On the Regularity in the Distribution of the Satellites of Spectrum Lines

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VI. *On the Regularity in the Distribution of the Satellites of Spectrum Lines.* By PROF. H. NAGAOKA, Imperial University, Tokyo.

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§ 1. The nature of the satellites of spectrum lines is not yet thoroughly known, nor are the positions with respect to the principal well determined, except for a few lines of a small number of elements. I have no doubt that there exists regularity in the distribution of the satellites, in their order of intensity, and in their behaviour when acted upon by strong electric and magnetic forces. The usual view as regards the satellites is to consider them as belonging to the principal line, which is the strongest among an assemblage of lines of various strengths clustered together within a narrow compass about the principal. It is still an open question whether such grouping of lines is swayed by the principal, or they have independent existence apart from the principal. There is as yet no observation of the effect of an electric field on the satellites, and on account of the difficulty of measurement and of the complexity of the separation the Zeeman effect is known only for a limited number of satellites. Experiments on the effect of the magnetic field on the satellites of mercury lines show that some of them are controlled by the principal or strong satellites in the neighbourhood, but there exist satellites which seem to lead an independent existence, the mode of magnetic separation being little affected by the principal. It may, therefore, be improper to call them satellites, while some of them are strongly influenced and bear the character of the name assigned to them.

§ 2. In a former communication to the Society,* we have noticed that the satellites of mercury lines are apparently distributed with some regularity and that there is gradation in intensity. Although no definite result could then be obtained, it was probable that the minute structure of the principal lines was not yet known, as most of the discrepancies among different observations were to be traced to the inaccuracy in the determination of the exact position of the principal lines. As suggested by Stansfield, the position of the satellites referred to one of the distinct components gave tolerably good concordance in the results of different observers with various

* Nagaoka and Takamine, "Proc." Phy. Soc., 25, p. 1, 1912.
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instruments, and it seemed quite likely that further study of the principal line would reveal the regularity in the distribution of the satellites, if such really exists.

§ 3. For this purpose, it is necessary to use instruments with high resolving power, in order that a strong line with rather vague boundaries may be separated into its components. A concave grating is generally inefficient, the echelon grating and Lummer-Gehrcke plate have definite resolving power for a given wave-length, so that we can not bring different powers to bear upon the lines to be analysed. The inconvenience is especially felt when the interval between successive spectra is so narrow that the lines are superposed and cause great confusion in discriminating them. The disadvantage arising from the definiteness of the resolving power can in no wise be remedied, so that it is generally expedient to resort to those interferometers in which the thickness of the air space contributing to interference can be easily changed, so as to suit the purpose of the experiment. The interferometers of Michelson and of Fabry-Perot belong to this type of instruments, as the air interspace is adjustable within wide limits. A simple device for changing the resolving power is a prime factor that makes the instrument indispensable in researches relating to the minute structure of a spectrum line, especially when it consists of a cluster of lines within narrow limits. The easiest way of arriving at a desired resolving power would be to increase the air interspace between the plates in a Fabry-Perot interferometer, the thickness being, of course, within the limit of interference for the line to be examined. The exact value of the resolving power cannot, however, be given, as it depends on the number of reflections from the semi-transparent silver films. One serious objection to a thick interspace is the crowding of the lines, as different interference fringes are intermixed with each other, and hence arises the trouble of discriminating the different lines, especially when there are numerous satellites. In fact, the appearance of interference rings is extremely vague, when the interspace is several centimetres thick, that it is almost impossible to tell even the position of the strong satellites. This difficulty can, however, be to a great extent overcome by the use of interference points, obtained by crossing it with another instrument, which may have either fixed or adjustable resolving power. The advantage* gained by crossing is especially great, when there is

* Nagaoka and Takamine, "Phil. Mag.," 27, p. 126, 1914.

ambiguity in the position of the satellites as observed with a single instrument; for it can easily be settled by examining the crossed spectra. With an adjustable interferometer, we can change the air interspace, such that the satellites to be analysed are brought distinctly under observation, without being intermixed with other components. I have tried the combinations of F.P. interferometer with echelon grating, L.G. plate, wedge plate, and another F.P. interferometer of the fixed type; the last method of crossing was found to have a disadvantage of losing too much light in passing through four semi-transparent silver films, so that most of the measurements were made with the two other combinations. In addition to this, I have found that the combination of echelon grating or L.G. plate with a wedge plate forms an excellent pair for work with fixed resolving power.

§ 4. Preliminary study of the minute structure of mercury lines was made in conjunction with Mr. Takamine,* and published in connection with the Zeeman effect of the mercury line $\lambda 4,359$; but subsequent experiments showed that there were many points still to be improved in the method of investigation, especially in increasing the order of interference, for making the fine structure of the principal line easily visible on the photograms.

§ 5. It was first shown by Janicki† that the principal line of the green line of mercury is multiple, by combining wedge and L.G. plates, which had a resolving power of 680,000 for the said line. Although Janicki denies the multiple character of other lines of mercury, it seemed quite likely that fine structure of the principal lines could not be placed under a strict experimental test, inasmuch as the resolving power of his instrument was fixed. With sliding F.P. interferometer, the resolving power can be raised to several times the above value, and the structure more accurately analysed.

§ 6. In the present experiment, the air interspace was much greater than in the former and generally ranged between 6 cm. and 12 cm.; with the thickness of 7 cm. or 8 cm., the composite character of the principal of the green line was distinctly visible, but owing to the overlapping of different spectra, it was necessary to cross it with a L.G. plate whose $d\lambda_{max.} = 0.1066$ A.U.; the resolving power thus utilised may be estimated at 2.3×10^6 for 8 cm. air space, assuming the number of reflections

* Nagaoka and Takamine, "Phil. Mag.," 29, p. 241, 1915.

† Janicki, "Ann. d. Phys.," 33, p. 438, 1912.

on the front face to be 8. The spectra were drawn out more than with the echelon grating; but, owing to the crowding of interference points resulting from numerous satellites, it was somewhat difficult to discriminate them; from our previous knowledge of the position of the satellites, we can, however, avoid their coincidence with the principal line, by calculating the value of the air interspace suitable for the experiment. In addition to this, the intensity of the principal line was far greater than that of the satellites, so that there was no trouble in following the curves of interference arising from the components of the principal line.

§ 7. The mercury lamp was of the Arons-Lummer type, water-cooled, and run with a current of 2.5 to 3 amperes at 100 volts; in the circuit was included a big electromagnet having sufficient inductance to maintain the current steadily. Great care was taken to keep the temperature of the air interspace constant; the resistance and other parts, which are liable to be heated, were kept outside the room. The interferometer was enclosed in a wooden box, leaving only a passage for light and photographic apparatus. Wratten-Wainright panchromatic plates were generally used for the green and yellow lines, and process plates for the violet lines. The interference points were measured by means of a micrometer made by Repsold, or with a dividing engine constructed by Société Genèvoise, having attached microscopes and an arrangement for eliminating screw error.

