.00
Rigidity ratio n_2/n_1		1.00	.924	.829
Rigidity at 21° C., 90×10^6 grammes weight per sq. cm.				

To control the absolute value found, a large number of measurements were made on a longer wire with a lighter vibrator; length 238.7 centim., mass 88 gm., density 11.4, moment of inertia of vibrator 3350 gm. centim.², temperature 17° C., and half-period .7365 sec. These values give a rigidity 90×10^6 at 17° C., in close agreement with the value found from the shorter wire.

To make still surer of getting this absolute value of the rigidity of lead thoroughly tested, I carried out some experiments on a much larger scale, using a piece of stout lead pipe instead of wire. For vibrator a rectangular plank 179.7 centim. long and 14.6 centim. wide was taken; a hole was bored at its centre through which the pipe was pushed till a short length projected which was slit and hammered back in four flaps, then strongly nailed to the plank; a conical plug being driven into the pipe, there was no doubt about the rigidity of the connexion between pipe and vibrator. The plank weighed

2649 gm. The other end of the pipe was similarly secured to a plank which was clamped as rigidly as possible to the beams of a roof. In the case of a hollow cylinder of external radius r_2 and internal radius r_1 rigidity is given by the formula

$$n = \frac{8\pi^2 I l^3 \rho}{t^2 m (r_2^2 + r_1^2)}.$$

The moment of inertia of the plank could be increased by adding weights.

l .	m .	ρ .	θ .	r_2 .	r_1 .
120.9 cm.	1699 gm.	11.4	23°	.9014 cm.	.6482 cm.

With the plank alone, whose moment of inertia was 7176000 gm. cm.², the period was .717 sec.; the moment being increased to 10577000 the period became .881 sec., and with the moment again increased to 20684000 the period rose to 1.222 sec. These values give for the rigidity the values 89.3×10^6 , 87.2×10^6 , and 88.6×10^6 respectively, the differences being due doubtless to imperfect rigidity in the supporting of so heavy a vibrating system. The mean value 88.4×10^6 at 23° is close enough to the former value 90×10^6 at 17° to make it pretty certain that the absolute value of the rigidity of lead at 20° can differ but little from 89×10^6 grammes weight per sq. centim.

Zinc.—To get a wire of zinc I had a strip about .3 centim. wide cut from a sheet of rolled zinc .3 centim. thick. The rod of square section was then roughly trimmed to circular section with a file, and, finally, by longitudinal scraping while kept revolving it was converted into a fairly uniform circular wire.

l .	λ .	m .	ρ .	I .				
73.7	5.1	36.3	7.04	586500				
θ C....	18°	48°	63°	82°	97°	67°	52°	21°
$t/2$...	1.338	1.367	1.398	1.446	1.495	1.405	1.382	1.340
n_2/n_1 ,	1.0	.952	.906	.842	.784	.897	.930	.997
Rigidity at 18°, 311×10^6 grammes weight per sq. centim.								

Tin.—Again I had to make a wire, this time by casting a rod and hammering out to three times its original length. I did not succeed in getting uniformity or true circularity of section, so that a reliable absolute value of the rigidity of tin is not obtainable from these experiments, but the rigidity ratio being independent of the form can be accurately got.

<i>l.</i>	λ .	<i>m.</i>	ρ .	<i>I.</i>	
71.7	2.5	37.45	7.29	586500	
θ C.....	16°	51°	66°	86°	103°
<i>t</i> /2	2.15	2.35	2.42	2.54	2.65
n_2/n_1	1.0	.854	.804	.727	.666

The rigidity at 16° is 109×10^6 , an uncertain value.

To get a more accurate absolute value of the rigidity of tin, I carried out some experiments on a large scale on a piece of tin pipe in the same manner as on the lead pipe.

<i>l.</i>	<i>m.</i>	ρ .	θ .	r_2 .	r_1 .
116.8	795.5	7.29	18°	.689	.429

With the three moments of inertia 7176000, 10577000, and 20684000 the periods were .851, 1.029, and 1.418 sec., and the corresponding values of the rigidity are 154.3×10^6 , 156.1×10^6 , and 160.9×10^6 ; the mean rigidity of tin at 18° being therefore 157×10^6 . This is a more reliable determination than the last.

Magnesium.—The specimen used was a piece of commercial fine wire.

<i>l.</i>	λ .	<i>m.</i>	ρ .	<i>I.</i>
54.6	0	.25 gm.	1.74	525 gm. cm. ²
θ	21°	100°		
<i>t</i> /2	1.42	1.45		
n_2/n_1	1.0	.955		
Rigidity at 21°, 130×10^6 .				

To check this value of the absolute rigidity another determination was made on a longer wire with a heavier vibrator; *l* 200.6, *m* .891, ρ 1.74, *I* 3395, θ 21°, *t*/2 7.10. Rigidity 131×10^6 , a result in good agreement with the last.

