XXXV. The change of the order of absolute viscosity encountered on passing from fluid to solid

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The sharp doublet ($m=2$) is nearly coincident with the two lines which hitherto have been measured. The wave-lengths of these lines are, viz :—

After Lecoq de Boisbaudran.	After Deville and Mermet.
$\left. \begin{array}{l} 4170 \\ 4031 \end{array} \right\} \nu = 826.9$	$\left. \begin{array}{l} 4171 \\ 4033 \end{array} \right\} \nu = 820.3$

The accordance is even better than could well be expected.

Finally, I will remark that the hypotheses of Mr. Lockyer on dissociation of the elements are quite incompatible with the results of my researches. The observations of Lockyer within the spectra of Na and K prove only that, with luminous atoms as with sounding bodies, the relative intensity of the partial tones may vary under different circumstances. For the lines in question belong, without doubt, to the same system of vibrations.

Lund, February 1890.

XXXV. *The Change of the Order of Absolute Viscosity encountered on passing from Fluid to Solid.* By CARL BARUS.

1. **T**O my knowledge nobody has thus far defined the difference between the solid state and the fluid states quantitatively. In the case of liquids and gases viscosity can be absolutely expressed with facility, and the data have therefore to be stated with considerable rigour. This is not true for solids, where the results are relative throughout.

The present paper submits two methods for the coordination of the viscous behaviour of solids and of liquids. In describing the differential method applied by Dr. Strouhal and myself†, we incidentally pointed out the way in which the relative viscosity of two solids may be stated in terms of the respective sectional areas, by which the motion at the junction of the two counter-twisted wires or rods is annulled. Suppose that one of the wires is a solid, whereas the other is a very viscous liquid. Then if the viscosity of the latter can just be measured by transpiration-methods (§§ 4, 5), it follows that the viscosity of the true solid with which it was counter-twisted may also be absolutely expressed. This indicates the first method of the present paper (§§ 8, 9).

* Communicated by the Author.

† Barus and Strouhal, *American Journal*, xxxiii. p. 29 (1887).

My second method for measuring the viscosity of solids absolutely is direct. It is capable of yielding results of any desirable accuracy, supposing the necessary facilities for an adjustment free from vibratory disturbances are given (§§ 12, 13).

Gases and Vapours.

2. Results for the viscosity of gases and vapours are given by many observers*. The data present some curious aspects; thus, for instance, the viscosity of vapours† is frequently smaller than the viscosity of gases. The order of viscosity (η) is about 10^{-4} g/cs (cf. § 18.). Results for the viscosity of substances in the neighbourhood of Andrews's critical temperature are not available. Such an investigation would be highly desirable (§ 18).

Liquids.

3. The literature contains extensive researches and is systematized in Landolt and Boernstein's Physical Tables‡. The data are easily expressed absolutely, by aid of Slotte's§ values of the viscosity of water between 0° and 100° . If water at 20° be taken as a type liquid, $\eta = \cdot 01$ (g/cs) (cf. § 18).

Viscous Liquids.

4. Data are wanting.

My experiments were made with marine glue||. The method is simple. A fairly wide capillary tube, abc , provided with a cylindrical open reservoir, a , at one end, was half filled with marine glue, the thread extending from the reservoir, a , as far down as the middle, b , of the tube. The other (open) end of the tube was inserted into the flask, $dddd$, from which the air was then exhausted. Thus a pressure of about 1 atm. is continually brought to bear on the reservoir a of the tube, the other end, c , being *in vacuo*. A lateral arm, e , closed by fusion, facilitates the exhaustion. A small vacuum-

* Summarized in O. E. Meyer's *Die kinetische Theorie der Gase*, Breslau, 1877, pp. 138 *et seq.*

† The most complete set of data are those of Puluj, Phil. Mag. [5] vi. p. 157 (1878). Lothar Meyer, E. Wiedemann, and others have contributed to the subject.

‡ *Physikalisch-chemische Tabellen*: Berlin, J. Springer, 1883, p. 153.

§ Slotte, Wied. Ann. xx. p. 267 (1888). References to other researches (O. E. Meyer, Poiseuille, Rosencranz) are there given.

|| A valuable cement being (nominally) a specially prepared mixture of rubber and shellac. It has a pitchy consistency, and can be thickened by adding shellac.

gauge, *g g*, held in place by a layer of paraffin, *h*, shows the observer to what degree the vacuum is maintained in the lapse of time. A number of tubes, *abc*, may be inserted side by side, and it is always necessary to coat the stopper with cement.

