laden the scale pan should be moved along the scale beam until it reaches a point where the weights are in equilibrio. The reading of the scale beam gives the average date of the purchases.

The entire process requires so little thought that one can determine a large number of averages without fatigue, and can verify the work by simply glancing a second time over the platform and pan for re-assurance as to the proper location of the weights. After the solution of any problem the weights are dumped and screened, thus preparing the machine for the next solution.

It is curious to note that the same machine can be used for solving a great variety in simple and inverse proportion, besides many others of a more intricate character, all of which may be determined with the greatest rapidity.

RADIO-DYNAMICS. II.

By PLINY EARLE CHASE, LL.D.

Abstract of Lectures delivered before the Franklin Institute, March 10 and 17, 1881.

Continued from page 65.

We have confined ourselves thus far to the simple dynamic laws of radial action to or from centres of energy and of inertia. I hope to satisfy you that those laws are sufficient for the explanation of all varieties of physical phenomena, and that radio-dynamics is, therefore, the foundation of all dynamics, the UNIVERSAL physical science, of which photo-dynamics, thermo-dynamics, electro-dynamics, cosmo-dynamics, moleculo-dynamics and chemistry are branches.

Following the Baconian method, I will first lay before you some general principles, followed by some additional FACTS to which I have been led by applying mathematical principles to the study of radio-dynamics, and then interpret those facts, in the hope of making them helpful towards the strengthening of hypotheses and the discovering of new facts or laws.

The elements of physical energy are mass, m , and velocity, v . The total energy of any force is called its *vis viva*, or living force. It is measured by the *work* that it is able to accomplish against uniform resistance, and is represented by $\frac{mv^2}{2}$.

La Place defined velocity as "the ratio of the space to the time

employed in describing it." In recent dynamical treatises the space traversed, or length, is represented by l ; time of traverse, by t ; velocity, by $\frac{l}{t}$; energy, by $\frac{m l^2}{t^2}$.

The greatest or controlling energy must, of course, be represented by the product of the greatest mass by the square of the greatest velocity.

The greatest mass of which we have any practical knowledge is the mass of the Sun, M_0 ; the greatest centripetal acceleration is the force of gravity at Sun's surface, G_0 ; the greatest velocity of wave propagation is the velocity of light, or the velocity of radiation, V_0 , at the seat of greatest centripetal acceleration.

Action and reaction being equal and in opposite directions, we may reasonably look for some simple relation between the centripetal and centrifugal maxima, G_0 and V_0 .

If we regard Sun's radiating energy as an action, its reaction must be dependent upon its inertia, or mass.

The velocity of projection against uniform gravitating resistance is represented by gt , t being one-half the time of flight. The height of projection is $\frac{gt^2}{2}$. The height of projection which would give the velocity

of wave propagation in an elastic medium is one-half the "height of a homogeneous atmosphere" of the medium.

Every particle of the Sun's surface is continually solicited by gravitating tendencies, G_0 and G_u , towards its own centre and towards the centre of the universe. In each solar rotation the particles are alternately projected from and drawn towards the centre of the universe, C_u .

We may, therefore, let $T_0 = \frac{1}{2}$ solar rotation, represent the time of cyclical equality of action and reaction between solar inertia and universal inertia, or between solar gravitation and æthereal undulation.

Then $L_0 = G_0 T_0^2$ is the height of a homogeneous æthereal atmosphere, at Sun's surface, which would have a velocity of wave propagation equivalent to the velocity of light; $\frac{L_0}{T_0} = V_0$ is the velocity of light.

All forms of energy, mechanical, thermal, photic, electric, magnetic

or chemical, are derived from and can be compared with the maximum energy, $\frac{M_0 I_0^2}{T^2}$.

1. All astronomical, barometric or other mechanical estimates of solar mass and distance involve the proportionality, $g \propto \frac{m}{r^2}$.