2. *Compilation of Data as to Variation of Rigidity with Temperature.*—The most extensive work yet done on this subject is that of Pisati (Wied. Beibl. i.), who studied silver, copper, gold, aluminium, iron, steel, and platinum from zero to 200°, and in some unannealed specimens up to 300°; but the results at high temperatures on unannealed specimens are of no use for our present purpose, because mixed up with the pure temperature effect there is an annealing effect. However, from zero up to 100° the proportional change of rigidity of the metals with high melting-point is nearly the same in the unannealed as in the annealed state. The other sources

of knowledge on this subject are Kohlrausch and Loomis (Pogg. Ann. cxli.), Napiersky (Pogg. Ann., *Erganzbd.* iii.), and Tomlinson (Proc. Roy. Soc. xl.), who do not go beyond 100°. The simplest way of comparing the results of different experimenters will be to tabulate the ratio of the rigidity at 100° to that at zero as found by each. I will also give from Pisati the ratio for rigidity at 200°. Where a specimen is known to have been unannealed it will be marked (*u.*)

TABLE I.

Metal.	100°.	200°.	Authority.
Copper (<i>u.</i>)	·969	·911	Pisati.
Copper	·971	·917	Pisati.
"	·945		Kohlrausch.
"	·971		Tomlinson.
Silver (<i>u.</i>)	·960	·858	Pisati.
Silver.....	·956	·898	Pisati.
"	·962		Napiersky.
"	·961		Tomlinson.
Gold (<i>u.</i>)	·971	·912	Pisati.
Magnesium	·943		Sutherland.
Zinc	·843		Tomlinson.
"	·749		Sutherland.
Aluminium (<i>u.</i>).....	·932	·803	Pisati.
"	·943		Tomlinson.
Tin	·638		Sutherland.
Lead	·773		Sutherland.
Iron	·979	·942	Pisati.
"	·960		Napiersky.
"	·950		Kohlrausch and Loomis.
"	·973		Tomlinson.
Nickel	·974		Tomlinson.
Platinum (<i>u.</i>)	·984	·964	Pisati.
Platinum	·982	·934	Pisati.
"	·993		Tomlinson.

The discrepancies in the above table are such as have hitherto proved inevitable in elastic experiments on account of differences in purity of specimen and elastic state. But in Pisati's experiments up to 200° for the metals with high melting-point, and in my own up to 100° for those with low melting-point, there is a large enough variation of rigidity to be only slightly perturbed by accidental circumstances, as will be seen when the law of variation is under discussion. In the case of zinc, where there is pronounced disagreement between Tomlinson's result and my own, Tomlinson's is to be preferred as he experimented on a true wire.

3. *Compilation of Data as to Absolute Values of Rigidity at Ordinary Temperatures* (between 0° and 20°).

The sources of these data are, in addition to those mentioned in the last compilation, Kupffer (*Mém. de l'Acad. des Sc. de St. Pétr.* 6 sér. t. v.), Tomlinson (*Phil. Trans.* 1883, and *Proc. Roy. Soc.* xlii.), Sir W. Thomson (*Encyc. Brit.* art. Elasticity), and Kiewiet (*Wied. Ann.* xxix.).

The rigidity is given in grammes weight per square centimetre.

TABLE II.

Metal.	$n/10^6$.	Authority.	Metal.	$n/10^6$.	Authority.
Copper	390	Kohlrausch and Loomis.	Aluminium (<i>u</i>)	234	Pisati.
	397	Pisati.		250	Tomlinson.
	441	Tomlinson.		267	Tomlinson.
	420	Tomlinson.		241	Thomson.
	457	Tomlinson.	Tin	121	Tomlinson.
(mean of many)	449	Thomson.		150	Kiewiet.
	440	Kiewiet.	(hammered)	109	Sutherland.
Copper (<i>u</i>).....	436	Pisati.	(pipe)	157	Sutherland.
Silver	283	Kupffer.	Lead	74	Tomlinson.
	257	Pisati.		89	Sutherland.
	272	Tomlinson.		88	Sutherland.
	270	Thomson.	Iron	742	Kupffer.
Silver (<i>u</i>)	331	Pisati.		713	Kupffer.
Silver (<i>u</i>)	276	Tomlinson.		694	Kohlrausch.
Gold	271	Kupffer.		811	Pisati.
	281	Thomson.		773	Tomlinson.
Gold (<i>u</i>).....	265	Pisati.		751	Tomlinson.
Magnesium ...	172	Tomlinson.		766	Tomlinson.
	130	Sutherland.		790	Thomson.
Zinc (<i>u</i>)	340	Tomlinson.	Nickel.....	758	Tomlinson.
	360	Thomson.	Platinum ...	637	Kupffer.
	380	Kiewiet.		628	Pisati.
Zinc (rolled anneal.)	311	Sutherland.		693	Tomlinson.
				662	Tomlinson.
				622	Thomson.

