358. Note on Sturm's Theorem in the Theory of Equations

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which will repay the adventurous purchaser. This chapter is well worth careful study by the young teacher. In the treatment of irrational number, Prof. Tannery adopts Dedekind's fundamental idea, in which the irrational number is defined by stating the rational numbers respectively greater and less than it. A few pages on aggregates and their elements, with upper and lower limits, leads to the general elementary treatment of limits and the conditions for convergence and divergence. Considerations of space forbid further comment. We must content ourselves with the expression of our opinion that few students of methodology will rise from even the casual perusal of this volume without the feeling that their time has not been wasted. Of this, however, the names of Profs. Tannery and Darboux will be in themselves an adequate guarantee.

MATHEMATICAL NOTES.

358. [A. 3. d.] *Note on Sturm's theorem in the theory of equations.*

There is considerable likeness between Sturm's method of testing the reality of the roots of an equation, and the method by which the reality of the roots of the discriminant of $S + \lambda(x^2 + y^2 + \dots)$ is proved, S being a quadratic in $x, y, \dots$; in each case a series of functions is formed such that when one member of the series vanishes its two neighbours have opposite signs.

The object of this note is to point out that the two methods are in essence identical, the fact being that Sturm's process enables us to express any polynomial in x as a symmetrical determinant involving x in the diagonal only.

Let f be the polynomial of the n th degree and $f, f_1, f_2, \dots f_m$ the series of Sturm's functions, $q_1, q_2, \dots q_m$ the quotients, so that

$$\begin{aligned} f &= q_1 f_1 - f_2, \\ f_1 &= q_2 f_2 - f_3, \\ &\dots\dots\dots \\ &\dots\dots\dots \\ f_{m-1} &= q_m f_m. \end{aligned}$$

Eliminating $f_1, f_2, \dots f_{m-1}$, we have

$$f = f_m \begin{vmatrix} q_1 & -1 & & & & \\ -1 & q_2 & -1 & & & \\ & -1 & q_3 & -1 & & \\ & & \dots\dots\dots & & & \\ & & \dots\dots\dots & & & \\ & & & -1 & q_{m-1} & -1 \\ & & & & -1 & q_m \end{vmatrix}$$

In general $m=n$, $q_1, q_2, \dots$ are linear in x and f_m is a constant, so that f is expressed as a symmetrical determinant, involving x linearly in the diagonal only.

The successive principal minors $A_{11}, A_{1122}, \dots$ are here $f_1, f_2, f_3, \dots$, and the identity

$$\Delta_x A_{1122} = A_{11} A_{22} - A_{12}^2$$

of the symmetrical determinant theory becomes

$$f \times f_2 = f_1(q_1 f_2) - f_2^2.$$

The roots of f are all real if the coefficients of x in $q_1, q_2, \dots$ are all of one sign. In the case of q_1 this sign must be positive, and we have the usual rule that all the roots are real when the leading coefficients in Sturm's functions are all of the same sign.

To reduce f to a symmetrical determinant in this way we may take any $(n-1)^{\text{th}}$ for f_1 , but for the purpose of placing the roots it is essential that the roots of f_1 should separate those of f ; then the roots of each principal minor are separated by those of the next in succession.

One exceptional case is that in which f has one or more repeated roots: f_m is not a constant, but it can be expressed as a symmetrical determinant, and thus f can be so expressed, but zero will occur in the lines next to the diagonal.

The series of Sturm functions may also be shortened through there being a drop of more than one in degree from one function to the next. In this case one of the quotients $q_1, q_2, \dots$ will not be linear; f must have imaginary roots.

A. C. DIXON.

359. [K. 1; L¹, 13. b.] With reference to Mr. W. Gallatly's note on the "Orthopole" in the *Gazette* for May last (Vol. VI., No. 92, p. 89) the following simple proof of its characteristic property (1) may be noticed:

Let Ap, Bq, Cr be $\perp$ rs on a given line L . Draw $q\omega, r\omega \perp$ rs to AC, AB respectively meeting in ω . It is required to shew that $p\omega$ is $\perp$ r to BC . Produce BA, Cr to meet in D , and draw AU parallel to BC to meet DC in U . Then the triangle ACD is similar to the triangle ωgr , corresponding sides being at right angles. Also, since $CU:UD=BA:AD=qp:pr$, therefore U, p are corresponding points on the bases CD, qr . Therefore $p\omega$ is $\perp$ r to AU , or to BC .

Under what circumstances is it possible to describe a parabola having L for its directrix in such a manner that ABC shall be a self-conjugate triangle?

It is a well-known property that the joins of the middle points of such a triangle are tangents; hence the orthocentre of the medial triangle (the circumcentre of ABC) must lie on L , that is to say, L must be a circumdiameter. Since Ap, Bq, Cr are then diameters of the parabola, the polars of the points p, q, r on the directrix L are parallel to the sides. Hence the perpendiculars through p, q, r to the respective sides conintersect in the focus ω , which lies on the circumcircle of the medial triangle, which is the N.P. circle of ABC .

R. F. DAVIS.

360. [I. 18. c.] On p. 158 of the *Gazette* a question of priority is raised as to the discovery of the identities

$$4^4 + 6^4 + 8^4 + 9^4 + 14^4 = 15^4 \quad \text{and} \quad 4^5 + 5^5 + 6^5 + 7^5 + 9^5 + 11^5 = 12^5,$$

the former being due to Mr. David S. Hart, and the latter to Mr. Artemas Martin, the signatory of the note.