2. The arbitrary units of thermo-dynamics are based upon the work done against the centripetal accelerations of superficial terrestrial gravity. The unit of acceleration, or the sum of accelerations in unit of time, at unit of distance, is proportioned to mass. Therefore, if we designate Earth's mass by m_3 , we have the proportion

$$M_0 : m_3 :: V_0 : u_3$$

V_0 is the velocity acquired during the cyclical actions and reactions of solar condensation and æthereal elasticity, at Sun's surface; $u_3 = 56558$ mile is the velocity acquired during the cyclical actions and reactions of water congelation and vaporization at Earth's surface. Dividing by 1.180 for the Fahrenheit scale, or by 1.100 for the Centigrade scale, we have the arbitrary units of velocity, .042156 mile for 1°F., .056558 mile for 1°C. The equation $V = 1.2\sqrt{gh}$ gives $h = 772$ ft. for 1°F., or 1389.6 ft., = 424 metres, for 1°C.

Combining these heights with the arbitrary units of mass, we have $J_1 = 772$ ft. lbs., for the English thermal unit, and $C_1 = 424$ kilogrammetres, for the calorie or French thermal unit.

3. Thermal, mechanical and photo-dynamic energies may be compared with energies of chemical combination, through the ratios

$$M_0 : m_3 :: V_0 : u_3 :: h_0 : h_c$$

Earth's mean distance from Sun, or height of solar projection, is represented by h_0 ; h_c is $\frac{1}{4}$ of $\frac{2}{3}$ of the height to which water vapor would be thrown, against the retardation of gravity, by the combining energy of oxygen and hydrogen, the two constituents of water; $\frac{1}{4}$ is the length of a conical pendulum which would vibrate in the same time as a linear pendulum of unit length; $\frac{2}{3}$ is the ratio of *vis viva* of wave propagation to the mean *vis viva* of the oscillating particles which originate the waves.

4. Cosmical, electrical and photo-dynamic energies may be compared by means of the ratio

$$M_0 V_0 c_3 : m_3 V_0^2 :: m_3 V_0^2 : m_3 V_0 c_3$$

Sun, M_0 , is at the centre of nucleation in the solar system; Earth,

m_3 , at the centre of condensation ; Jupiter, m_3 , at the nebular centre ; v_3 is Earth's aphelion or " nascent " orbital velocity.

5. The electro-static, magnetic and electro-kinetic units of energy can all be derived from the above expression for Earth's photo-dynamic energy, $m_3 V_0^2 = \frac{m_3 I_0^2}{T_0^2}$, through the equations $[e E] = [m \Omega] = [p C] = \frac{M I_0^2}{T_0^2}$.

The bracketed symbols represent, respectively, quantity of electricity ; line integral of electromotive force, or electric potential ; quantity of free magnetism, or strength of a pole ; magnetic potential ; electro-kinetic momentum of a circuit ; electric current.

6. Atomic energy, or energy of unit volume, can be compared with Earth's photo-dynamic energy of unit volume, $\frac{m_3 I_0^2}{r_3^3 T_0^2} = \frac{m_3}{L_1 T_0^2}$, and with corresponding electric energies, through the equations

$$[D E] = [B H] = [C V] = \frac{M}{L T^2}$$

The bracketed symbols represent, respectively, electric displacement (measured by surface density) ; electromotive force at a point ; magnetic induction ; magnetic force ; current electric intensity at a point ; vector potential of electric current.

7. Electro-chemical and electro-magnetic energies may be compared with thermal, photo-dynamic and other energies through the proportion

$$z : \mu :: M_0 t_a^2 \times M_0 v_3^2 : m_3 t_n^2 \times m_3 V_0^2$$

I designate Weber's units of electro-chemical and electro-magnetic force by z and μ , respectively ; t_a is the time of acquiring orbital velocity, or incipient associative energy, at Laplace's limit of equal velocities of rotation and revolution ; t_n is the time of acquiring nucleal nascent or dissociative velocity. The ratio of t_a to t_n is the same as the ratio of the diameter of a circle to its circumference, $1 : \pi$.

8. Total magnetic force, ζ_0 , can be compared with the reactions of terrestrial magnetic force, ζ_3 , by the proportion

$$M_0^2 : \pi^4 m_3^2 :: \zeta_0 : \zeta_3$$

The reactions of orbital tendency are $t_a M_0^{1/2}$, $t_n m_3^{1/2}$, respectively. Centripetal undulation varying as the fourth power of orbital velocity, we have the ratio $t_a^4 M_0^2 : t_n^4 m_3^2$, or $M_0^2 : \pi^4 m_3^2$.