5. The transpiration-equation due to Poiseuille, theoretically corrected by Hagenbach*, has the form

$$\eta = \frac{\pi P r^4}{8 v l} t - \frac{m}{2^{10/3} \pi l} \frac{1}{t}, \quad (1)$$

where η is the absolute viscosity of the liquid, v the volume, and m the mass transpiring through the capillary length l and radius r , in the time t . In the present case of excessively slow transpiration the correction may be neglected. P is the pressure-excess, in dynes per square centimetre. All magnitudes are to be expressed in C.G.S. units.

In the above form of apparatus (fig. 1) l is not constant, and v is measured in terms of the increase of length of the capillary thread *ab* (fig. 1). Hence the differential equation corresponding to (1), since $dv = \pi r^2 dl$, is

$$\eta = \frac{P r^2}{8 l} \frac{dt}{dl} \quad \dots \dots \dots (2)$$

Integrating between 0 and t , and between l_1 and l_2 ,

$$\eta = P r^2 t / 4 (l_2^2 - l_1^2). \quad \dots \dots \dots (3)$$

Here l_1 is the original length ($t=0$), and l_2 the final length

* Pogg. *Ann.* cix. p. 358 (1860). This reference is incidental, or I should have to refer to Poisson, Navier, Stokes, Stefan, Helmholtz, and others.

Fig. 1.—Apparatus for the Secular Transpiration of Viscous Liquids.

of the thread of marine glue. The effect of the cylinder of viscous fluid in the reservoir *a* may be neglected from its large radius.

5*a*. By way of digression I may add another similar case; viz. that of a viscous liquid like glycerine, transpiring in a vertical capillary tube, in virtue of the weight of its own column only. In this case $P = (L + l)\delta g$, δ being the density of the liquid, g the acceleration of gravity, L the initial head of the column. Inserting these quantities and integrating, equation (2) becomes

$$\eta = \frac{\delta g r^2}{8 \left(l_2 - l_1 + L \ln \frac{L + l_1}{L + l_2} \right)} t.$$

An example of results obtained in this way is given in the following Table. The observations, though made upon a single descending thread at the end of each half minute, are grouped in two batches. The tube was dry; perhaps a moist tube (adhering thin film of glycerine) would have been preferable.

EXAMPLE.—Viscosity of Glycerine.

$r = .0505$ centim.; $L = 1.35$ centim.; $\delta = 1.26$; $\Delta t = 120$ sec.

<i>t</i> .	<i>l</i> .	η .	<i>t</i> .	<i>l</i> .	η .
sec.	cm.	<i>g/cs.</i>	sec.	cm.	<i>g/cs.</i>
30	10.07	5.05	270	29.82	5.53
150	20.30		390	38.60	
60	12.72		300	31.95	
180	22.75	5.07	420	40.83	5.46
90	15.30		330	34.20	
210	25.10		450	43.04	
120	17.80	5.11	360	36.40	5.44
240	27.43		480	45.22	

To account for the difference of η in the two sets of results is beyond the present purpose. It is noteworthy* that each measurement of η occupies but two minutes, even admitting that the tube is unfavourably wide; for in the case of viscous liquids the temperature-effect is of great importance, and this expeditious method therefore has some advantages. The experiment may be varied by observing the descending upper

* Cf. Graham, Phil. Trans. cli. p. 382 (1861):—"The liquid (glycerine) is too viscous to be transpired by means of the bulb and capillary employed in these experiments." I am not aware that the viscosity has been measured since.

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meniscus, while the lower end of the capillary is submerged in glycerine.

6. By applying equation (3) I obtained the η expressed in the following Table:—

TABLE I.—Transpiration of Marine Glue.

$P = 10^6$ dynes ; $r = .0406$ centim. ; temperature (say) = 25° .

Date.	t .	l .	$\eta \times 10^{-6}$.
	sec.	cm.	c/gs.
May 7, 1889. 12 ^h	0	3.7	
Dec. 7, 1889, 12 ^h	18.5×10^8	7.2	200

This is a striking result, remembering that a sphere of the cement, if placed on a plane surface, will run out to a flat cake in a few months. Nevertheless this type of viscous fluid is 20 billion times as viscous as water at the same temperature.