§ 8. It will be necessary to give a short explanation of the locus of interference points obtained by the combination of interferometers above mentioned. For the F.P. instrument, the order of interference P for light of wave-length λ subtending a ring of small angle 2θ is

$$P = \frac{2e \cos \theta}{\lambda} = \frac{2e}{\lambda} \left(1 - \frac{\theta^2}{2}\right),$$

e denoting the thickness of the air interspace. Referred to rectangular co-ordinates x and y through the centre of the ring in the focal plane of the observing telescope, and supposing that the wave-length λ is different by a small amount $\delta\lambda$ from λ_0 , we get

$$\frac{x^2 + y^2}{-} = 1 - \frac{P}{2e}(\lambda_0 + \delta\lambda) = \left(1 - \frac{P}{P_0}\right) - \frac{P}{P_0} \frac{\delta\lambda}{\lambda_0},$$

where P_0 stands for $2e/\lambda_0$.

Since P is nearly equal to P_0 and very large,

$$\frac{x^2 + y^2}{2} = \frac{\delta P}{P_0} - \frac{\delta \lambda}{\lambda},$$

where $\delta P = P_0 - P$.

The order h for L.G. plate is given by

$$h = \frac{2t\sqrt{(\mu^2 - \sin^2 i)}}{\lambda},$$

where t is the thickness, μ the index of refraction for light of wave-length λ , and i the angle of exit. As observations are generally made near grazing exit, we can put

$$i = 90^\circ - \alpha$$

where α is small. Thus,

$$h^2 \lambda^2 = t^2 \{4(\mu^2 - 1) + 2\alpha^2\}.$$

Putting $\lambda = \lambda_0 + \delta \lambda$, where λ_0 is the wave-length of the reference line, and $\delta \lambda$ is the difference in wave-length of the satellites from it, we get

$$2h^2 \lambda_0 \delta \lambda = \{4t^2(\mu^2 - 1) - h^2 \lambda^2\} + 2t^2 \alpha^2,$$

the quantity within the parenthesis being constant.

§ 9. On combining the plate with F.P. interferometer, the centre of the ring does not always coincide with the direction of grazing incidence: we have, therefore, to put $a = a + x$; consequently $2h^2 \lambda_0 \delta \lambda = 2h_0^2 \lambda_0 \delta \lambda + 2t^2(a + x)^2$ nearly.

Equating $\delta \lambda$ in these two equations for F.P. and L.G. interferometers, and using the relation $\frac{4t^2}{h_0^2 \lambda_0^2} = \frac{1}{\mu^2 - 1}$, we get the equation

$$x^2 + \frac{(a+x)^2}{2(\mu^2 - 1)} + y^2 = 2\left(\frac{\delta P}{P_0} - \frac{\delta h}{h_0}\right),$$

which is evidently an equation to an ellipse. Thus the interference points to be obtained by the said combination lie on ellipses with the centre on the x -axis. If we now adjust the position of the plate, such that the ray at grazing exit passes through the centre of the rings, $a = 0$, and the equation reduces

$$\text{to } \frac{2\mu^2 - 1}{2(\mu^2 - 1)} x^2 + y^2 = 2\left(\frac{\delta P}{P_0} - \frac{\delta h}{h_0}\right).$$

The major axis of the ellipse is evidently on the y -axis, and the excentricity $= 1/\sqrt{2\mu^2 - 1}$. In the present experiment for $\lambda 5,461$, the excentricity amounts to $1/1.994$ and the ratio of the axes 1.156 , being nearly $= \sqrt{4/3}$. For the same value of δP , the

axes of the ellipse become smaller as the air interspace is increased, so that the ellipses for successive orders of P become denser, till they are obliterated by coming into contact, as the limit of interference is reached. With the green and the violet line of mercury, the interference is still visible for $c=12$ cm., corresponding to the values of $P=440,000$ and $550,000$ respectively. From the measurements of the breadth of interference points, I found the order at the limit of interference to be 750,000 and 810,000 respectively, nearly coinciding with the values found by Fabry and Perot by another method.

§ 10. For a given wave-length, the value of h in the L.G. plate is fixed, as the angular interval of the successive spectra cannot be changed. With F.P. interferometer, this is nevertheless the case. With increased air space, the interval for the same value of P decreases, as the angular difference for successive spectra is given by the relation $\Delta\theta = -\frac{\lambda}{2e\theta} = -\frac{1}{P_0\theta}$. Consequently

the value of $\frac{\delta P}{P_0} - \frac{\delta h}{h_0}$ becomes smaller as the interspace is widened. As the axes of the ellipse on which the interference points for the same orders of P and h lie are determined by the above term, the curve for the same difference of P and h occupies a smaller space as the resolving power of the interferometer is increased. The experiments in this direction indicate that the disadvantage thus caused is of little consequence, provided the temperature of the interspace does not vary during observation or photographing.

§ 11. Another important question in connection with the present combination is the variation of x and y co-ordinates, as we pass from one interference spectrum to that succeeding it. For simplicity's sake, I shall make the supposition that the ray at grazing incidence passes through the centre of the rings, i.e., $a=0$. Keeping λ constant,

$$h^2 - h_0^2 = \frac{2f^2}{\lambda^2} x^2$$

for L.G. plate and $P - P_0 = -\frac{c}{\lambda}(x^2 + y^2)$

for F.P. interferometer. For finite differences of h and P , we get approximately

$$\lambda^2 h \Delta h = 2f^2 x \Delta x$$

and

$$\lambda \Delta P = -2c(x \Delta x + y \Delta y).$$

FIG. 1A.
 λ 5,461.

FIG. 1B.
 λ 5,461.

FIG. 1C.
 λ 5,461.

$e=8.8\text{ cm.}$
L.G.+F.P.

$e=8.0\text{ cm.}$
L.G.+F.P.

$e=9.5\text{ cm.}$
L.G.+F.P.

FIG. 3A.
 λ 4,359.

FIG. 3B.
 λ 4,359.

$e=5.0\text{ cm.}$
L.G.+F.P.

$e=6\text{ cm.}$
L.G.+F.P.

In these combinations, let us pass from P to $P+\Delta P$ by keeping h constant, then

$$\Delta x=0$$

and
$$\Delta y = -\frac{\lambda}{2e\eta} \Delta P = -\frac{1}{P_0\eta} \Delta P,$$

giving the change in y . Next vary h and keep P constant; then x and y are to be found from the above two equations by putting $\Delta P=0$; evidently

$$\Delta x = \frac{\lambda^2 h}{2t^2 x} \Delta h = \frac{2(\mu^2 - 1)}{x} \frac{\Delta h}{h}$$

$$\Delta y = -\frac{\lambda^2 h}{2t^2 y} \Delta h = -\frac{2(\mu^2 - 1)}{y} \frac{\Delta h}{h}$$

approximately.

These simple relations will help us to understand how the curves of interference points are crowded, as we pass outwards from the centre of the rings, showing that the variations of the co-ordinates for successive spectra are inversely proportional to them.

§ 12. From the above discussion, it will be seen that for calculating the difference in wave-length of interference points referred to the L.G. plate, there is great inconvenience in the gradual decrease in the intervals between the successive spectra as the order h increases from that at grazing exit. Interpolation by simple proportional parts is inadmissible, so that we have to take the second differential into account. This is a rather tedious process, so that for interpolation, the graphical method is perhaps the easiest and the least liable to error. This process was followed in evaluating the readings of the micrometer.