9. In the actions and reactions between the centre of nucleation, M_0 ,

and the centre of condensation, m_3 , there are continual tendencies towards the centre of gravity, the centre of lineal oscillation and the centre of conical oscillation, as well as to centripetal and orbital motions. These are all satisfied by the proportion

$$M_0 : m_3 :: (2 \times 3 \times 4)^4 : 1^4 :: 331776 : 1$$

In order to show the closeness of accordance between the theoretical results and those which have been deduced from observation or experiment, I make the following comparisons:

1. The theoretical oscillatory value of Sun's mass, (9), gives 92,-785,700 miles for Sun's distance; 8.809'' for Sun's parallax; 25.496 days for Sun's rotation; 299,943 kilometres per second for the velocity of light. These values differ by less than $\frac{1}{40}$ of one per cent. from Faye's, Michelson's and La Place's estimates.

2. The difference between the theoretical and experimental thermodynamic velocities is less than $\frac{3}{4}$ of one per cent. Perhaps, when proper allowances are made for the temperature at which the experiments were performed, the accordance will be found to be exact.

3. The difference in the combining energy of hydrogen and oxygen is less than $\frac{1}{35}$ of one per cent.

4. The difference in Earth's "nascent" velocity is less than $\frac{1}{11}$ of one per cent.

5, 6. The difference which is indicated by the latest electro-dynamic and electro-magnetic investigations is less than $\frac{1}{30}$ of one per cent.

7. Weber's experimental determination of the electro-chemical unit was $\frac{2}{5}$ of one per cent. greater than the theoretical value.

8. The experiments of Joule and others, in Great Britain, upon magnetic action and terrestrial reaction, indicate a difference of about $1\frac{1}{4}$ per cent. in the estimate of Sun's mass, and $\frac{1}{2}$ of one per cent. in the estimate of Sun's distance.

I have been obliged to use many technical terms and equations which are doubtless new to most of you, but you can all see how close is the agreement of the values which are derived mathematically from simple radio-dynamic action, with those which have been obtained experimentally by the most careful observers. You will have noticed how often I have called your attention to evidences of the fourth fundamental law, which is generally known as "Fourier's Theorem," viz.; every periodic vibrating or orbital motion can be regarded as the sum of a certain number of pendulum vibrations.

To illustrate this law I have prepared a stretched cord, to which I

have suspended three balls, at equal intervals, by strings of equal length. I set the middle one swinging, and you see how it gradually imparts its motion to the two lateral balls, and comes to rest itself. Now the side balls are losing their motion, and the middle one takes it up again, until it swings almost as far as at first, and its companions hang motionless. These alternations continue until the resistance of the air overcomes the motion, and the balls are all quiet, as at the beginning. The motion, however, has not ceased; it has merely been imparted to the air, and by the air to the æther, so that we do not see it, and we are unable to follow it through all its changes.

When the regularly recurring vibrations of an elastic medium are transmitted to the ear through a suitable medium, they produce the sensation of sound: if the interval between the successive vibrations is short enough, the sound is musical. We may distinguish between the music in the sounding body and the music which we hear. A bell may ring when swung by the wind, but it will give no sensation of sound unless its vibrations reach the drum of the ear with energy enough to make it vibrate. I strike this tuning fork, but you hear nothing except a single ring. I bring it near the opening of this wooden box and the oscillations of the fork are imparted, first to the air, then to the wood, and the note of the fork is sounded, loud, clear and musical.

It is not necessary that the unison should be exact, or, in other words, that the fork and the resonator should make the same number of pendulum swings in the same time. If the vibrations coincide at intervals which are so short that the ear cannot distinguish between them, one body, for example, making three vibrations and another two in the hundredth part of a second, both sounds are said to be harmonious. Whenever any elastic medium is vibrating, its waves tend to produce harmonic vibrations in all bodies which they strike.