After long continued transpiration the thread of marine glue shows two strata. The advance portion is amber-coloured and less viscous, the rear portion brown, clearly containing most of the shellac. Hence the observed η is probably somewhat low. (*Cf.* § 10.)

7. Paraffine (melting-point 55°) treated in the same way showed no transpiration. The tube ($2\rho = .082$ centim.) was nearly of the same bore as above, and the length of thread in the most favourable case 2.2 centim. The motion, if any, must have been below .01 centim. Hence the viscosity of the paraffine must be greater than 2×10^{11} g/cs at about 25° . It is nearly impossible to insert a capillary paraffine thread in a tube, free from the vacuum-bubbles due to contraction on cooling.

Solids.

8. Having obtained the result in § 6, I proceeded to use it for obtaining the viscosity of hard steel by the method of comparison indicated in § 1. The apparatus was essentially like that of my earlier experiments*. To secure additional safety† I introduced the principle of substitution, whereby the steel wire and the rod of marine glue were consecutively subjected to known torsion-couples, in ways otherwise identical. From the enormous viscosity of steel, as compared

* From the context it appears that the method of substitution is probably necessary.

† American Journal, xxxiv. p. 2 (1887).

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even with marine glue, a beam of this substance, say a decimetre thick, would show about the same deformation initially as a steel wire only a millimetre thick, *cæt. par.* So large a mass of the cement is inconvenient to handle. I therefore used a rod about 4 centim. thick and suitably apportioned the torsion-couples in the two cases. The results are given in Tables II. and III., where 2ρ is the diameter, l the length, of the similarly twisted rods. This twist τ is given in the first column, and is proportional to the applied couples. N denotes the viscous motion at the index, at the time specified, and is given in scale-parts (distance between mirror and scale 200 centim.). Reduction of N is not necessary, since the present purposes are comparative. The steel wire is twisted alternately in opposite directions to allow for the accommodation. In Table II. this precaution is superfluous, the substance being a fluid. The three sets of data are merely given to show the behaviour more fully. Other experiments made with the cement may be omitted here.

TABLE II.—Viscous Deformation of Hard Steel. Wire No. 1.

$\rho = 0.40$ centim. ; $l = 30$ centim.

τ .	Time.	$N \times 10^2$.	τ .	Time.	$N \times 10^2$.	τ .	Time.	$N \times 10^2$.
$+5.7$	minutes.	cm.	-5.7	minutes.	cm.	$+5.8$	minutes.	cm.
	0	Twisted.		0	Twisted.		0	Twisted.
	2	0		2	— 0		2	0
	3	70		3	— 71		4	40
	4	115		4	— 125		3	72
	6	187		6	— 200		6	122
	8	235		8	— 258		8	160
	10	278		10	— 301		10	190
	12	315		12	— 340		12	212

TABLE III.—Viscous Deformation of Marine Glue.

$\rho = 2$ centim. ; $l = 30$ centim.

τ .	Time.	$N \times 10^2$.	τ .	Time.	$N \times 10^2$.	τ .	Time.	$N \times 10^2$.
$*1.4$	minutes.	cm.	1.4	minutes.	cm.	1.4	minutes.	cm.
	0	0		0	0		0	0
	2	260		2	250		2	220
	3	480		3	450		3	420
	4	710		4	700		4	610
	5	910		5	900		5	810
							6	1010

* This refers to the applied couple. The initial twist of the bar of marine glue is practically zero.

9. From these Tables the mean rate of motion of marine glue appears as 2 centim. per minute, whereas the corresponding mean motion for hard steel is (say*) .4 centim. per minute, for the three cases. The couple acting on the cement is $1.4x$, while the couple in the other case is $5.7x$. Since the cement is a fluid, the rate of motion for identical couples would be 8 centim.

