Si può anche ottenere l'ellitticità calcolando il rapporto medio dei due assi nel modo usato per le osservazioni del 1883: si trova allora

$$\log \frac{b}{a} = 9.96534 \pm 0.00280$$

$$\frac{a-b}{a} = 0.07670 \pm 0.00596 = \frac{1}{13.04 \pm 1.01}$$

Le differenze fra questi risultati e quelli dell' anno scorso sono intieramente nei limiti degli errori accidentali possibili.

Se si considerano a parte le tre prime osservazioni, nelle quali ambidue i diametri sono stati misurati nella posizione parallela e nella posizione perpendicolare alla

linea degli occhi, si trova, facendo il calcolo col secondo metodo,

$$\log \frac{b}{a} = 9.96029 \pm 0.00686$$

$$\frac{a-b}{a} = 0.08738 \pm 0.01459 = \frac{1}{11.44 \pm 1.92}$$

il quale risultato coincide cogli altri nei limiti degli errori probabili, e dimostra, non esser possibile di ricavare dalle tre osservazioni sopradette alcun indizio positivo di una differenza sistematica fra le misure eseguite nella direzione parallela e quelle eseguite nella direzione perpendicolare.

Anche questa volta si è potuto constatare l'esistenza di macchie sulla superficie del pianeta, ma non si vedevano abbastanza bene per poterne fare oggetto di studio.

R. Osservatorio di Brera in Milano 1884 Luglio 4.

G. V. Schiaparelli.

Note on the Reduction of Transit Observations for Time and Longitude.

By T. H. Safford.

The interesting article of Director Dr. Braun in A. N. No. 2595 affords a graphical solution of the equations resulting from a set of time observations which will be very welcome to many observers who are better skilled in drawing than in computation.

Some years ago I published in the Report of the U. S. Engineers upon Geographical Surveys West of the 100th Meridian in charge of Lieut. (now Captain) George M. Wheeler, Vol. 2, pag. 344-347, a method of preliminary computation which is practically identical in results with Dr. Braun's graphical method.

I give here the equations which Dr. Braun's example affords, formed according to the ordinary method, after allowing for the diurnal aberration.

(A)

$$\begin{aligned} 0 &= 3^s.776 + 0.464 a - 1.086 c + u \\ 0 &= 3.909 + 0.827 a - 1.007 c + u \\ 0 &= 3.794 + 0.653 a - 1.010 c + u \\ 0 &= 3.839 + 0.633 a - 1.014 c + u \\ 0 &= 3.734 + 0.410 a - 1.120 c + u \\ 0 &= 3.678 + 0.856 a - 1.014 c + u \\ 0 &= 4.11 - 6.16 a - 10.42 c + u \\ 0 &= 0.85 - 6.16 a + 10.42 c + u \\ 0 &= 3.324 + 0.645 a + 1.011 c + u \\ 0 &= 3.297 + 0.598 a + 1.025 c + u \\ 0 &= 3.415 + 0.861 a + 1.015 c + u \end{aligned}$$

(B)

$$\begin{aligned} 0 &= 3^s.788 + 0.640 a - 1.042 c + u & (1) \\ 0 &= 4.11 - 6.16 a - 10.42 c + u & (2) \\ 0 &= 0.85 - 6.16 a + 10.42 c + u & (3) \\ 0 &= 3.345 + 0.701 a + 1.017 c + u & (4) \end{aligned}$$

$$0 = -0^s.322 + 6.800 a + 9.378 c \quad (5)$$

$$0 = -0.047 + a + 1.379 c \quad (6)$$

$$0 = +2.495 + 6.861 a - 9.403 c \quad (7)$$

$$0 = +0.364 + a - 1.371 c \quad (8)$$

$$a = -0.159$$

$$c = +0.149 \text{ from the azimuths}$$

$$0 = 3^s.818 - 1.925 c + u \quad (9)$$

$$0 = 3.087 + 1.990 c + u \quad (10)$$

$$c = +0^s.187 \text{ from the clock-corrections reduced to the zenith}$$

$$u = -3^s.459$$

The equations (1) and (4) are derived respectively from a combination with equal weight of those derived from the 6 time stars preceding reversal, and the 3 following; (5), (6), (7), (8) are readily obtained from these, (9) and (10) from combination of (1) and (6) and (4) and (8) respectively.

The value of u thus found, however, would be improved and brought nearer to the result of a least square solution by combining, as Dr. Braun has done, the two values of c ; his graphical process gives $0^s.232$ instead of $0^s.187$ from the clock-corrections; my method here differs from his by the previous reduction to the zenith in (9) and (10). I should be inclined to assign the two values of c which I have obtained equal weights in this case, and to use $0^s.168$, as its final value. We then have from (9)

$$u = -3^s.495 \quad (6 \text{ stars})$$

and from (10)

$$u = -3^s.421 \quad (3 \text{ stars})$$

and can, without serious error, give the former value double weight, thus obtaining the final value

$$u = -3^s.470.$$

It is obvious, however, that the elimination by reversal of constant error would perhaps be more necessary than the increase of weight obtained by the latter process; so that the result $u = -3^s.459$ whose weight is roughly that of $\frac{4.6 \cdot 3}{6+3} = 8$ stars is to be preferred to $u = -3^s.470$ with a weight of 9 such observations. Of course the introduction of the same observations of a polar star into each result in either position tends to equalize these numbers.