If the elastic force of the æther is more than a million million times as great as that of the air, its musical rhythm, if our ears were sensitive enough to hear it, should be of a far higher order than that of any earthly choir. Chemical atoms and molecules, the particles of heated or electrified bodies, satellites, planets, suns, stars and nebulae, all intercept the luminous waves. Hence there should be a continual tendency to relative positions which would make all the motions harmonic, or rhythmical. Such a tendency is shown in atomic weights, spectral lines, mechanical equivalents of heat, the various phenomena

of electro-dynamics, the relative positions of satellites, asteroids and planets, and the bonds of union between our own system and the nearest of the fixed stars. The truth of the radio-dynamic hypothesis has been further confirmed by the indication or prediction of harmonic nodes beyond Neptune, as well as between Mercury and the Sun, fourteen of which have been subsequently corroborated by the calculations of various European and American astronomers.

If you throw a ball, or fire a projectile upwards into the air, it will continue to rise until the resistance of the air and the attraction of gravitation have overcome the velocity of projection. If there were no air, and no resistance to the upward flight except gravitation, the relation of projectile velocity to gravitation and height of projection would be represented by the equation

$$v = 1/2gh \quad (1)$$

The relation of velocity to time of rise, or to the equal time of fall would be represented by

$$v = gt \quad (2)$$

In solar rotation each particle is alternately projected from and drawn towards the universal centre of gravity. Substituting the equatorial values of g and t in equation (2), we find that v is the velocity of light.

The circular-orbital velocity of a particle at Sun's surface, v_s , is connected with the velocity of rotation, v_r , and the velocity of light, c , by the equation

$$\pi r_s^2 = v_r c \quad (3)$$

The velocity of a particle, at any distance whatever from any attracting centre, can be deduced from equation (3), by means of the eleventh fundamental law, that centripetal accelerations vary directly as the mass and inversely as the square of the distance. Hence we see that *all gravitating velocities are dependent on the velocity of light.*

The distribution and motions of the principal planetary masses are also dependent upon the same velocity. The largest planet is Jupiter, which is more than twice as large as all the other planets. The mean centre of gravity of the solar system is therefore the same as the mean centre of gravity of Sun and Jupiter. The next planet in point of magnitude is Saturn, which is more than two and three-fourths times as large as all the remaining planets, and which is placed at the nebular centre of planetary inertia. When the primitive planetary belt was successively divided into inter-asteroidal and extra-asteroidal belts,

two-planet belts and single-planet belts, the amount of inertia remained unchanged. But in these divisions the rectilinear propagation of luminous waves has been changed into the synchronous oscillations of conical pendulums, introducing the length-ratio of 4 to 1; the synchronous oscillations have been changed into orbital oscillations, in which the time varies as the $\frac{3}{2}$ power of the distance; nebular radii have given place to radii of subsidence-collision, which are only $\frac{2}{3}$ as great; and centripetal tendencies vary as the fourth power of orbital tendencies. If we designate Saturn's mass by m_6 , and the mean velocity of rotation of the centre of gravity of the solar system by v_s , these changes and their dependence on the velocity of light are shown by the proportion

$$4^{\frac{3}{2}} M_0 : (\frac{2}{3})^4 m_6 :: v_\lambda : v_s$$

Bessel's estimate of the comparative masses of Sun and Saturn was $M_0 = 3501.6 m_6$. Substituting this value in the above proportion, we find

$$v_\lambda = 141815 v_s$$

The most accurate estimate that we are able to make of v_s is 1.31405 miles per second. This velocity, which is so readily deduced from the velocity of light, is also the *wave-velocity which represents the conversion of water into vapor*.