§ 13. Figs. 1 (A, B, C) show the photograms of the line $\lambda 5,461$ obtained by the above-mentioned combination for three values of e . The principal line is evidently a triplet with a strong satellite on the positive and negative sides. These points are connected by deep shades, as the resolving power of the plate is not sufficiently high; it is on account of the high resolution of the F.P. interferometer that these points appear distinctly. Several photographs for different values of the air space were taken, all leading to the same conclusion; their distances are as follows:—

$$-19.9 \quad -8.4 \quad 0 \quad +8.2 \quad +18.4 \quad \text{m.}\overset{\circ}{\text{A.}}\text{U.};$$

the remaining points are due to satellites, of which the outermost one -23.7 is conspicuous, and may be used as a reference

line for comparing the results of different observers, as suggested by Stansfield. For finding the position of other satellites the air space of about 1 cm. is preferable to that above mentioned. The following table will show how the results can be reconciled with each other, if the lines be referred to the said satellite, and not to the principal line whose position is very vague in ordinary experiment, owing to its multiple character. The last column gives the difference in wave-number per centimetre referred to the principal line :—

Von Baeyer.	Nagaoka & Takamine.		Janicki.	Nagaoka.	
L.G. + L.G.	Echel. + L.G.		L.G. + Wedge	F.P. + L.G.	ν .
0	0	0	0	0	+0.793
136	133	134	134	135	+0.341
169	166	168	168	167	+0.234
188	191	188	188	190	+0.157
213	220	216	214	216.9	+0.066
...	...	...	227	228.4	+0.027
237 ?	242 ?	246 ?	236	236.8	0
...	...	...	245	245.0	-0.029
...	...	...	254	255.2	-0.062
319	318	320	320	322	-0.285
362	364	365	364	365	-0.431
448	450	452	450	451	-0.717

The above results were all obtained by crossing the two instruments mentioned under the names of the observers ; the comparison shows that the position of the satellites and their number are now well settled for the green line.* The mutual agreement of these numbers affords the proof of the accuracy of the method of crossed spectra. By crossing the echelon grating with L.G. interferometer and utilising the method of coincidences† between the interference points of these satellites, we have measured the distance between the satellites 0 and 365 accurate to four figures and found it to be 365.0. Relying on the echelon spectrum only, which in the present case was better defined than in the former experiments, I have, after repeated micrometric measurements, found it to be 365.0, which exactly coincides with the number obtained by a more accurate but complicated process. It further lends weight to the reliability of echelon spectroscope in analysing the structure of spectrum lines, although the apparatus is more complex and susceptible to changes in the instrumental constants than

* See Note added to the present Paper (p. 114).

† Nagaoka and Takamine, " Phil. Mag., " 27, p. 123, 1914.

a L.G. plate, as is evident from the frequent appearance of ghosts in the spectra produced by it.

§ 14. In our former communication, no attempt was made to arrange the satellites according to the difference in wave-number. If they are arranged according to the numbers of the last column in the Table (p. 98), we find that there is a remarkable symmetry in the distribution of the satellites in different lines. In Fig. 2, the first row represents the general aspect of

FIG. 2.

the green line, with the intensity taken proportional to the length of the line. Arranging them in groups as indicated in the following four rows, we find a remarkable symmetry in the distribution of the satellites.

The principal line forms a triplet with -0.062 and $+0.066$ as components; the satellite -0.029 then forms another triplet with the components -0.263 and $+0.256$ referred to it, being nearly four times the distance between the components of the principal triplet; then $+0.027$ forms another triplet having the components -0.744 and $+0.766$ referred to it, the interval being about three times that in the second triplet and 12 times that in the first triplet. The remaining satellites -0.431 , $+0.157$ and $+0.341$ have apparently no regularity. Further, the components of the triplet have symmetry in the intensity of lines in the first and second groups, but in the third this is no longer the case. The wave-number in the last row is in a singular ratio. The difference in the wave-number intervals counted from the principal of the first positive and negative

component = 0.274, and the difference between the first and the second positive components = 0.184; these numbers are in the ratio 3:2.

In addition to this, we find that the interval between the satellites

$$-0.717 \text{ and } +0.341 = 1.058$$

$$-0.285 \text{ and } +0.793 = 1.078.$$

The mean of these two numbers = 1.068. It will be afterwards shown that for several mercury lines, the difference in wave-numbers of satellites is constant and about equal to the above number, so that this is not characteristic of the green line only.

§ 15. The complexity of the violet line $\lambda 4,359$ is well known, but the multiple structure of the principal line was not yet studied. The arrangement being the same as before, the crossed echelon spectrum obtained by short exposure showed at once that the principal line is complex, but the components are so close together, that 10 m.Å.U. were difficult to separate. Crossing the plate with the F.P. interferometer, the complex structure was made out as shown in Figs. 3 (A, B). The interference points are arranged in a manner similar to those in the green line, the principal forming a triplet, of which the middle component is rather weak compared with the side components, each of them being associated with a strong satellite. Perhaps the middle of the triplet, though weak, can be taken as the principal line. The lines of the group are distant by the following amount :—

$$-17.9 \quad -5.8 \quad 0 \quad +5.7 \quad +16.2 \quad \text{m.Å.U.}$$

For determining the position of other satellites, I have used crossed echelon spectra. The table given on next page gives the comparison referred to the strong outermost satellite which is easily oriented. The difference of wave-numbers in the last column are not calculated from what appeared to be the principal line, but the initial point is laid at the middle of two strong lines 157.7 and 163.4, so that they form a strong doublet.

§ 16. In our former experiments with an echelon grating and a L.G. plate, the exposure was limited to 20 or at most 30 minutes, but in the present experiment with an echelon and F.P. instrument, the satellites already found were distinctly visible with exposure of 20 minutes; when it was prolonged to 40 minutes, additional lines appeared, which evidently were not to be identified with ghosts; by exposure for two hours,

FIG. 4A.
 λ 4,359.

FIG. 4B.
 λ 4,359.

FIG. 4C.
 λ 4,359.

Ech. + L.G.
Exp. 60 min.

(Compare with Fig. 22 of the former communication, Vol. 25 of the "Proceedings.")

FIG. 4D.
 λ 4,359.

e = 7.0 cm.
Ech. + F.P.
Exp. 40 min.

e = 5.0 cm.
Ech. + F.P.
Exp. 20 min.

FIG. 4E.
 λ 4,359.

e = 8.0 cm.
Ech. + F.P.
Exp. 120 min.

