XIX.—*On the Expansion of Liquids.*

By D. MENDELEJEFF, Foreign Member of the Chemical Society.

(Translated from the Russian by B. BRAUNER.)

THOUGH every liquid has its own peculiar coefficient of expansion, it has long been a *desideratum* (as may be seen for example from abstracts published in the *Fortschritte der Physik* since 1845) to find a general expression for the expansion of all liquids. The generalisation which I now lay before the Society is connected with the discussion of this question which has already taken place, and with the additional experimental material which has been collected by chemists, chiefly for the purpose of studying the specific volumes of liquids at their boiling temperatures. The present paper is, however, limited to the discussion of the physical side of the question of the expansion of liquids. At some future time I may direct attention to the history of the subject, and to the consideration of the connection between the expansion of liquids, their composition, and other properties.

From all that we know about the expansion of liquids (Pierre, Kopp, Thorpe, Elsässer, and others) we can now easily see that the change of volume V with increase of temperature, t , takes place uniformly and regularly for liquids of different composition and properties, but the usual empirical formula

$$V = 1 + At + Bt^2 + Ct^3 + \dots \quad (I)$$

by which the connection between V and t is represented, does not allow a direct conclusion of this kind to be drawn. To make this clear, the adjoining table has been drawn up containing the data* for the expansion of the 47 liquids studied by Thorpe (*Chem. Soc. J.*, 1880, Trans., 141, 327); they are arranged in the order of increase of expansion, and show distinctly the regularity and quantitative uniformity in the expansion of liquids of the most different natures.

* For PBr_3 and AsF_3 Thorpe's experimental data were recalculated by me, and in the table are given numbers following from my calculations, whereas for all other bodies numbers are given, following from Thorpe's own interpolation (formula I). According to Thorpe's calculations, the volumes of PBr_3 at 10° , 30° , 60° , 100° , and $150^\circ = 1.0085$, 1.0258 , 1.0528 , 1.0914 , and 1.1448 . Their accordance with the formula given below ($K = 0.000841$) is still greater than in the case of numbers recalculated by me. Thorpe's data for PBr_3 agree very closely with those of Pierre, and this allows the greater confidence to be put in the numbers referring to this liquid.