But in general the groups will contain a more nearly equal number of stars than 6 and 3; so that the effect of the differences between the two values of c will usually disappear almost entirely.

I cannot say that I quite agree with Dr. Braun's views as to the relative weights. The polar star, it is true, gives the collimation with rather more accuracy than the polar deviation, because its assumed place is not absolutely accurate; but I fancy that the polar here used, Bradley 402 (not 422), is so well determined that its right ascension is probably not more than $\pm 0^s.15$ in error. We can hardly estimate the probable error of the simple Vienna observation, even in reversed positions, as less than $\pm 0^s.15\sqrt{2}$ or $\pm 0^s.21$; so that the weight of the observations as applied to the determination of the azimuth will be roughly more than two-thirds that which it should receive as applied to the determination of collimation. Dr. Braun says 'ein sehr viel grösseres Gewicht in Hinsicht auf die Bestimmung einer Unbekannten (Collimation), als in Hinsicht auf eine andere (Azimut).' I do not infer that he here refers to the greater coefficient of the collimation, but to the uncertainty of the computed right ascension.

On the other hand I should fear the effect of a slight jar in reversal or of molecular gradual change afterwards quite as much as the possible variability from day to day of personal equation between slow and quick stars; and if the places of all the polar stars used at Vienna and Kalocsa are taken from recent good observations made at the same place and uniform in character, the probable error of their positions will be lessened in its effects.

That is to say, if in a longitude campaign the polars employed are all observed with good meridian instruments simultaneously with such employment, the systematic errors of these results will disappear from the longitude work at each station; and the azimuths will be simply relative to an adopted system.

In the previous calculation I ought perhaps to have taken the collimation $0^s.187$ as derived from the clock-stars only, or at least to have given it more weight than the value $0^s.149$; for the former value already includes a certain proportion of the effect of the observations upon the polar; and Dr. Braun's value, $0^s.232$, derived from the direct comparison of the two sets of time-stars reduced to their average parallel, has a probable error of about $\pm 0^s.02$ only; it seems to me quite doubtful whether the value $0^s.150$, derived from the polar star alone, can be said to have any less probable error.

In this country the first extensive list of standard polars employed for longitude work was that contained in

the American Ephemeris. They are mainly in the declinations 70° to 80° ; and were largely drawn from a list given in Sawitch's Practical Astronomy (Goetze's edition) many years ago, originally selected by Struve in connexion with his teaching and work at Dorpat. Owing to our more southern latitudes (25° - 49°) this selection has generally been adhered to, and supplemented mainly from the stars between 70° and 80° in the 'Mittlere und scheinbare Oerter u. s. w.'; and the least square solution of the final equations generally accomplished. My own longitude work has been pretty extensive, and done for geographical purposes, frequently with very bad instruments, over a very wide extent of country. Most of it has been published in official documents; the only published series of mine made with a thoroughly good instrument is given in the 'Report of the Chief of Engineers of the United States Army for 1879 and 1880'; the observations were taken at Ogden in July to October 1878. The method of reduction was quite elaborate; I first made a preliminary solution by the method given above, and afterwards determined the corrections Δu , Δa , Δc to the preliminary values of clock-correction, azimuth, and collimation; I extract from the work just mentioned the final equations for a set upon which 11 stars were observed before signals were exchanged

$$a_0 = +0^s.40 \quad c_0 = +0^s.20 \quad u_0 = -2^m.22^s.46$$

$$o = +0^s.099 + 8.38 \Delta u + 0.85 \Delta a + 0.61 \Delta c$$

$$o = +0.273 + 0.85 \Delta u + 3.94 \Delta a - 2.20 \Delta c$$

$$o = +0.430 + 0.61 \Delta u - 2.20 \Delta a + 19.75 \Delta c$$

$$\Delta u = -0^s.001 \quad u = -2^m.22^s.461 \quad \text{weight} = 8.13$$

$$\Delta a = -0.087 \quad a = +0.313 \quad \text{weight} = 3.59$$

$$\Delta c = -0.032 \quad c = +0.168 \quad \text{weight} = 18.38$$

The weights were taken equal to

$$p = \frac{1.3}{1 + 0.3 \sec^2 \delta}$$

and the probable error for weight = 1, an eye-and-ear observation at the equator, not far from $\pm 0^s.06$. The average Δu was about $0^s.012$; so that not very much was gained by the least square solution.

So large a part of the computation was done in blank forms — some of which were ruled out by a soldier of the U. S. Army who was detailed to help me — that it was very easy to accomplish.

Tables of A , B , C ; A^2p , ACp , C^2p , were computed for every degree of declination, and interpolated as I needed them; so that there was no difficulty whatever in keeping the reductions up to date with a ratio of one successful exchange of signals to every three days. The weather in Utah was such that 90 per cent of the evenings allowed observation; but the telegraphic communications, over 1000-3000 km. of line, were very precarious.

The use of least squares in telegraphic longitudes has become so much a matter of official routine in this country that many hundreds of sets of observations were so reduced before any attempt was made to discriminate between the weight for equatorial and polar stars.

Williamstown, Mass., 1884 June 23.

T. H. Safford.