Von Baeyer.	Nagaoka and Takamine.		Nagaoka.	
L.G. + L.G.	Echel. + L.G.		L.G. + or Echel. + F.P.	
...	...	...	-78	+1.254
...	...	...	-57	+1.082
0	0	0	0	+0.844
52	53	53	53	+0.566
66	67	66	65	+0.502
114	116	115	114	+0.245
142	142	140	139.8	+0.110
...	...	...	151.9	+0.047
159 ?	161 ?	163 ?	157.7	+0.015
...	...	...	163.4	-0.015
178	181	180	179.6	-0.089
187	193	189	190	-0.154
203	206	206	207	-0.244
273	267	268	268	-0.566
...	...	...	318	-0.834
344	344	345	343	-0.956
...	...	...	383	-1.176
...	...	...	398	-1.251

the group about the principal was completely darkened, but new lines became distinctly visible. Trying similar experiment with L.G. plate and echelon, I noticed traces of faint lines apparently near the strong lines of the principal group, and though much obscured by them, the course of the interference

Hg. 4359

FIG. 5.

points indicated that the faint dots belong to echelon spectra different from the principal. The photographs are shown in Figs. 4 (A, B, C, D, E). These satellites are nearly coincident with those found by Wood with a 6 in. concave grating. Although the identification is not perfect, the difference is probably to be ascribed to the error of observation. These are -78, -57, +318, +383 in the above Table.

§ 17. Plotting the differences of wave-number (Fig. 5), we get two rows of which the upper one shows remarkable symmetry. The intensity of -0.099 is somewhat less than that of +0.110, and for these lines the symmetry is not so exact as for

other satellites in the same row. The components near the initial point, *i.e.*, -0.244 , -0.099 , -0.015 and $+0.015$, $+0.110$, $+0.245$, evidently form a sextet, and the outer components -1.251 , -0.834 , -0.564 and $+0.566$, $+0.844$, $+1.254$ another sextet, which is more symmetrical than the inner. The wave-number interval of the outer sextet of $\lambda 4,359$ is so related that the mean interval of the inner components 0.274 is to the mean of the outer 0.414 in the ratio 2:3. This regularity is indeed remarkable and will no doubt have some intimate connection with the atomic structure. It must not, however, be forgotten that the principal line, which is evidently represented by the triplet -0.015 , $+0.015$ and $+0.047$ is split into a sextet in magnetic fields, so that the structure here exhibited may have an important relation to the Zeeman effect of the lines. It is also to be noted that the strong satellites -0.099 and $+0.110$ show the same mode of separation.

§ 18. The lines in the second row have apparently no symmetry, but some of them show striking coincidences in the frequency difference with components of both rows. As to the row of satellites having no apparent symmetry, we can detect the same regularity as for the green line. The difference in absolute wave-number for the first positive and the second negative components $=0.909$, and the same quantity between the first two positive components $=0.455$; these numbers are in the ratio 2:1. Further the interval between the principal -0.015 and the second negative component of the same row $=1.161$, which is nearly double the difference in wave-number of the second and third positive satellites $=0.580$.

The wave-number interval between

$$\begin{array}{rcl} -1.176 \text{ and } -0.099 & = & 1.077 \\ -0.956 \quad , \quad +0.110 & = & 1.066 \\ -0.834 \quad , \quad +0.245 & = & 1.079 \\ -0.564 \quad , \quad +0.502 & = & 1.066 \\ -0.244 \quad , \quad +0.844 & = & 1.088 \\ +0.015 \quad , \quad +1.082 & = & 1.067 \end{array}$$

$$\text{Mean} \dots\dots\dots = 1.074$$

A similar interval in $\lambda 5,461$ (§ 14) has a mean interval 1.068 , which is almost equal to the number found for $\lambda 4,359$. Moreover, the interval between

$$\begin{array}{rcl} -1.251 \text{ and } -0.244 & = & 1.007 \\ +0.245 \quad , \quad +1.251 & = & 1.009 \end{array}$$

$$\text{Mean} \dots\dots\dots = 1.008$$

The difference between these two numbers = 0.066, which is the interval between $+0.566$ and $+0.502$, and also corresponds to the interval in the triplet found in the green line. These numerical coincidences are apparently not a mere chance, as will be further shown for the line $\lambda 4,047$.

§ 19. Experiments similar to those on the lines $\lambda 5,461$ and $\lambda 4,359$ were made on the violet line $\lambda 4,047$. The optical constants of the L.G. plate were such that several satellites came on the same line with the principal, so that even with crossed spectra, the structure was not distinctly observed as with the lines above described. Crossed echelon and F.P. interferometer spectra were more definite, and gave points, which showed that the principal line is a triplet accompanied by a satellite. The wave-number differences counted from the principal are as follows:—

$$-0.715, -0.362, -0.025, 0, +0.024, +0.075, +0.290, \\ +0.375, +0.708.$$

It is rather difficult to estimate the resolving power used in the experiment, as the reflection is much enfeebled for such

Fig. 4047

FIG. 6.

short waves and the absorption in the echelon is much increased, but at the distance of 6 cm. or 7 cm. it will be over 1.5×10^6 , which is no doubt sufficient to effect the above resolution. Plotting the above number in Fig. 6 we find that there is a triplet at the middle, and two lines on both sides are symmetrically situated about the principal. There are two other lines which are evidently not included in the same group. With this line, the interval in wave-numbers between

$$-0.715 \text{ and } +0.375 = 1.090$$

$$-0.362 \text{ and } +0.708 = 1.070$$

$$\text{Mean} \dots\dots = 1.080$$

The occurrence of this number in $\lambda 5,461$ and $\lambda 4,359$ was already noticed; moreover, the ratio between the mean components 0.368 and 0.712 to 1.080 is as 1:3 and 2:3 respectively.

§ 20. The similarity in the distribution of the satellites and the occurrence of the same wave-number intervals may be attributable to the character of the second subordinate series to which the three lines here mentioned belong. Arranging the wave-number intervals in another manner, we find that there are many numbers which are common to two of them or to all the three lines. As to the initial position, we shall take $+0.793$ for $\lambda 5,461$, $+0.844$ for $\lambda 4,359$, and $+0.708$ for $\lambda 4,047$. Tabulating the lines we find the following coincidences indicated by dots :—

5,461	4,359	4,047
	+410	
	+238	
0.....	0.....	0
	-278	
	-342.....	-333
-452		-418
-559		
	-599	
-636.....		-633
		-684
		-708
-727.....	-734.....	-733
-766		
-793.....	-797	
-822.....	-828	
-855.....	-859	
	-943	
-1,078.....	-1,088	-1,070
-1,224		
	-1,408	-1,423
-1,510		
	-1,673	
	-1,800	
	-2,020	
	-2,095	

These numbers are expressed in units 1,000 times smaller than those already used. The table shows that the three lines are not independent of each other, and the satellites are not arbitrarily distributed. It is a question why such coincidences are not found with reference to the principal lines.

Owing to the symmetry in the distribution of the satellites in $\lambda 4,359$, the coincidence above stated can be found in the same manner by taking the initial point at $-1,678$ in the above schedule. Most of the numbers are reproduced, so that by proceeding backwards, we meet with most of the numbers found for $\lambda 5,461$ and $\lambda 4,047$ already given.

§ 21. Turning our attention to the lines of the other series, we see that in the mercury line λ 5,769 (Fig. 7) there is a triplet about the principal with the wave-number difference -0.139 and $+0.147$. It is also the case with the line λ 4,078 (Fig. 8), in which the difference amounts to -0.283 and $+0.285$; this interval is nearly double that between the outer components in the triplet of λ 5,769. We notice that the interval between the two outer negative components of λ 4,078 is 0.146 , which is equal to the distance of the first positive component from the principal in λ 5,769. Moreover, the second positive component of the line is distant from the principal by 0.360 , which is nearly double the distance of the principal line λ 4,078 from the first negative component.