Let θ represent the latent heat of steam, then well-known laws of thermo-dynamics give us the equations

$$v_s = 1 \sqrt{\frac{2gh}{\theta}}$$

Solving these equations, we find

$$h = 750,098 \text{ feet.}$$

$$\theta = 539.794 \text{ C.}$$

Among the various experimental values which have been obtained for θ , the following are generally regarded as the most trustworthy:

Favre and Silbermann,	.	.	535.77°
Andrews,	.	.	535.90°
Regnault,	.	.	536.67°
Tyndall,	.	.	537.20°
Despretz,	.	.	540.00°
Dulong,	.	.	543.00°

According to the kinetic theory of gases the internal movements of the particles of steam are rectilinear. When the steam is condensed, in the form of water or ice, the internal energies tend to maintain a spherical figure. The synchronous oscillations of the two conditions may be represented, according to Fourier's theorem, by linear and conical pendulums. The height to which the vapor would be projected above the water level being represented by 750,098 feet, or 142.064 miles, the length of the conical pendulum is $\frac{1}{3}$ as great, or 47.355 miles, and the length of the linear pendulum is $\frac{4}{3}$ as great, or 189.419 miles. The heat of sphericity should be, therefore, $\frac{1}{3}$ of the latent heat of steam, or 179.93° . Deducting 100° for the expansion of water from the freezing to the boiling point, we have 79.93° for the "latent heat" of ice, or more properly speaking, for the heat which is required to overcome the crystallizing energies of water. The following values have been obtained experimentally:

Desains and De la Provostaye,	79.25°
Black,	79.44°
Person,	80.00°
Hess,	80.34°

The combined influence of photo-dynamic *vis viva* and the nebular "subsidence" which was pointed out by Herschel, is shown in the proportion

$$1.01555^2 : \left(\frac{5}{9}\right)^2 :: m_5 : m_6$$

The radius of incipient subsidence for Neptune's orbit (Law 10, p. 61) is 1.01555; $\frac{5}{9}$ is the ratio of the *vis viva* of wave propagation to the mean *vis viva* of oscillating particles (Law 22, p. 62). If we substitute Bessel's estimate for m_6 in the above proportion we find

$$1.03136 : \frac{25}{81} :: \frac{1}{1047.9} : \frac{1}{3501.6}$$

If we take Stockwell's estimate of Neptune's radius of incipient subsidence, 1.0145, we get $M_0 = 1050 m_5$. Bessel's estimate was 1047.879; Leverrier's, 1050. The difference between the two estimates is only $\frac{1}{3}$ of one per cent. Bessel's is the one which is adopted by the British and American Nautical Almanacs, but the reputation of Stockwell and Leverrier seems to render it probable that the true values may be intermediate between the two which are here given. The photo-dynamic relations of mass and distance become still more striking when we find that the incipient subsidence at the nebular centre, Jupiter, is at a mean proportionate distance between the centre

of condensation, Earth, and the incipient subsidence of the primitive planetary belt, Neptune. This is shown by the proportion

$$1 : 5\cdot52 :: 5\cdot52 : 30\cdot47$$

If we represent Earth's semi-axis major by 1, Jupiter's secular aphelion is 5·52, and Neptune's secular aphelion is 30·47.

These comparisons might be extended almost without end. We have now surveyed the whole field of physical science, and have found, in every direction, that all possible physical energies can be expressed in terms of the greatest and most pervasive energy, through the mass of the Sun and the velocity of light. Simple gravitation, solar rotation, nebular subsidence, orbital revolution and all other gravitating motions, cosmical aggregation, the distribution of planetary masses, the establishment of centres of inertia, condensation and nucleation, evaporation, crystallization, heat, mechanical work, barometric pressure, atomic energy, chemical combination, electricity and magnetism, are all so simply connected by the universal laws of action and reaction in elastic media, that they all furnish ready methods for estimating the mass and distance of the Sun and the velocity of light.

THE PROPERTIES OF AIR RELATING TO VENTILATION AND HEATING.

By ROBERT BRIGGS, C.E.

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The surface of the earth is covered by a gaseous body, some forty or fifty miles in depth, which is called the atmosphere. Chemistry has discovered and isolated various gases, some of which, so far as further separation is concerned, may be deemed elementary, while some are chemical compounds of definite proportions of other elementary gases and bodies. In some cases bodies which in their elementary form, at temperatures subsisting in nature, are solid, become portions of chemical combinations as gases at similar temperatures.

The atmosphere is composed mainly of a mixture of two elementary gases, together with small but appreciable quantities of two other gaseous bodies, products of combustion; beside other gaseous bodies of various kinds, in nearly inappreciable quantities, the latter varying somewhat in character in inhabited localities. Its substance, as a