The discrepancies in this table must be due mainly to differences in purity and elastic state. But on taking the mean of the above values for each element we must obtain values near to the true ones at 15°; these are now given.

TABLE III.

Mean values of 10^{-6} times the Rigidity at 15° C.

Cu.	Ag.	Au.	Mg.	Zn.	Al.	Sn.	Pb.	Fe.	Ni.	Pt.
430	280	270	150	350	250	135	84	750	760	650

4. *General Law of the Variation of Rigidity with Temperature.*

The method of procedure in discovering this law was as follows:—For no single metal was the temperature range large enough to show clearly any simple law, but if the molecules of the different metals behave like similar dynamical systems, then, in passing from the rigidity of rest at absolute zero to the vanishing point of rigidity at the melting-point, the different systems must show close similarity in behaviour. The first thing to do, then, was to find some empirical formula with which to extrapolate for a first approximation to the rigidity at absolute zero. In addition to the experimental data, there was the valuable fact that in each case the rigidity is known to vanish at the known melting-point, so that, although only a small arc of the whole curve of rigidity and temperature was known in each case, the curve could be pretty safely completed by sentiment down to the point of zero rigidity at the melting-point. The curves thus drawn looked as though they could be fairly represented by the empirical formula

$$\frac{1}{n} = a + b 10^{\frac{c\theta}{T-\theta}},$$

T being the melting-point. This makes the rigidity vanish at the melting-point, and allows the three constants a , b , c to be determined from three values of the rigidity at different temperatures: the formula therefore represented a large stretch of curve, and could with some confidence be used for extrapolating over about 300° down to absolute zero. In this manner a preliminary series of values of N , the rigidity at absolute zero of the various metals, was found. Then it was possible to express the value of the rigidity of each metal at all temperatures in terms of the value at absolute zero as unit, and, further, it was promising to express temperature for each metal in terms of its melting-point as unit. When the rigidity ratios so found for the different metals were plotted as ordinates to the temperature ratios as abscissæ, the points for all the metals were found to lie on one curve, a parabola with the equation

$$\frac{n}{N} = 1 - \left(\frac{\theta}{T}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The temporary exponential form can now be discarded; and we will proceed at once to the proof that the simple relation just given is the actual law of the variation of rigidity with temperature. The best way to proceed with the data

already given in this paper will be as before to measure rigidity for each metal in terms of that at common temperatures (about 15°) as unity, and then according to the data given in the earlier part of this paper for lead, zinc, tin, and magnesium, and in Table I., after Pisati, to calculate a value of N/n_1 from equation (1) for each value of n given for the different values of θ .

For example, in the case of tin we have the following values, $T=503^\circ$ (absolute):—

$\theta \dots$	289°	324°	339°	359°	376°
$n_2/n_1 \dots$	1.0	.854	.804	.727	.666

which give respectively the following values of N/n_1 :—

1.492	1.461	1.472	1.480	1.510,
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with a mean value 1.483, the departures from which are within the limits of experimental error.

In the case of zinc, as already mentioned, there is discrepancy between Tomlinson's results and my own; Tomlinson gives for the rigidity of zinc at any temperature the formula

$$n_t = n_o (1 - .00108t - .0000049t^2),$$

by means of which I have calculated the rigidities at the same temperatures as my own determinations were made at in terms of that at 18° as unit in the following, $T=690^\circ$:—

$\theta \dots \dots \dots$	291°	321°	336°	355°	370°
n_2/n_1 (Toml.)	1.0	.955	.932	.896	.865

These give the following values of N/n_1 :—

1.217 ₁	1.218	1.221	1.218	1.215;
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while at the same temperatures my own experiments give

$n_2/n_1 \dots \dots$	1.0	.952	.906	.842	.784
$N/n_1 \dots \dots$	1.217	1.215	1.188	1.145	1.101

Thus, while Tomlinson's values confirm the formula with wonderful accuracy, my own fail to do so, the reason being I believe that my specimen of zinc was not a genuine wire, whereas Tomlinson worked with a properly drawn wire; the mean value of N from his experiments is 1.218, with a maximum departure in the individual experiments of .25 per

cent., which I take to be a very satisfactory confirmation of the law.

The following table needs no explanation now.

TABLE IV.