The principle given in § 1 may now be applied, whereby viscosities are inversely as the fourth power of the radii, *cæt. par.* Let η' and η be the viscosities of marine glue and hard steel respectively. Then

$$\eta = \eta' \left(\frac{2.00}{.04} \right)^4 \frac{8.0}{.4} = \eta' \times 10^9 / 8. \quad . \quad . \quad . \quad (1)$$

From Table I., $\eta' = 200 \times 10^6$; hence the mean viscosity of originally untwisted glass-hard steel, during three consecutive alternate twists of 12 minutes each, just within the limits of rupture is at least

$$\eta = 25 \times 10^{15} \text{ g/sc.} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

10. The present results subserve their chief purpose in furnishing an estimate of the order of viscosity of glass-hard steel; for the unavoidable errors encountered in a comparative method like the present are not insignificant. In the first place the thread of marine glue observed for transpiration in § 6 during half a year is not identical with the rod of this material which is twisted (§ 8), neither as to temperature nor composition (*cf.* remarks end of § 6). Possibly better results might be obtained with old pitch. Again, the rod of marine glue, while it is being twisted, is also being appreciably stretched by its own weight. This complicates the angular measurement. Finally, the sectional error of an opaque rod, not necessarily free from air-bubbles, is considerable. Equation (3) must therefore be looked on as assigning an order to the viscosity of hard steel above 10^{16} g/cs.

11. Before giving the results of the following direct method, it is expedient to insert a paragraph on the viscosity of solids generally. In § 8, while the steel wire is strained by the couple almost as far as the limits of elasticity permit, the rod of marine glue is initially scarcely strained at all. In the latter case, however, the unstable configurations essential to viscous motion are supplied in relatively great numbers by the ever changing distribution of the heat-agitation within the body. In a homogeneous solid free from strain (soft steel)

* The viscosity of solids being a function of time, strain, and temperature. This distinction is beyond the purposes of the above paragraph, but will be fully carried out below, in §§ 11 *et seq.*

such configurations practically vanish as to number. I doubt whether in soft steel the viscous deformation due to instantaneous exceptionally intense molecular motion at different points within the solid, could be recognized at ordinary temperatures. The case is different when the solid is under stress. Any twist, no matter how small (?), is accompanied by a proportionate amount of permanent strain, with which a corresponding amount of instability is necessarily associated. Hence it must be borne in mind that the viscosities expressed absolutely below refer to steel in a condition of strain just within the elastic limits.

I can here merely allude to the very recent and suggestive paper of C. H. Carus-Wilson (*Nature*, xli. p. 213, 1890), in which the behaviour of steel near the elastic limits is interpreted in analogy with the well-known circumflexed isothermals of condensable gases, due to James Thomson.

Now it is clear that if stressed steel be left to itself, the number of unstable configurations becomes rapidly less as the time after twisting increases. Energy is dissipated. The viscosity of solids is therefore essentially a time-function as well as a strain-function.

12. The apparatus with which the following absolute results

Fig. 2.—Apparatus for measuring the Absolute Viscosity of Solids.

were obtained is shown in fig. 2. AA and BB are two massive torsion-circles, the hollow cylindrical shafts, *oo*, of

which are firmly clamped to supports projecting from the wall of the room. The steel wire, ww , to be tested is tensely drawn between the cylindrical cores, al , $a'l'$, and clamped without torsion. To accomplish this the ends of the wire are bent hook-shaped, and inserted in flat fissures at the ends of the rods al , $a'l'$, the clamp-screw (shown at a') passing through the hooks of the wire. The middle part of ww carries a pair of plates, bc , held together by four screws in such a way as to clamp the wire firmly between the plates. A brass rod of screw-wire, dd , is soldered to the upper plate, and when in adjustment is horizontal and at right angles to the axis of ww . Thus the rod dd is virtually the beam of a balance, and two small nuts, mm' , symmetrically placed near the ends of dd , are provided with hooks from which a scale-pan, hh , may be suspended. The latter carries a light horizontal disk, vv , to be submerged in water (not shown), with the object of deadening vibrations when the apparatus is in adjustment. The upper plate bc also carries an adjustable mirror, N , for angular measurement, as well as an index, fg , moving across a stationary graduated dial (not shown) by which the amount of twist holding the weight, P , in equilibrium is roughly registered.

In adjusting the apparatus, the scale-pan hh is removed and the rods al so placed that the lever dd is vertical. After clamping al (care being taken to avoid twisting ww) the scale-pan is attached at m' , and additional weights P added to hold the lever dd horizontal. Hence the two halves of the wire ww are twisted 90° each for the whole length. Other twists may be applied by rotating the torsion-circles together (it would be necessary to connect them rigidly for this purpose) and adding suitable weights P .

To fasten the wire securely between the plates the lower one is clamped horizontally in a vice. If there were rotational sliding possible here it would be detected in examining very viscous wires. Some difficulty is experienced in fixing ww in place quite free from initial torsion. There is also some flexure; but for wires of the radius given these discrepancies may be disregarded. When AA and BB are rigidly connected, the twist of the wire ww may be expressed in terms of the load P .

