are also certain points to which attention must be drawn, in view of future editions of the book.

When dealing with plasmolysis (p. 37), the author omits to state that the animal or vegetable cells used must contain living protoplasm, and the reader is led to infer that artificially-coloured, instead of naturally-coloured cells are employed for plasmolytic observations. Although certain stains are known which are not immediately fatal to living cells, there is no record of their use in plasmolytic experiments. It is also made to appear that red blood corpuscles contain a semi-permeable membrane, despite the conclusive observations of Hamburger to the contrary. In connection with this subject, it is misleading to state (p. 38) that De Vries "established the most important generalisation" that solutions of the same molecular concentration are isotonic, for inasmuch as by far the greater number of his solutions were electrolytic, his results clearly contradict this statement. On p. 34, the credit of preparing semi-permeable membranes is given to Pfeffer, whereas M. Traube first described their preparation and properties. As regards the more general treatment of the first section, it is noteworthy that although the solubility of mixed substances is to some extent discussed, no notice is taken of the work of Roozeboom, and Gibbs' phase-rules, which apply to all cases of heterogeneous equilibrium in solution, are not even mentioned.

In the section on electrolysis, some inkling might have been given of the wide field opened up for the verification of the ionic hypothesis by its application to the operations of analytical chemistry. Among smaller points, it may be noted that, on p. 128, potassium platinichloride should be sodium platinichloride, and in a somewhat vague paragraph, on p. 164, we read that the introduction of oxygen, sulphur, or a halogen, which raises the affinity of a weak acid, "has no effect on the affinity of these strong acids." Since the strong acids quoted by the author are hydrochloric, nitric, &c., the student may be pardoned if he is puzzled to understand how the introduction is to be brought about, or what acids would result if it were possible.

A novel feature in a book of this kind is an attempt made by the author to reconcile the Hydrate Theory with the Newer Theory of solutions. Of course it has all along been apparent that the latter does not preclude combination between solvent and dissolved substance. What the upholders of the newer theory assert, however, is that at the present time there is no definite evidence that, in general, such combination exists. An attempt to reconcile the two views should therefore involve a careful study of the experimental data in favour of combination. It is for this reason unfortunate that the author gives but a very brief statement of the results of the extensive work of Pickering in this field.

As an appendix to the book is given part of the list of the conductivity, migration, and fluidity data of solutions compiled by Fitzpatrick for the British Association Report of 1893. For the sake of chemical readers it is to be regretted that most of Ostwald's observations on the conductivity of organic substances have been omitted, since it is in the case of such substances that the close connection between the electrolytic properties of solutions and the chemical nature of the dissolved substances can be most conveniently traced. J. W. RODGER.

THE THEORY OF ALGEBRAIC FORMS.

An Introduction to the Algebra of Quantics. By E. B. Elliott, M.A., F.R.S. Pp. xiv. + 424. (Oxford: Clarendon Press, 1895.)

THE history of the theory of algebraic forms gives a striking example of the fact that the germ of a mathematical doctrine may remain dormant for a long period, and then suddenly develop in a most surprising way. The principles of the calculus of forms are to be found in the arithmetical works of Lagrange, Gauss, and Eisenstein; but the great expansion of the theory, with which we are now so familiar, practically dates from the publication of the papers of Boole, Cayley, and Sylvester, about fifty years ago.

It is well known that the theory of forms has advanced upon two distinct lines: one method being derived mainly from the differential equation of sources, supplemented by generating functions and the theory of equations; the other, from the symbolical representation of a quantic, invented by Aronhold, and applied with such power by Clebsch and Gordan. Until quite lately, the symbolical method might not unjustly claim to be superior in respect of organic unity, as it must still be admitted to be in compactness and geometrical suggestiveness; but the other method has now undergone a remarkable transformation at the hands of Hammond, MacMahon, Hilbert, and others, and has led to results of the highest interest and value, which the symbolical calculus could not easily or naturally supply.

With the exception of three pages, devoted principally to Cayley's hyperdeterminant notation, Prof. Elliott does not refer to the symbolical method. With his reasons for not using it we must reluctantly acquiesce. It is quite true, as he says, that a mere outline of the method would have been worse than useless; and by omitting it altogether, he has been enabled to give a very lucid and thorough account of the subject from one consistent point of view, without that excessive condensation which is so often a defect rather than a merit.

It is not necessary to say much of the earlier chapters, except that, like the rest of the book, they are very clear and pleasant to read; in particular, the proof that every covariant of a covariant is a covariant of the original form is easier to follow than that given by Salmon. It is when we come to chapters vi. and vii., which deal with seminvariants and their annihilators, that the influence of recent discoveries begins to be felt. Thus the notions of *excess* and *extent* are introduced, and the annihilators of invariants and covariants of systems of quantics are indicated.

Chapter viii. discusses generating functions, and is a very good introduction to this part of the subject. It does not profess to be exhaustive; and it is perhaps as well that the author has refrained from giving the detailed reduction of the generating functions for forms higher than the quartic. This would have taken up a good deal of space; and the full discussion for the lower forms, which is given, is quite enough to illustrate the general procedure. The results for the quintic are also stated, and references are given to the memoirs of Sylvester and Franklin, which ought to be easily understood by any one who has mastered this chapter.

