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LX. On the nebular hypothesis.—II. Interaction

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all star-masses at a stage of their development exhibit a phase of volcanic activity. And it is possible that during this period many of those masses which are of comparatively trifling magnitude may have been partially or wholly broken up and resolved into small fragments.

LX. *On the Nebular Hypothesis*.—II. *Interaction*. By PLINY EARLE CHASE, *Professor of Philosophy in Haverford College*.

[Continued from p. 319.]

THE tendency of vibrations in elastic media to produce harmonic vibrations, combined with the tendency to geometrical and harmonic as well as to arithmetical progressions, should prepare us to look for relations of various complexity when we study the mutual actions and reactions of planetary masses.

The triangular numbers, 1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, appear as indices in the following equation among powers of the masses of the five principal bodies in the solar system :—

$$\left(\frac{\Psi}{\odot}\right)^1 \times \left(\frac{6}{\odot}\right)^3 \times \left(\frac{4}{\odot}\right)^6 = \left(\frac{1}{2}\right)^{10}.$$

This appears to be the first equation ever discovered which introduces the products of powers of masses in so systematic a form*. Although its full interpretation may be at present beyond our reach, we can catch glimpses of its meaning, and we may feel a reasonable assurance that it represents some important functional law of equilibrating tendencies between centripetal and centrifugal forces. The truth of the equation is, of course, independent of any assumption with regard to the proper unit of comparison; but the dominance of the solar influence lends interest to the æsthetic harmony afforded by its introduction. This interest is increased by the accordance between the order of position and the magnitude of the indices in the left-hand member of the equation, and by the fact that the nebular centre of planetary inertia ($\sqrt{\Sigma mr^2 \div \Sigma mr}$) is in Saturn's orbit.

There is still some uncertainty as to the masses of Neptune and Uranus; so that it is impossible to tell how close this agreement may be; but the deviation from precise accuracy cannot be large. According to Newcomb's latest determinations of

* Laplace, however (*Méc. Céleste* II. vol. viii. pp. 65–69; VI. vol. ii. pp. 12–16, &c.), investigated inequalities depending on squares and products of the disturbing forces. In his discussions of the Jovian and Saturnian systems he introduced terms containing the third and fifth dimensions of eccentricities and inclinations.

those masses*, from Neptune's satellite and from perturbations of Uranus, the equation gives two values for Saturn's mass, one of which is slightly larger, the other slightly smaller than Bessel's value. There are, however, other relations of a similar character which can be measured with great accuracy.

Laplace found that if the mass of each planet be multiplied by the product of the square of the eccentricity and the square root of the mean distance, the sum of all the products will always retain the same magnitude; also that if each of the masses be multiplied by the product of the square of the orbital inclination and the square root of the mean distance, the sum of the products will always remain invariable. Now the square root of the mean distance varies inversely as the velocity of circular revolution at the mean distance, or inversely as the square root of the velocity of nuclear rotation at the same distance. It is therefore probable that the primitive nuclear undulations, to which I have hypothetically attributed both the relative positions and the relative masses of the planetary orbs, may have left their record in many other directions than those which I have already pointed out.

Stockwell has found † the following relations :—

"I. *The mean motion of Jupiter's perihelion is exactly equal to the mean motion of the perihelion of Uranus; and the mean longitudes of those perihelia differ by exactly 180°.*

"II. *The mean motion of Jupiter's node on the invariable plane is exactly equal to that of Saturn; and the mean longitudes of these nodes differ by exactly 180°.*"

If the four great planets were ranged in a line, Jupiter on one side of the sun and the remoter planets on the other, the tidal influences, relatively to the nuclear centre of inertia, would drive Jupiter, Uranus, and Neptune to their respective aphelia. Those positions would accord with Stockwell's two theorems, they would approximate the planetary centre of inertia very closely to Saturn's mean radius vector, and they would make the above equation of the products of triangular powers applicable to vector radii as well as to masses; for the logarithms of mean vector radii and of their designated triangular powers, according to Stockwell ‡, are :—

Neptune, mean aphelion ...	1·481951	(Neptune) ¹	1·481951
Uranus, " " ...	1·301989	(Uranus) ³	3·905967
Jupiter, " " ...	·734588	(Jupiter) ⁶	4·407528
			10)9·795446
Saturn, mean	·979496	Saturn	·979545

* "The Uranian and Neptunian Systems," Washington Obs. for 1873, App. I.

† Smithsonian Contributions, 232, p. xiv.

‡ Ibid. p. 5.

The difference between the actual value of $\log h$ *r. vec.* and the value as found by the equation $\Psi^1 \times \mathfrak{S}^3 \times \mathfrak{U}^6 = h^{10}$, is therefore only .000049, representing a numerical difference of only $\frac{1}{80}$ of 1 per cent.

When the hypothetical nebular condensation had proceeded so far as to show the controlling planetary influence of Jupiter's mass, the mean perihelia of Saturn and Uranus were so fixed as to establish the following relationships of harmonic powers (mean perihelion, mean, and mean aphelion* being designated by the subscript figures 1, 2, 3 respectively).