13. To complete the results obtained with this apparatus, let 2ϕ be the amount of angular viscous motion*, in radians, of any right section of the wire relative to another right section, whose distance from the first is the unit of length. Suppose, furthermore, that in the plane of a right section

* The factor 2 is supplied conformably with my earlier notation.

there is no shearing. Then $2d\phi/dt=2C_t$ is the common angular velocity, per unit of length, with which the shear is increased. Consider an elementary cylindrical shell whose length l is the effective length of either half of the wire ww (fig. 2), and whose right section is $2\pi r dr$. Then the amount of force distributed uniformly over $2\pi r dr$ will be

$$df = \eta_t \frac{2\pi r dr}{l} (l2C_t r). \quad (4)$$

Multiplying by r , and integrating between 0 and ρ , the radius of the steel wire,

$$\frac{1}{2} P\lambda = \eta_t \frac{\pi \rho^4}{2} 2C_t, \quad (5)$$

where $P/2$ is the weight at the end of the lever-arm λ , per effective half of the wire. In any given adjustment $P\lambda/\pi\rho^4 = A = \text{constant}$. Hence

$$\eta_t = A/2C_t. \quad (6)$$

In these equations it has been assumed that $\int \eta r^3 dr = \eta_t \int r^3 dr$. Hence under all circumstances η_t is a mean value defined by these integrals.

14. Results obtained with the apparatus § 12, are given in Tables IV. to VII. Here ρ is the radius, l the length of each effective half of the steel wire. P is the total force (being the weight plus the weight of the scale-pan) acting at the end of the lever λ (see fig. 2). The rate of twist is given under τ , and 2Φ is the total yield during the interval of experiment. N denotes the actual scale-reading observed by the telescope (scale-distance from mirror, $R = 200$ centim.) at the time given; and 2ϕ is the viscous angular motion between two right sections of the steel wire 1 centim apart.

In the second half of the tables, $2C_t$ is the time-rate of change of 2ϕ , computed for consecutive intervals of 500 seconds each. The final column contains the corresponding absolute viscosity. All times are reckoned from the moment of twisting. To obtain $2C_t$ I availed myself of graphic methods*, these being in conformity with the mean accuracy of the work. Table IV. contains results for the wire No. 1, already tested in Table I. Three twists are applied, alternately in opposite directions. In Table V. the same wire is similarly examined, after it has been softened by heating to

* Of course a set of smoother values might be obtained by computing the constants of the curves ϕ from Kohlrausch's exponential formula, and the tangents from the values obtained.

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redness and cooling in air. Table VI. contains data for a new glass-hard steel wire, originally free from stored twist. Table VII., finally, contains the viscous behaviour of a wire tempered at 450°, which was used as a normal in my former differential experiments.

TABLE IV.—Absolute Viscosity of a Glass-hard Steel Wire.
No. 1. Fourth Twist.

$l=12.9$ centim.; $\rho=.0405$ centim.; $P=82 \times 981$ dynes;
 $\lambda=9.43$ centim.

$\frac{\tau}{2\phi}$.	Time.	$N \times 10^2$.	$2\phi \times 10^6$.	Time.	$2C_t \times 10^9$.	$\eta_t \times 10^{-15}$.
Radians.	Sec.	Centim.	Radians per centim.	Sec.	Radians per centim. per sec.	$g/cs.$
.12	0	Twisted.	0	0		
	420	0	*0	500	735	122
	720	104	202	1000	462	194
	1020	180	349	1500	310	291
	1860	320	621	2000	222	405
	2220	360	698	2500	183	491
	2820	417	808	3000	150	600
	3480	465	901			
No. 1. Glass-hard. Fifth Twist.						
-.12	0	Twisted.	0	0		
.009	480	*0	*0	500	*910	99
	840	120	233	1000	474	190
	1320	235	456	1500	347	259
	1860	329	638	2000	270	333
	2400	402	779	2500	230	391
	2940	464	900	3000	210	428
	3480	520	1008			
No. 1. Glass-hard. Sixth Twist.						
.12	0	Twisted.	0	0		
-.009	360	0	0	500	467	193
	960	120	233	1000	265	339
	1260	160	310	1500	193	466
	1800	210	407	2000	156	577
	2340	250	485	2500	138	652
	3120	306	593	3000	126	715
	3900	353	684			

* The minus signs, being without interest in this and the following similar series, are omitted for brevity.