	10°.	30°.	60°.	100°.	150°.
$\text{PCl}_6\text{H}_5\text{Cl}_2$	1·0083	1·0250	1·0507	1·0869	1·1368
PBr_3	1·0085	1·0259	1·0530	1·0916	1·1450
$\text{NC}_6\text{H}_5\text{H}_2$, aniline....	1·0087	1·0262	1·0535	1·0925	1·1473
$\text{SO}_2(\text{OH})\text{Cl}$	1·0091	1·0273	1·0552	1·0941	1·1465
$\text{C}_2\text{H}_5\text{Cl}$	1·0095	1·0285	1·0578	1·0994	1·1585
S_2Cl_2	1·0094	1·0285	1·0584	1·1016	1·1618
CHBr_3	1·0094	1·0287	1·0584	1·1012	—
ICl^*	1·0093	1·0283	1·0586	1·1027	—
$\text{C}_2\text{H}_4\text{ICl}$	1·0094	1·0286	1·0587	1·1023	—
$(\text{CH}_2\text{Br})_2$	1·0096	1·0293	1·0605	1·1061	—
$\text{S}_2\text{O}_2\text{Cl}_2$	1·0098	1·0299	1·0616	1·1074	—
CrO_2Cl_2	1·0097	1·0298	1·0618	1·1086	—
VOCl_3	1·0097	1·0298	1·0618	1·1087	—
TiCl_4	1·0099	1·0301	1·0619	1·1084	—
NC_6H_7 , picoline.....	1·0098	1·0299	1·0620	1·1097	—
POBrCl_2	1·0101	1·0307	1·0630	1·1098	—
AsCl_3	1·0100	1·0306	1·0631	1·1104	—
PSCl_3	1·0102	1·0310	1·0639	1·1121	—
$\text{C}_3\text{H}_5\text{OCl}$, epichlorhydrin.....	1·0103	1·0315	1·0653	1·1155	—
$\text{C}_3\text{H}_5\text{OH}$, allyl alcohol.....	1·0103	1·0314	1·0660	1·1204	—
$\text{P}(\text{C}_2\text{H}_5\text{O})\text{Cl}_2^\dagger$	1·0104	1·0324	1·0680	1·1204	—
$\text{C}_2\text{Cl}_3\text{OCl}$	1·0110	1·0332	1·0682	1·1201	—
CBrCl_3	1·0109	1·0332	1·0686	1·1206	—
POCl_3	1·0109	1·0333	1·0693	1·1232	—
$\text{C}(\text{NO}_2)\text{Cl}_3$	1·0111	1·0337	1·0694	1·1226	—
Br_2	1·0108	1·0335	1·0698	—	—
$\text{C}_2\text{Cl}_3\text{OH}$	1·0111	1·0347	1·0719	—	—
SnCl_4	1·0117	1·0356	1·0736	1·1302	—
$\text{CH}_2\text{ClCH}_2\text{Cl}$	1·0116	1·0356	1·0742	—	—
CS_2	1·0116	1·0360	—	—	—
$\text{C}_8\text{H}_{18}^\ddagger$	1·0119	1·0360	1·0745	1·1331	—
SOCl_2	1·0117	1·0360	1·0752	—	—
PCl_3	1·0117	1·0360	1·0755	—	—
$\text{C}_8\text{H}_{18}^\S$	1·0120	1·0368	1·0771	1·1401	—
CCl_4	1·0121	1·0372	1·0778	—	—
$\text{C}_7\text{H}_{16}^\parallel$	1·0122	1·0376	1·0792	1·1439	—
SO_2Cl_2	1·0124	1·0381	1·0797	—	—
$\text{C}_7\text{H}_{16}^\parallel$	1·0125	1·0386	1·0815	—	—
CHCl_3	1·0125	1·0387	1·0818	—	—
NC_3H_5 , propionitril....	1·0125	1·0388	1·0820	1·1479	—
CH_3CHCl_2	1·0130	1·0404	1·0855	—	—
CH_2Cl_2	1·0134	1·0417	—	—	—
SiCl_4	1·0136	1·0425	1·0904	—	—
AsF_3	1·0144	1·0433	1·0885	—	—
$\text{CO}(\text{CH}_3)_2$, acetone....	1·0138	1·0433	1·0920	—	—
$\text{C}_2\text{H}_3\text{OCl}$	1·0139	1·0433	—	—	—
$(\text{NO}_2)_2$	1·0157	—	—	—	—

* For a body like iodine monochloride, we can assume with De Heen, that it begins to dissociate on heating already when liquid, sooner than it distils.

† $\text{PCl}_3 + \text{C}_2\text{H}_5\text{OH} = \text{HCl} + \text{P}(\text{C}_2\text{H}_5\text{O})\text{Cl}_2$.

‡ Octane from methyl-hexyl-carbinol, boiling at $125\cdot46^\circ$, sp. gr. at $0^\circ = 0\cdot71883$.

§ Diisobutyl, boiling at $108\cdot5^\circ$, sp. gr. at $0^\circ = 0\cdot71110$.

|| From *Pinus sabiniana*, boiling at $98\cdot4^\circ$, sp. gr. at $0^\circ = 0\cdot70048$.

¶ Ethyl-amy, boiling at $90\cdot3^\circ$, sp. gr. at $0^\circ = 0\cdot69692$.