Stockwell's logarithmic values are:—

Neptune ...	Ψ_3	1.481951 α	$\frac{1}{3}$	1.473327 α'
Uranus ...	$\mathfrak{S}_1$	1.262996 β	$\mathfrak{S}_3$	1.301989 β'
Saturn ...	h_1	.957973 γ	h_2	.979496 γ'
Jupiter ...	$\mathfrak{U}_3$	.734588 δ	$\mathfrak{U}_2$	.716237 δ'

$$\frac{1}{3}(\alpha - \beta) + \frac{1}{2}(\beta' - \gamma) + (\delta - \gamma') = .000085 = \log 1.0002$$

$$\frac{1}{3}(\alpha' - \beta) + \frac{1}{2}(\beta' - \gamma) + (\delta' - \gamma) = .000382 = \log 1.0009$$

These results represent the following equations of distance:—

$$\left(\frac{\Psi_3}{\mathfrak{S}_1}\right)^{\frac{1}{3}} \times \left(\frac{\mathfrak{S}_3}{h_1}\right)^{\frac{1}{2}} \times \left(\frac{\mathfrak{U}_3}{h_2}\right)^{\frac{1}{6}} = 1; \quad \left(\frac{\Psi_2}{\mathfrak{S}_1}\right)^{\frac{1}{3}} \times \left(\frac{\Psi_3}{h_1}\right)^{\frac{1}{2}} \times \left(\frac{\mathfrak{U}_2}{h_1}\right)^{\frac{1}{6}} = 1.$$

The theoretical differ from the actual values by less than $\frac{1}{30}$ of 1 per cent. in the first, and by less than $\frac{1}{11}$ of 1 per cent. in the second equation. The closeness of these agreements may, perhaps, induce a glance at a few that are somewhat more intricate.

If we substitute for the theoretical primitive figurate exponents (1, 3, 6, 10) the present actual vector radii ($a = \Psi_2$; $b = \mathfrak{S}_2$; $c = h_2$; $d = \mathfrak{U}_2$), we may form an equation for Saturn's mean perihelion:—

$$\Psi_2^d \times \mathfrak{S}_2^{b-d} \times \mathfrak{U}_2 = h_1^{a+b}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If a, b, d represent the mean *aphelion* vector radii, the equation represents Saturn's mean distance:—

$$\Psi_3^d \times \mathfrak{S}_3^{b-d} \times \mathfrak{U}_3^a = h_1^{a+b}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If we take powers of the masses instead of powers of the vector radii, equation (2) yields two values for Saturn's mass, according as we use Newcomb's greatest or least value of Neptune's mass—

the greatest ($\frac{1}{19380}$), deduced from Ψ 's satellite, . . . (3)

or the least ($\frac{1}{19700}$), „ „ perturbations of $\mathfrak{S}$. (4)

These four equations give approximations to precise accu-

* Smithsonian Contributions, 232, p. 38.

racy, varying between $\frac{3}{8}$ and $\frac{1}{20}$ of 1 per cent., as will be seen by the following solutions :—

(1)

$$\begin{array}{rcl}
 \log 30\cdot03386^{5\cdot202798} & . & 7\cdot687712 \\
 \text{,, } 19\cdot183581^{19\cdot183581-5\cdot202798} & . & 17\cdot936362 \\
 \text{,, } 5\cdot202798^{30\cdot03386} & . & 21\cdot511361 \\
 & & \hline
 & & 47\cdot135435 \\
 \text{,, } 9\cdot077645^{30\cdot03386+19\cdot183581} & . & 47\cdot148979 \\
 (47\cdot148979-47\cdot135435)\div(30\cdot03386+19\cdot183581) & & \\
 = \log 1\cdot00051. & &
 \end{array}$$

(2)

$$\begin{array}{rcl}
 \log 30\cdot33551^{5\cdot427351} & . & 8\cdot043068 \\
 \text{,, } 20\cdot044183^{20\cdot044183-5\cdot427351} & . & 19\cdot030955 \\
 \text{,, } 5\cdot427351^{30\cdot33551} & . & 22\cdot284102 \\
 & & \hline
 & & 49\cdot358125 \\
 \text{,, } 9\cdot538852^{30\cdot33551+20\cdot044183} & . & 49\cdot346714 \\
 (49\cdot358125-49\cdot346714)\div(30\cdot33551+20\cdot044183) & & \\
 = \log 1\cdot0005. & &
 \end{array}$$

(3), (4)

If $10 = \log$ of sun's mass, the logarithms of the assumed mass are :—

Ψ (Newcomb, from satellite)	. .	5·712646
Ψ („ „ perturbations)	. .	5·705534
δ („)		5·645892
$\frac{1}{2}$ (Bessel)		6·455734
$\frac{1}{4}$ („)		6·979689

Substituting these logarithms in equation (2), we get for logarithm of Saturn's mass, by using

$$\begin{array}{rcl}
 \text{Satellite value of } \log \Psi & . & (3) \quad 6\cdot458198 \\
 \text{Perturbation value of } \log \Psi & . & (4) \quad 6\cdot456439 \\
 6\cdot458198-6\cdot455734 & = & \cdot002464 = \log 1\cdot0057 \\
 6\cdot456439-6\cdot455734 & = & \cdot000705 = \log 1\cdot0016.
 \end{array}$$

These relations are suggestive of the many familiar equations between the *sums* of periodic times. The substitution of *products* for sums, and powers for products, in the distribution of mass and position, seems to indicate the early organizing activity of constant forces, acting with reference to nuclear centres, in elastic media.

Philadelphia, March 30, 1876.