TABLE V.—Absolute Viscosity of Soft Steel, being Wire No. 1, heated to redness and cooled in air. First Twist.

$l=13.0$ centim.; $\rho=.0405$ centim.; $P=82 \times 981$ dynes;
 $\lambda=9.43$ centim.

$\frac{\tau}{2\phi}$	Time.	$N \times 10^2$	$2\phi \times 10^6$	Time.	$2C_t \times 10^9$	$\eta_t \times 10^{-15}$
Radians.	Sec.	Centim.	Radians per centim.	Sec.	Radians per centim. per sec.	$g/cs.$
.12	0	Twisted.	0	0		
-.007	420	0	0	500	220	409
	900	40	77	1000	89	1010
	1500	63	121	1500	63	1430
	2100	80	154	2000	47	1920
	2700	92	177	2500	34	2650
	3380	98	188	3000	26	3460
	3780	103	198			
No. 1. Softened. Second Twist.						
.12	0	Twisted.	0	0		
-.010	360	0	0	500	164	550
	1020	50	96	1000	116	776
	1500	70	135	1500	59	1520
	1983	82	158	2000	40	2250
	2520	93	179	2500	30	3000
	3120	100	192	3000	15	6000
No. 1. Softened. Third Twist.						
.12	0	Twisted.	0	0		
-.007	240	0	0	200	138	652
	840	42	81	1000	70	1290
	1560	64	123	1500	40	2250
	2100	74	142	2000	28	3210
	2520	76	146	2500	17	5300
	2640	78	150	3000	14	6400
	3120	81	156			

TABLE VI.—Absolute Viscosity of Glass-hard Steel. Wire No. 2. First Twist.

$l=13.0$ centim.; $\rho=.0405$ centim.; $P=82 \times 981$ dynes;
 $\lambda=9.43$ centim.

τ . 2 Φ .	Time.	$N \times 10^2$.	$2\phi \times 10^6$.	Time.	$2C^2 \times 10^9$.	$\eta_t \times 10^{-1}$
Radians.	Sec.	Centim.	Radians per centim.	Sec.	Radians per centim. per sec.	$g/cs.$
.13	0	Twisted.	0	0		
-.009	120	0	0	500	660	136
	420	186	358	1000	325	277
	720	270	520	1500	230	391
	1090	335	644	2000	180	500
	1620	410	789	2500	140	643
	2100	458	881			
	2580	495	952			
	3000	518	996			
No. 2. Glass-hard. Second Twist.						
-.13	0	Twisted.	0	0		
.010	240	0	0	500	683	131
	600	144	277	1000	392	229
	900	220	423	1500	285	316
	1260	288	554	2000	240	375
	1800	365	702	2500	205	439
	2100	404	777			
	2460	450	865			
	2700	470	904			
	3000	502	965			
No. 2. Glass-hard. Third Twist.						
	0	Twisted.				
	240	0	0			
	600	117	225			
Wire breaks spontaneously.						

TABLE VII.—Absolute Viscosity of Steel, annealed at 450°. Wire No. 15. Twisted indefinitely.

$l=13.6$ centim. ; $\rho=.0405$ centim. ; $P=82 \times 981$ dynes ; $\lambda=9.43$ centim.

$\frac{\tau}{2\phi}$.	Time.	$N \times 10^2$.	$2\phi \times 10^6$.	Time.	$2C_t \times 10^9$.	$\eta_t \times 10^{-15}$
Radians.	Sec.	Centim.	Radians per centim.	Sec.	Radians per centim. per sec.	$g/cs.$
$\cdot 12$ $-.002$	0 240 720 1200 1860 2520 2000	Twisted. 0 51 71 86 95 101	0 94 131 158 175 186	0 500 1000 1500 2000 2500 3000	177 69 40 32 25 20	508 1900 2250 2810 3600 4500
No. 15. Tempered at 450°. Next Twist.						
$-.12$ $-.003$	0 240 600 1200 1740 2580 3240	Twisted. 0 26 43 54 68 81	0 48 80 100 125 149	0 500 1000 1500 2000 2500 3000	111 46 40 34 31 30	811 1950 2250 2650 2910 3000
No. 15. Tempered at 450°. Next Twist.						
$-.12$ $-.003$	0 240 660 1260 1740 2520 3120	Twisted. 0 48 78 92 112 126	0 88 143 169 206 232	0 500 1000 1500 2000 2500 3000	180 82 57 48 45 42	500 1100 1570 1870 2000 2140