FIG. 7.

FIG. 8.

Further, we notice that the intervals between the three lines -0.137 , $+0.147$ and $+0.360$ in λ 5,769 being 0.284 and 0.213 are in the ratio 4:3. Obviously the components of λ 4,078, -0.183 , 0 , $+0.285$, $+0.466$ ($=0.285 + 0.181$) form a quartet. Further experiments on the principal line of 4,078 showed that it consists of three other satellites on the side of lower frequency, at a constant interval of about 0.039 , but as these components were difficult to observe with the L.G. plate and F.P. interferometer combination, the position is not indicated in the figure. The structure of the principal line λ 5,769 was difficult to analyse on account of the low sensitivity of photographic plate; some photograms showed that it is a close triplet.

§ 22. The structure of the mercury lines λ 3,663, 3,655, 3,650 belonging to the first subordinate series is shown in Fig. 9, and

is taken from the measurements of Mr. Takamine.* Here we notice the same constant difference in the wave-number.

For 3,663, interval between -0.678 and $+0.395=1.073$

„ 3,655 „ „ -0.546 „ $+0.518=1.064$

„ 3,650 „ „ -0.374 „ $+0.712=1.086$

Mean $=1.074$

This interval was already noticed in connection with the lines belonging to the second subordinate series, and seems to

FIG. 9.

be characteristic of most mercury lines. In the above-mentioned lines there is another interval which is common to them; the difference in wave number

For 3,663, between -0.678 and $0=0.678$

„ 3,655 „ -0.718 „ $0=0.718$

„ 3,655 „ -0.172 „ $+0.518=0.690$

„ 3,655 „ $+0.518$ „ $+1.222=0.704$

„ 3,655 „ 0 „ $+0.712=0.712$

Mean $=0.700$

The ratio of this number to another constant 1.07 is about 2:3. We meet with a similar number in 2 4,047, in which the

* Takamine, "Proc.," Tokyo Math. Phys. Soc., 8, p. 309, 1915.

outermost components have the mean value 0.711 which stands to 1.07 in the ratio 2:3 more accurately than for the three lines just discussed.

It is worth noticing that the intervals given by $l\frac{c}{3}$, $m\frac{c}{4}$, and $n\frac{c}{5}$, where $c=0.71$ and l, m, n are whole numbers, are found scattered among the satellites of the eight mercury lines above mentioned, for $l=1$ to 4, $m=1$ to 7, and $n=1$ to 7.

The second positive satellite of the line 3,663 is nearly three times as distant from the principal as from the first, and the

FIG. 10.

negative satellite is five times the same amount. The first positive satellite of the line 3,655 is distant from the principal by three times as much as the first negative, and the second negative satellite appears as the difference between the first and the third satellite, thus forming a quartet. The difference between the first and the second satellite on the positive side is 0.704 and is nearly equal to the distance of the first positive satellite in 3,650. The components -0.172 , $+0.518$, $+1.222$ form a triplet.

§ 23. According to the investigation of Mr. Takamine,* the structure of the cadmium line is simpler, and the regularity more apparent. The lines λ 5,086, λ 4,800, λ 4,678 belong to a triplet series, corresponding to the mercury lines first discussed. These lines have satellites as shown in the difference of wave-number in Fig. 10.

* Takamine, "Proc." Tokyo Math. Phys. Soc., 8, p. 51, 1915.

As already noticed, we meet with a constant interval in the wave-number of the satellites as follows :—

For 5,086, interval between -0.297 and $+0.097=0.394$

„ 4,800 „ „ -0.256 „ $+0.143=0.399$

„ 4,678 „ „ -0.144 „ $+0.260=0.401$

Mean $=0.398$

This number evidently corresponds to 1.07 and 0.71 already found for mercury lines. Further, we have to remark that for $\lambda 5,086$, the interval between -0.297 and 0 is equal to 3×0.099 , while the interval between the principal and the positive satellite is 0.097 ; similarly for $\lambda 4,800$ the interval between the

FIG. 11.

negative and the first positive component $=0.399$ is nearly double the interval 0.204 between the components. With $\lambda 4,678$, the distribution of the satellites is reverse to that for $\lambda 4,800$, for in place of -141 and $+260$ in the former, we find $+143$ and -256 in the latter. Thus the satellites of these three lines seem to be mutually dependent, as already noticed for the mercury lines. This singular fact would have been noticed by Mr. Takamine, if he had reduced his measurements to wave-numbers. It is customary to give the position of the satellites in wave-lengths, but from the theoretical point of view it is advisable to give the wave-number.

§ 24. Similar features are shown by the bismuth lines $\lambda 4,722$ and $\lambda 4,122$. They are shown in Fig. 11: the interval between the components -1.27 and $-1.07=0.20$ and that between -0.46 and $-0.25=0.21$, while that between -1.07 and $-0.46=0.61$, being three times the above amount. $\lambda 4,122$ consists of a strong doublet at an interval of 1.89 , having the

satellites at equal intervals of $+0.34$ and -0.34 from each of them ; it is also to be noticed that the interval between the doublets in the latter line is nearly three times that of the former.

§25. It would be premature to inquire into the cause of the regularity of the satellites mentioned in the preceding pages, as the data on which the discussion rests are as yet scanty, and the effects of the electric and magnetic fields on the behaviour of spectrum lines are not yet well known. The presence of magnetic and electric fields in atoms is a matter beyond doubt, and changes in interatomic fields will be felt in the vibrations of electrons. It is only recently that Stark has been able to establish definitely the effect of external electric field on emission lines, while that of the magnetic field, although well known for about 20 years, has not yet been satisfactorily explained, excepting that of the normal triplet, which forms only one of the numerous types of separation. If the said effect had been more accurately investigated, it would perhaps not be difficult to found a theory of the existence of the satellites, as the most plausible explanation seems to lie in the change of the quasi-elastic force giving rise to light vibrations, due to the variation of the interatomic fields. The theory of the effect of an electric field on the spectrum line has been discussed by Voigt,* and of the joint action of the electric and magnetic forces by Zeeman.† The investigation of positive rays by Sir J. J. Thomson has revealed the existence of atoms of mercury, which have lost one and sometimes eight electrons at one time. The change in the electric field of mercury atoms by the liberation of so many electrons will have sufficient influence on the radiation by producing an appreciable disturbance of the field, that the wave-length is slightly altered ; most probably the change thus caused in the electronic configuration will alter the atomic magnetic field to such a degree, that it will sensibly affect the radiation sent out by the mercury atoms. If the investigation begun by Thomson be extended to different elements, it will shed much light on the nature of the spectrum lines, but in the present stage of our knowledge it is rather difficult to advance a new hypothesis as regards the presence of satellites on the basis of the change of atomic electric and magnetic fields.

* Voigt, "Ann. der Phys.," 4, p. 197, 1901; "Magneto-u. Elektro-optik," 1908.