Another very interesting chapter follows. This contains Hilbert's proof of Gordan's celebrated theorem, that the number of irreducible concomitants is finite. Compared with Gordan's original proof, this is simplicity itself; and it is unlikely that the demonstration can be essentially improved upon in this respect, although no doubt some simplification in detail may be effected.

Chapters x. and xi. are also well brought up to date. They deal with protomorphs and perpetuants, and the connection of seminvariants with non-unitary symmetric functions. It is needless to say that they are based principally on the researches of MacMahon and Hammond. The deduction of the annihilator of non-unitary symmetric functions of the quantic

$$b_0 x^p + \frac{b_1}{1!} x^{p-1} + \frac{b_2}{2!} x^{p-2} + \ldots + \frac{b_p}{p!}$$

seems rather artificial, as it is made to depend upon the transformation

$$-\frac{\partial}{\partial s_1} = b_0 \frac{\partial}{\partial b_1} + 2b_1 \frac{\partial}{\partial b_2} + \ldots pb_{p-1} \frac{\partial}{\partial b_p}$$

But this is a small matter, and the chapters are full of interest. One remarkable novelty is a differential operator which annihilates any rational integral function whatever of the coefficients of a finite quantic. Here is, indeed, a universal solvent. It should be added that the examples at the end of chapter xi. give a synopsis of Stroh's verification of MacMahon's brilliant conjecture that the generating function for perpetuants of degree i is

$$\frac{x^{2^{i-1}} - 1}{(1 - x^2)(1 - x^3) \ldots (1 - x^i)} \ldots (i > 2)$$

The remaining chapters (xii.–xvi.) treat of canonical forms, the binary quintic and sextic, systems of binary quantics, orthogonal invariants, and the ternary quadratic and cubic. The chapter on the quintic and sextic does not go into detail, but gives complete lists of the concomitants, and in particular the explicit forms (supplied by Mr. Hammond) for the quintic $(a, b, 0, 0, e, f)(x, y)^5$. The other chapters do not seem to call for special remark; suffice it to say that they maintain the high standard of those which precede them.

Prof. Elliott states in his preface that the book is an expansion of a course of lectures delivered annually for some years past at Oxford. To this fact, no doubt, may be attributed, in some measure, the lucidity and symmetry of the treatise. Another good feature, perhaps due to the same cause, is the occasional statement of what a theorem does *not* imply. To the well-informed reader this may seem superfluous, but it is by no means so in the case of a learner, who not infrequently reads into a theorem a degree of generality which it does not really contain.

To return to the symbolical method of Clebsch and Aronhold. Prof. Elliott admits that an English work on this calculus is a desideratum; will he not be persuaded to supply this want himself? It would be a great boon to have an English book something after the kind of the Clebsch-Lindemann "Geometrie," including, at least, the theory of plane quadratic and cubic curves, and of surfaces of the second order, with perhaps an introduction to the theory of cubic surfaces. This is, no doubt, a heavy task, but it is well worth attempting; the theory of forms is infinitely more interesting in its geometrical applications than as a mere branch of analysis, and it is here, above all, that the power of the symbolic method shows itself. Such a work would do much to avert the danger of divorcing the theory of forms from analytical geometry; a danger which is encouraged by the present regulations of the mathematical tripos, which place these cognate subjects in two different divisions. G. B. M.

SURFACE-COLOURS.

Die Oberflächen- oder Schiller-Farben. Von Dr. B. Walter. 1 vol., with 8 woodcuts and 1 plate. Pp. vi. + 122. (Braunschweig: F. Vieweg und Sohn, 1895.)

THIS work is primarily addressed to zoologists, mineralogists, and chemists, appealing in only a subordinate measure to physicists. On this account the mathematical developments most desirable for the physicist are reserved for appendices, while the text itself contains only such matter as is vital to the theory of surface-colours, together with very simple and well-established formulæ given without proof.

The importance to the first-named classes of an acquaintance with the physical basis of these colours immediately appears, says the author, from the facts that, on the one hand, to this class of colours belong the tints of many butterflies and birds, and also those of a series of crystals exhibiting the most gorgeous natural phenomena; and on the other hand, the technologist, if he desires artistically to imitate these colours, must naturally, first of all, obtain a true insight into their manner of production. Now, although this treatise contains no startling additions to our physical knowledge of surface-colours, it may yet be expected to render a most acceptable service to this branch of physics, since in many minds there still linger hazy, or even discordant, conceptions of these colours, and until now no work seems to have appeared devoting to the subject even any approach to an exhaustive treatment.

Of the experiments, which form the basis of the calculations and statements contained in this book, those which are new have been carried out by the author in the State Physical Laboratory at Hamburg.

The first chapter is a brief introduction to the subject of the work. The second and third chapters treat of the surface-colours of colourless materials and of metals respectively. In the fourth chapter, embracing about a third of the entire treatise, the author discusses the dichroic substances proper; solid fuchsine and diamond green, also solutions of these, and fluorescein solution being specially dealt with. This chapter is unusually rich in experimental results. It is pointed out that the body-colour and the surface-colour are only approximately complementary, and are not exactly so, as stated in Haidinger's law. We have also here the following important statement: "The coefficient of reflexion of a particular ray from a given substance depends not only upon the absorption coefficient of the substance for that ray, but also upon its refractive index for the ray in question, the relative importance of these two factors varying with circumstances, so that in the case of the feebly-absorbed rays of coloured substances the refractive index is prac-