Some irregularities are apparent, largely introduced by vibrations against which our laboratory facilities afford no adequate protection. Vibrations increase the rate of twist

periodically, and thus supply a greater number of instabilities than correspond to the mean twist (position of equilibrium) in question. They may also have a more direct molecular effect in decreasing viscosity. When the rate of twist in the two wires is unequal, viscous motion will also be in excess of the true value. This also applies in case of slightly different lengths, l , of the two effective halves of the wire. In the case of No. 15, which had become somewhat worn by use, the load P probably strained the wire too near the elastic limits. This I infer from the observed successive reduction of viscosities from twist to twist, clearly indicating that an excess of instability is being supplied. It is therefore better to accept the behaviour of No. 2 (soft) as typical of No. 15; for the normal viscosities of the two states of temper are identical.

Discussion.

15. Turning first to the values of ϕ , it is seen at once that they substantiate the inferences of my earlier papers throughout*. They therefore need no further comment here. Tables V., VII. contain data for my normal wire, by aid of which the differential results of the earlier papers may be reduced to absolute values. I have endeavoured to make the angles τ of nearly the same value in all cases, so that ϕ/τ may be comparable.

16. The spontaneous breaking of the glass-hard wire in Table VI. deserves mention. It occurred during the third twist, and at a time when the wire was in no way interfered with. This corresponds to the spontaneous explosion of hard projectiles frequently observed. It also corresponds to the spontaneous rupture of stressed glass, whether the stress be stored internally or applied externally. I have observed this interesting phenomenon in glass under a great variety of

* Phil. Mag. xxvi. p. 183 (1888); *ibid.* xxvii. p. 155 (1889).

In the deductions of the former paper (p. 189) I neglected the elastic motion, in virtue of which, at every stage of viscous yielding, the rate of twist is maintained constant throughout the system. After correcting for this,

$$L\psi = 2l' \{ \phi_3(\beta - l') + \phi(a - \beta) + \phi_1(L - a) \} - 2l\phi'l';$$

which under the simplified conditions of experiment becomes $\psi = l(\phi - \phi')$. Hence it follows that the data $(\phi - \phi')$ of the two papers cited are relative, and must be multiplied by $\ln 2$ to reduce them to the dimensions defined in the text. Beyond this the error is without importance, the data applying at once to rods initially somewhat harder than those actually observed.

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conditions. In all cases there is gradual molecular change, and the solid finally breaks without any apparent cause.

17. The chief results of Tables IV. to VII. are the values η , whether they be regarded as time-functions, or as exponents of the *solidity* of steel. In fig. 3 these data are constructed graphically. An inspection of the figure shows at once that both solid viscosity and its rate of increase with time are magnitudes which increase together (Tables IV., VI. compared with Tables V., VII.). After about an hour has elapsed since twisting, the viscosity of hard steel may be quite as large as the initial viscosity of soft steel; but at corresponding times the viscosity of soft steel continues to be more and more pronouncedly the greater.

The curves are not consistent as to curvature. This may be best inferred from the hard wires, where the viscosities are smaller, the deformations large, and the observations therefore as a whole much more accurate (Tables IV., VI.). Hence I regard the promiscuous irregularities of fig. 3 as due to errors of observation and construction. According to the law of F. Kohlrausch*, $-d\phi/dt = a\phi/t^n$, where a and n are constants. In case of small torsions or short times, $-d\phi/dt = a\phi/t$ suffices. Thus for the purpose of deducing the time-rates, κ , of increase of solid viscosity, it is permissible to regard the curves of fig. 3 as essentially linear.

This is done in the following table.

TABLE VIII.—Mean time-rate of increase of the absolute Viscosity of Steel, hard and soft.

Hard Steel.			Soft Steel.		
	Twist No.	$\kappa \times 10^{-12}$.		Twist No.	$\kappa \times 10^{-12}$
Table IV....	4	g/cs^2 .	Table V...	1	g/cs^2 .
	220	220		1200	1200
	—5	140		1400	1400
Table VI...	6	240	Table VII.	3	1900
	1	230		1	1500
	—2	140		—2	1200
				3	1000
Mean rate κ		194×10^{12}	Mean rate κ		1400×10^{12}
Mean viscosity at 500 sec.		140×10^{15}	Mean viscosity at 500 sec.		570×10^{15}

* Kohlrausch, Pogg. Ann. cxxviii. p. 216 (1866).