† Zeeman, "Akad. v. Wetens.," Amsterdam, May 27, 1911.

§ 26. In his theory of the vibrations of electrons in an isotropic quasi-elastic electric field, Voigt has shown, under the assumption of the existence of potential, which contains terms of higher order than that usually assumed in elasticity, that the change of frequency is proportional to the square of the field : observed transverse to the field, the original line is separated into two components of the same frequency, and parallel to the field there is a displacement three times as large as that of the line vibrating perpendicular to the field. Extending the analogy of the Zeeman effect in an electric field, Voigt has further shown that we shall have a quartet. In the application to mercury atoms it is natural to assume that the result will conform to the case in which the electrons are coupled, because the green line is divided into a nonet in a magnetic field, the violet line 24,359 into a sextet, and 24,047 into a triplet of double the normal separation, all of which can be explained from the standpoint of the coupled electrons. If we now assume that such coupling takes place in the electric field in the same way as for the magnetic, we can without much difficulty explain the regularity observed in some lines of mercury, if not the whole of it. In other cases, Zeeman's calculation, based on a simple Lorentz's hypothesis as regards the action of magnetic force, lends support to the explanation of the appearance of the satellites in cadmium and bismuth lines.

§ 27. The discussion of the effect of an electric or magnetic field indicates the nature of the polarisation to be observed if the direction of the field be fixed, but in case the seat of such fields lie in the radiating atoms, the polarisation observed becomes promiscuous and cannot be discriminated. With the green line (Fig. 2), taking -0.06 as the initial point, we notice that the zeros of the lines as arranged in the third and fourth row in the figure are distant from it by 0.03 and 0.09 , being in the ratio $1:3$ as expected on Voigt's theory. Starting from these lines, we have the lines ± 0.26 in the third row and ± 0.75 in the fourth, thus affording a solution in the light of coupled electrons. The last row consists of an unsymmetrical triplet, which can be caused by the joint action of the electric and magnetic forces as indicated by Zeeman. Another suggestion that we can make of these three symmetrical triplet rows in the diagram is that they are all due to the Zeeman effect, the magnetic field producing these separations being caused by the disturbance in the configuration of electrons in the mercury atom on the expulsion of electrons and their subsequent return

to it. But it seems to me that the former way of seeking the solution of the problem is more plausible than the latter; for there are different instances which favour the view that the satellites are due mostly to electric fields. The feeble satellites -0.137 and -0.395 in $\lambda 3,663$, $+0.172$ and $+0.546$ in $\lambda 3,655$, and $+0.104$ and $+0.299$ in $\lambda 3,650$ are all in the ratio 1:3 and conform to Voigt's simple theory. This is, however, only one aspect of the phenomena; as, for the line $\lambda 3,655$, we may consider the group of lines -0.718 , -0.546 , -0.172 , and 0 as forming a quartet having a constant interval of 0.172 at both ends; moreover, the lines -0.172 , $+0.518$ and $+1.222$ may be looked upon as a triplet with a mean interval of 0.697 ; as to the line $\lambda 3,650$, the said lines may form a group of gradually evanescent lines; thus various arrangements of lines may be considered as possible, for which different solutions can be suggested. If the said lines in $\lambda 3,655$ be considered as arranged in a quartet, then they may be due to the action of the electric and magnetic forces which are at right angles to each other. The satellites belonging to the bismuth lines $\lambda 4,722$ and $\lambda 4,122$ belong to the same type, as indicated in Fig. 11.

As to the line $\lambda 4,047$ of mercury (Fig. 6), the middle line may represent the position in the undisturbed state, while the other four lines may correspond to the state in which the coupled electrons vibrate in the electric field, or may be subject to the action of electric and magnetic forces. Near by the principal are two lines $+0.024$ and $+0.075$ in the ratio 1:3; they thus seem to belong to the lines conformable to Voigt's theory, but the four lines in the upper row are symmetrical about the principal and not about these two; whence it seems plausible that they are produced in the uncoupled condition. The presence of two symmetrical sextets in $\lambda 4,359$ (Fig. 5) is not easy to explain; on Zeeman's theory the presence of a dissymmetrical triplet is made possible, and when the presence of the line in zero field is considered as possible, there results a symmetrical quartet when the magnetic force is perpendicular to the electric. If the theory of the Zeeman effect be perfected, so that it can explain different modes of separation, such as a quartet, sextet or nonet, we shall be in a position to explain the distribution of the satellites in $\lambda 4,359$. If we exclude the outermost components, which are very faint, and not observed by most experimenters, we can consider the lines -0.834 -0.564 and $+0.566$, $+0.844$ as forming a symmetrical quartet, and -0.244 , -0.099 , and $+0.110$, $+0.245$ as

forming another quartet ; these may be supposed as due to the vibrations of coupled electrons in two different electric fields, which may arise during the liberation of electrons from the radiating atom or return to it. Similarly, if the outer satellites in the second row be not taken into account, the lines form a dissymmetrical triplet and may be due to the electric and magnetic forces. Good instances of a dissymmetrical triplet are found in the cadmium lines λ 5,086 and λ 4,678, while the mercury line λ 5,769 and the cadmium line λ 4,800 form dissymmetrical quartets ; singular enough is the fact that in the former the intervals among the satellites are in the ratio 3:4 and in the latter 1:2 ; this makes us consider the cause of these lines as due to some action or arrangement of electrons, which give rise to the above mentioned regularity.

I cannot as yet lay much importance on the theories above given ; they may be looked upon as having some significance in the theoretical aspect of the problem.

§ 28. When we consider the different causes which introduce the regularities above mentioned, we naturally fall back on the supposition that the primary cause is of electromagnetic origin. As stated before, the researches of Thomson seem to give some hints to the origin of such forces. As regards the mercury atom we know that it gives out a single electron or eight electrons at once ; the former is the usual mode of ionisation, and the emission of the principal lines will also be the consequence of the expulsion of the electrons giving rise to electric and magnetic disturbances. As to the latter way of ionisation, it is peculiar to mercury, and the disturbance arising from it will be of such an amount that the field in which the atomic constituents are placed will be subject to changes, by which the electrons emit intense electromagnetic waves. How often such expulsions of electrons take place compared with the number of expulsions of a single electron is not known, but most probably it will be less frequent, and though the emission may be intense, the total effect will not be so great as for the ordinary single electron emission. Owing to the violent change consequent upon the expulsion of several electrons, the electric field of the atom will be influenced by it, and the vibration of electrons modified to such a degree as to give rise to satellites. It will be at the moment of disruption that the electrons are subject to the most violent acceleration, and are in a state of giving out the most intense radiation. It therefore seems to me probable that the principal lines are due

to the consequence of the expulsion of an electron from the atom, and the satellites are due to the disruption of a ring of electrons. It may, however, be argued that on the gradual return of the electrons to the atom, there will be a new adjustment of the electron configuration in the atom, which calls forth radiation; this will inevitably be the case, but the disturbance caused by the return of the electrons to the atom will generally be small as compared with the expulsion of eight at one time, so that the result is not so great as to affect the photographic plate.