Temperature θ	273°	373°	473°	
Copper $\left\{ \begin{array}{l} n_2/n_1 \\ N/n_1 \end{array} \right.$	1·0 1·044	·971 1·054	·917 1·050	T=1330°. Mean 1·05.
Silver $\left\{ \begin{array}{l} n_2/n_1 \\ N/n_1 \end{array} \right.$	1·0 1·052	·956 1·053	·898 1·054	T=1230°. Mean 1·053.
Gold $\left\{ \begin{array}{l} n_2/n_1 \\ N/n_1 \end{array} \right.$	1·0 1·045	·971 1·056	·912 1·049	T=1310°. Mean 1·05.
Aluminium $\left\{ \begin{array}{l} n_2/n_1 \\ N/n_1 \end{array} \right.$	1·0 1·063	·932 1·047		T=1120°. Mean 1·055.
Temperature θ	294°	333°	375°	
Lead $\left\{ \begin{array}{l} n_2/n_1 \\ N/n_1 \end{array} \right.$	1·0 1·318	·924 1·338	·829 1·364	T=599°. Mean 1·34.
Temperature θ	294°		373°	
Magnesium $\left\{ \begin{array}{l} n_2/n_1 \\ N/n_1 \end{array} \right.$	1·0 1·090		·955 1·102	T=1023°. Mean 1·096.
Temperature θ	273°	373°	473°	
Platinum..... $\left\{ \begin{array}{l} n_2/n_1 \\ N/n_1 \end{array} \right.$	1·0 1·018	·982 1·016	·934 (·987?)	T=2050°. Mean 1·017.

In the case of iron, if the absolute melting-point 2080, as given by Carnelley, is taken, and Pisati's measurements of rigidity used to calculate N/n_1 , the agreement is not satisfactory ; thus

Temperature	273°	373°	473	573°
Iron $\left\{ \begin{array}{l} n_1/n_2 \\ N/n_1 \end{array} \right.$	1·0 1·017	·979 1·011	·942 ·993	·904 ·978

but if the melting-point is taken at 1650°, which is within the limit of some determinations, then the values of N come out

1·028 1·031 1·026 1·028

with a mean value 1·028 and a maximum departure of ·3 per cent.

Using the mean values of N/n_1 just found, and the values of n_1 given in Table III., we get the following values of N :—

TABLE V.

$10^{-6} N$ or 10^{-8} times the Rigidity at absolute Zero.

Cu.	Ag.	Au.	Mg.	Zn.	Al.	Sn.	Pb.	Fe.	Ni.	Pt.
452	295	284	164	426	264	200	118	771	781	661

The readiest way of clinching the whole demonstration now will be to draw the parabola $y=1-x^2$ from $x=0$ to $x=1$, and with the mean values of N/n_1 or N , calculating n/N as y and θ/T as x , to mark the separate points so determined. This is done in the figure.

For zinc Tomlinson's values of the rigidity are used, and for iron the melting-point is taken as 1650. To avoid excessive crowding of points on the figure all the points except one for iron, and all for platinum and magnesium are omitted. The course of the curve among the points proves that as regards the connexion between rigidity and melting-point the

molecules of the metals are similar dynamical systems, and also that

$$\frac{n}{N} = 1 - \left(\frac{\theta}{T}\right)^2.$$

This concludes the experimental inquiry as to rigidity. The result shows that rigidity in its essence is a kinetic phenomenon almost as simple in character as the elasticity of perfect gases.

[To be continued.]

VI. *Note on Kohlrausch's Theory of Ionic Velocity.* By
W. C. DAMPIER WHETHAM, B.A., *Coutts Trotter Student*
of Trinity College, Cambridge*.

KOHLRAUSCH calculates his numbers for the specific velocities of different ions from measurements of the conductivities of salt-solutions, and of their migration constants, on the supposition that all the molecules of the salt present in solution are actively concerned in conveying the current. The values thus obtained were found to agree with experiment in certain cases by Prof. Oliver Lodge, and an investigation I am now engaged in carrying out seems also to confirm them. It seems to be generally supposed that this is inconsistent with any theory (such as that of dissociation) which supposes only a certain part of the salt to be active (see Lodge, B. A. Report, 1886, p. 391), though some such form of theory seems to be required by the relations shown to exist by Arrhenius, Van't Hoff, Ostwald, and others. If we examine the matter a little more closely, however, I think the two suppositions can be reconciled. Suppose that the ratio of the numbers of the active and the inactive molecules (which is generally supposed to measure the "dissociation") represents in reality the average ratio of the time during which each molecule is active to the time during which it is inactive. Every molecule is in turn active, but *at any instant* only a certain fraction of the molecules are active. [In terms of the dissociation hypothesis, the dissociation ratio measures the ratio of the mean free time to the mean paired time of the ions.] This is, of course, equivalent to supposing a certain fixed fraction of the whole number of molecules to be active, as far as statical effects, such as osmotic pressure, are concerned, but when we consider the velocities of the ions the case is different.

* Communicated by the Physical Society: read May 9, 1891.