Fig. 3.—The Absolute Viscosity of Hard Steel (Tables IV., VI.) and of Soft Steel (Tables V., VII.), considered in its variation with the time elapsing after twisting.

The general explanation of these results has been indicated in § 11. In a liquid or a viscous fluid under moderate stress the instabilities are supplied by the mere thermal agitation at ordinary temperatures, at the same rate in which they are used in promoting viscous motion. Hence viscosity is constant at a given temperature. In a solid under stress the instabilities are expended at a rate decidedly greater than the small rate of continuous supply. Thus viscosity decidedly increases with time. Suppose, therefore, a solid of initially greater viscosity starts on its viscous deformation at an instant when the viscosity of a solid of initially smaller viscosity has been increased by time to the initial value of the former case. This can actually be realized by twisting a soft steel rod about an hour later than a hard steel rod (fig. 3). Then, in succeeding times, even though the substances start at a state where both are equally viscous, the viscosity of the initially more solid body will rapidly overtake the other; for the supply of instability due to temperature alone is continuously greater in one case than in the other.

18. Summarizing the results of the above paragraphs, the viscosities of the three states of aggregation may be expressed in terms of the absolute g/cs scale as follows:—

TABLE IX.—Viscosity, considered in its Variations with the State of Aggregation.

Gases and Vapours.		Andrews's critical state.		‡ Liquids.	
Substance.	η .	Sub- stance.	η .	Substance.	η .
* Ether, 0°	6.8×10^{-5}	?	?	Ether, 30° ...	9×10^{-4}
* H ₂ , 0	8.7×10^{-5}			Ether, 10 ...	1.9×10^{-3}
* Air, 0	1.75×10^{-4}			Water, 97 ...	3.0×10^{-3}
† O ₂ , 0	2.12×10^{-4}			Water, 20 ...	1.0×10^{-2}
				§ Glycerine ...	5
Range	10^{-5} to ?	?	to ?	Range	? to 10^{-2}

* Puluj, *l. c.*

† O. E. Meyer, *l. c.* p. 142.

‡ Landolt and Boernstein's Tables, *l. c.*, and Slotte, *l. c.*

§ A rough measurement of my own, § 5a. Graham's results, *Phil. Trans* 1861, p. 373, refer to dilute glycerine.

Table IX. (*continued*).

Viscous Fluids.		Solids.	
Substance.	η .	Substance.	η .
Marine glue	2×10^3	Paraffine at 20° (m.p. 55°).	$> 2 \times 10^{11}$
		† Hard steel, glass &c.	$\begin{cases} 10^{17} \text{ to} \\ 6 \times 10^{17} \end{cases}$
		† Soft steel	$\begin{cases} 6 \times 10^{17} \text{ to} \\ 6 \times 10^{18} \end{cases}$
Range	$10^3 \text{ to } 10^{11}$	Range	$10^{11} \text{ to } 10^{20}$

† During the first hour (500 to 3000 seconds) after twisting just within the elastic limits,

The limits here defined are somewhat arbitrary. They will be made more definite when a greater number of substances lying on the boundary between the classes have been examined. Information is lacking and particularly desirable in the neighbourhood of Andrews's critical temperature. From Table IX. it does not seem improbable that the critical temperature may be definable by a narrow limit of viscosity, quite apart from the substance operated on. What this limit may be I do not venture to assert, seeing that the viscosity of gases decreases on cooling, whereas that of liquids increases on cooling.

Table IX. gives the positively astounding range of variation of η , the chief variable of our material environment—a variable which throughout the whole enormous interval in question nowhere fails to appeal to our senses.

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XXXVI. *On the Effect of Permanent Elongation on the Cross Section of Hard-drawn Wires.* By Professors T. GRAY and C. L. MEES*.

IN the course of a series of experiments on the torsional rigidity of metals, forming part of the course of instruction in elasticity in the mechanical laboratory of the Rose Polytechnic Institute, we were somewhat surprised to find a

* Read before the Physical Society of Glasgow University, February 23, 1890. Communicated by Th. Shields, M.A., Secretary.