§ 29. The frequent occurrence of the common difference in the wave-number for lines of different wave-length calls for a new explanation. According to Bohr, the radiation emitted during a transition between two stationary states of an electron system in an atom is determined by

$$h\nu = A_1 - A_2,$$

where h is Planck's constant, ν the frequency, and A_1 and A_2 are the energies of the system in two stationary states. For a satellite of frequency $\nu + \delta\nu$, a similar equation must hold, so that we shall have

$$h\delta\nu = \delta A.$$

The above fact shows that for different values of ν , $\delta\nu$ has sometimes constant values, and consequently δA is common to these satellites. In other words, the energy difference between two stationary states which give out radiation corresponding to the satellites is constant. Assuming that the change in the stationary states of the emitting system is the effect of electric and magnetic fields, the energy possessed by the system in these states will be altered in such a way that the amount is still some multiple of a certain quantum, which is different from h .

On this hypothesis, the occurrence of such parts in the difference between the wave-numbers of satellites is a natural consequence. This is well exemplified in most of the lines discussed in the preceding pages; the numbers 1.07 and 0.70 found in the mercury lines, and 0.40 in the cadmium lines are remarkable instances of the existence of the quantity just discussed.

Judging from the fact that such common differences are characteristic of the lines belonging to the same series, it would appear that the lines belonging to the same series emit multiples of quanta, which are peculiar to them. If we suppose that the electrons are arranged in rings about a central nucleus,

and that they revolve in circular orbits, the above conclusion indicates that the stationary orbits do not vary continuously, but by jumps. This is necessarily due to the quantum change in energy, which will be emitted in the case of mercury atom, when an electron is liberated from it, or when eight electrons are expelled, or on subsequent return of electrons till the atom becomes ultimately electrically neutral. These considerations lead to the conclusion that the existence of numerous satellites in mercury lines is not at all singular, but is a natural consequence of the mode of ionisation peculiar to the element. If the experiment on the positive rays be extended to various elements and its consequence examined in the light of the theory here sketched, we shall perhaps be able to decide the question as regards the origin of the satellites. To account fully for the existence of the satellites and the regularity of their distribution, we are still in want of sufficiently accurate data as to the position of the satellites in order to test the consequences of the hypothesis which has been here advanced.

Note on the Structure of the Green Line of Mercury and the Terms of Correction in Using a Concave Grating.

The structure of the green line of mercury as revealed by Michelson's grating* ($9\frac{3}{4}$ in. $\times$ $2\frac{3}{4}$ in.) of 20 ft. focal length in the sixth order spectrum presents some points of discrepancy between that obtained by means of interferometers, echelon gratings, Lummer-Gehrcke plate, or Fabry-Perot apparatus. The principal difference is the obliteration of the strong satellites -19.9 and $+18.4$ m. $\text{\AA}$.U. from the principal line, which are fused into one line, although the resolving power of the grating would have been sufficient to separate them; the triplet structure of the principal would, however, have been difficult to resolve. This, I believe, is due to terms of correction, which, for gratings of such size account for the overlapping of the neighbouring lines.

Take the x -axis in the direction of the normal to the grating at its middle point, and y, z axes in the tangent plane at the point of the sphere, on which the lines parallel to the z -axis are ruled. Referred to these axes, let the co-ordinates of the source of light A be a, b, o ; of the point A' at which a spectrum of h th order is formed a', b', o ; and of a point P on the grating x, y, z . Then $AP^2 = (a^2 + b^2) - 2by - 2ax + x^2 + y^2 + z^2$;

* Michelson, "Proc." American Phil. Soc., 54, p. 137, 1915.

putting $r^2 = a^2 + b^2$, and introducing the condition that P lies on a sphere of radius ρ , we get

$$AP^2 = r^2 - 2by + \left(1 - \frac{a}{\rho}\right)x^2 + \left(1 - \frac{a}{\rho}\right)y^2 + \left(1 - \frac{a}{\rho}\right)z^2,$$

$$\text{and } A'P^2 = r'^2 - 2b'y + \left(1 - \frac{a'}{\rho}\right)x^2 + \left(1 - \frac{a'}{\rho}\right)y^2 + \left(1 - \frac{a'}{\rho}\right)z^2.$$

In the present problem, y may be considered as a small quantity of the first order, and x as of the second order, while in practice z is intermediate between x and y . Expanding in powers of y , we obtain

$$AP = r - \frac{b}{r}y - \frac{a}{2r}\left(\frac{a}{r^2} - \frac{1}{\rho}\right)y^2 - \frac{ab}{r^2}\left(\frac{a}{r^2} - \frac{1}{\rho}\right)y^3 + \dots$$

with similar expression for $A'P$. If we make the terms containing y^2 and y^3 evanescent in these two expressions, we arrive at the usual Rowland's arrangement $r^2 = a\rho$ and $r'^2 = a'\rho$, by which the points A and A' lie on a circle of half the radius of the sphere. When this condition is fulfilled,

$$1 - \frac{a}{\rho} = 1 - \frac{a^2}{r^2} = \frac{b^2}{r^2},$$

$$\text{and } AP^2 = r^2 \left\{ \left(1 - \frac{b}{r}y\right)^2 + \frac{b^2}{r^4}(x^2 + z^2) \right\},$$

which, on expansion, gives

$$AP = r - \frac{b}{r}\left(1 - \frac{b^2}{2r^4}z^2\right)y + \frac{1}{2}\frac{b^2}{r^3}(x^2 + z^2).$$

Putting the approximate value $x = \frac{ay^2}{2r^2}$, and denoting the angle of incidence by θ , and the angle of diffraction by θ' ,

$$AP = r - \sin\theta \left\{ 1 - \frac{\tan^2\theta}{2} \left(\frac{z}{\rho}\right)^2 \right\} y + \frac{\tan\theta \sin\theta}{2} \frac{z^2}{\rho} + \frac{1}{8} \tan\theta \sin\theta \frac{y^4}{\rho^3} \\ + \dots \text{ with similar expression for } P A'.$$

Confining our attention to a constant value of z , we get the equation of finite difference with respect to y .

$$\Delta(AP + P A') = \{ -(\sin\theta + \sin\theta') + \left(\frac{\tan^2\theta \sin\theta}{2} + \frac{\tan^2\theta' \sin\theta'}{2} \right) \left(\frac{z}{\rho}\right)^3 \\ + \left(\frac{\tan\theta \sin\theta}{2} + \frac{\tan\theta' \sin\theta'}{2} \right) \left(\frac{y}{\rho}\right)^3 \} \Delta y.$$

Let the successive intervals of the ruling along the chord be $\varepsilon = \Delta y$; then, for a spectrum of h 's order, we shall have

$$\varepsilon \left\{ (\sin \theta + \sin \theta') - \left(\frac{\tan^2 \theta \sin \theta}{2} + \frac{\tan^2 \theta' \sin \theta'}{2} \right) \left(\frac{z}{\rho} \right)^2 - \left(\frac{\tan \theta \sin \theta}{2} + \frac{\tan \theta' \sin \theta'}{2} \right) \left(\frac{z}{\rho} \right)^3 \right\} = \pm h \lambda.$$

According to the usual way of treating a grating spectrum, the corrections in the second and third terms are not taken into account, so long as they do not exceed $\pm \frac{\lambda}{4}$. If the line be not associated with satellites the assumption would be admissible, but in case the structure of the line is complex, the effect of these terms is to introduce admixture of different lines, and the feeble satellites are liable to be effaced; with strong satellites, the demarcation between the lines becomes indistinct, so that they are amalgamated into a wide line with vague boundaries.

Let λ_0 be the wave-length for $y=0$ $z=0$; then corresponding to the same angles, and for y and z , $\lambda = \lambda_0 + \delta\lambda$, where

$$\delta\lambda = \pm \frac{\varepsilon}{2h} \left\{ (\tan^2 \theta \sin \theta + \tan^2 \theta' \sin \theta') \left(\frac{z}{\rho} \right)^2 + (\tan \theta \sin \theta + \tan \theta' \sin \theta') \left(\frac{z}{\rho} \right)^3 \right\}.$$

The deviation from λ_0 increases very rapidly, as the portion of the grating at some distance from the middle is utilised, especially when the angle θ increases and the ruled space amounts to a few hundredth part of the focal length of the grating. The following tables, giving the corrections, at once show the importance of taking these terms into account.

Table of $\frac{\sin \theta \tan \theta}{2} \left(\frac{z}{\rho} \right)^2 \times 10^7$.

θ	$\frac{z}{\rho}$	1/500	1/400	1/300	1/250	1/200	1/150
10°		0	0	0	0	1	1
20°		1	1	3	4	6	10
30°		3	5	9	13	21	37
40°		9	14	25	36	57	101
50°		22	34	60	81	136	241
60°		52	81	144	208	324	577
70°		142	221	394	567	885	1,576
80°		634	991	1,760	2,534	3,977	7,039

Table of $\frac{\sin\theta \tan\theta}{2} \left(\frac{y}{\rho}\right)^3 \times 10^7$.

θ	$\frac{y}{\rho} =$	0.01	0.02	0.03	0.04	0.05
10°	0	1	4	10	19	
20°	1	5	17	40	78	
30°	1	12	39	92	180	
40°	3	22	73	173	336	
50°	5	37	123	292	570	
60°	8	60	202	480	937	
70°	13	103	348	830	1,615	
80°	28	223	754	1,786	3,491	

These tables show at once how the corrections arising from large values of y and z increase very steeply as the angle of incidence is increased. In addition to this, it must not be omitted that the presence of the term $\left(\frac{\tan\theta \sin\theta}{2} + \frac{\tan\theta' \sin\theta'}{2}\right) \frac{z^2}{\rho}$ in the expression for $AP + P.A'$ gives rise to the further intermixture of lines, if in practice we gather the light coming from different points of the ruled spacing.

An example may be cited for a grating of 20 in. focal length with 11,700 lines to the inch; for the green line of mercury λ 5,461 in the sixth order spectrum, a simple calculation shows that $\theta = \theta' = 48.2^\circ$, and for $y = 5$ in., $\delta\lambda = 24m.\text{\AA.U.}$, which is greater than the distance of the satellites lying close to the principal line; for $y = 10$ in., $\delta\lambda = 213m.\text{\AA.U.}$, which greatly surpasses the limit of error usually admitted in wave-length measurements.

It is therefore not at all surprising to find that in the photograph of the green line of mercury obtained with Michelson's 10 in. grating, the principal line is amalgamated with its neighbouring satellites into a single broad line, which far surpasses the other lines in intensity. The ambiguity in the position of the satellites arising from the overlapping of spectra of different orders usually observed with interferometers of high resolving power, and which can be discriminated by the method of crossed spectra, is entirely got rid of in spectra obtained with a concave grating. This is a great merit of the grating with large ruled spacing, and physicists are under great obligations to Michelson that he has brought his instrument to such perfection that no ghost can be detected, and the ruled space is so large that most of the satellites can be determined with accuracy and without ambiguity. What I want to indicate in this

note is that there are terms of correction, which make the resolving power of the instrument less than that calculated by the usual formula, and that large values of the angle of incidence and of diffraction in such a grating lead to considerable deviations.

These considerations lead to the conclusion that for the exact delineation of fine structure of lines, a concave grating with a large ruled spacing is not so effective as interferometers of high resolving power, due to the gradual intermixture of neighbouring lines. To remedy this defect, the only course that may be followed is to increase the radius of the sphere on which the lines are ruled. Another means would be to rule at smaller intervals, and thus have such a large number of lines within a narrow space as would make the terms of correction above discussed less disturbing in the result.

ABSTRACT.

The Paper describes a further development of the work done by the author and Mr. Takamine on the distribution of the satellites of the mercury lines. The apparatus employed was a Fabry-Perot interferometer crossed with an echelon or Lummer-Gehrke plate. An account of the theory governing the interpretation of crossed spectra is given.

It is shown that much of the discord between the results of various observers of these satellites is due to the unsatisfactory nature of the principal line as a datum from which to define their positions, and that if the distances be measured from one of the distinct satellites, good agreement is obtained.

If these separations be expressed as differences of wave-number, instead of wave-length, a remarkable symmetry in their distribution becomes apparent. For example, among the satellites of the green line, 5,461, can be found three groups of symmetrical triplets, of which the wave-number differences are in the simple ratios 1 : 3 : 12.

Similar results are obtained for other lines, the principal component of $\lambda 4,359$ being shown for the first time to consist of a triplet, of which the middle component is relatively weak.

A similarity in the distribution of the satellites exists for all the lines examined, and certain wave-number intervals are common to all.

It is observed that the intervals given by $l\frac{c}{3}$, $m\frac{c}{4}$, and $n\frac{c}{5}$, where $c=0.71$, and l , m , and n are whole numbers, are found among the satellites of the eight lines quoted for $l=1$ to 4, $m=1$ to 7, and $n=1$ to 7.

The possible origin of satellites is discussed in the light of the Zeeman and Stark effects, the magnetic or electric fields being electronic in origin, and many of the results are shown to be consistent with Voigt's theory of the vibrations of electrons in an isotropic, quasi elastic, electric field. The suggested explanations can only be tentative as yet, pending further development of our knowledge of the theory

of the Zeeman effect, and the extension of Sir J. J. Thomson's work on positive rays to a larger number of substances.

The second part of the Paper consists of a mathematical investigation of terms of correction in the theory of large sized concave gratings, which prevent the theoretical resolving power from being realised in practice.

DISCUSSION.

Dr. H. S. ALLEN thought the Paper contained many interesting and important points. That simple numerical relationships had been shown to hold for the separations of the satellites was likely to prove very helpful to those who were trying to solve the constitution of the atom. He was interested in the discussion of the part which might be played by the magnetic fields of atomic origin in these phenomena. He had himself held this idea for some time.

Dr. R. S. WILLOWS did not quite understand the statement made at the beginning of the Paper that there was, as yet, no observations of the effect of an electric field on the satellites, in view of the experiments of Stark, Lo Surdo and others.

Prof. NICHOLSON, in reply, said he presumed the statement was intended to refer to the mercury satellites only.