That there is a uniformity in the expansion of liquids may be inferred, not only from an arrangement of a similar kind, but also from the mutual comparison of the expansion of liquids. Compare for example the expansion of mercury with that of two liquids which are placed very far from each other in the table. We have, according to Thorpe, for phosphorus tribromide :

$t =$	10°	20°	30°	40°
$V =$	1.00847	1.01706	1.02577	1.03463

For mercury we have the following empirical expression, which was calculated by myself from Regnault's experimental data (*Russ. Chem. Phys. Soc. Jour.*, 1875) :

$$V = 1 + 0.0001801T + 0.00000002T^2.$$

According to this formula the same volumes as those given for PBr_3 correspond with the temperatures :

$T =$	46.8°	93.7°	140.9°	189.1°
and $T : t =$	4.68	4.68	4.69	4.72

In exactly the same way we have for nitrogen tetroxide :

$t =$	5°	10°	15°	20°
$V =$	1.00789	1.01573	1.02370	1.03196
$T =$	43.6°	86.5°	129.7°	174.6°
$T : t =$	8.72	8.65	8.65	8.70

From this follows that phosphorus tribromide expands 4.7 times, and nitrogen tetroxide 8.7 times more strongly than mercury, but in qualitative respect the expansion of these, as well as of other liquids, is uniform.

The uniformity in the expansion of liquids shown above may be represented by the formula :

$$V = (1 + \frac{k}{n} t)^n$$

which, according to Gay-Lussac's law, represents the uniformity in the expansion of gases. For gases, $n = +1$; for liquids we must assume $n = -1$.*

* The formula $V = eat$, which has been several times proposed as the expression of the expansion of liquids, though it answers pretty satisfactorily to the observed uniformity in the expansion of liquids, does not agree with the reality. Thus for example, I find for phosphorus tribromide, from Thorpe's data, that at $t = 40^\circ$, $a = 0.000851$; for $t = 80^\circ$, $a = 0.000866$; for $t = 120^\circ$, $a = 0.000885$; for $t = 160^\circ$, $a = 0.000907$; but this shows that the volume of the liquid increases more rapidly with increasing temperature than should follow from the formula $V = eat$.

The expression given above for liquids will therefore take the following form:—

$$V = (1 - kt)^{-1} = \frac{1}{1 - kt} \dots\dots\dots \text{II.}$$

But, as the specific gravities are inversely proportional to the volumes, it follows that if D represents the density at t° and D_0 that at 0° , we shall have:

$$D = D_0 (1 - kt) = D_0 - k D_0 t \dots\dots\dots \text{III.}$$

This requires the change of density with changing temperature, $\frac{dD}{dt}$ to be a constant $= -kD_0$.

In my recent investigation (*Russ. Chem. Phys. Soc. Jour.*, 1880—1883) of the products of distillation of Baku petroleum, I was often obliged to determine in different portions of the distillate the change of density with changing temperature, viz., $\frac{dD}{dt}$, and I found that it always remains very nearly constant* within wide limits of temperature, this being in accordance with III. I have further to remark that, as far back as 1860 (*Liebig's Annalen*, 114, 165), I found that the coefficient of change of density $\frac{dD}{dt}$ varies with changing composition, more simply than the coefficient of expansion of liquids with relation to volume, $\frac{dV}{dt}$. These circumstances have induced me to consider the expansion of all liquids in this sense, and the first result of this comparison is in accordance with the simple expression† of

If we expand this formula, we obtain the series $V = 1 + at + \frac{1}{2}a^2t^2 + \frac{1}{6}a^3t^3 + \dots$. As a rule, in the empirical formula $V = 1 + At + Bt^2 + Ct^3 + \dots$ the value $B > \frac{1}{2}A^2$ and $C > \frac{1}{6}A^3$.

* Miss E. K. Gutkowska has determined (1881) in my laboratory the specific gravity of American kerosine (petroleum), and found it: at $0^\circ = 0.8056$; at $16^\circ = 0.7940$; at $32^\circ = 0.7824$; at $48^\circ = 0.7708$; from which $\frac{dD}{dt} = -\frac{0.0116}{16} = -0.000725$.

For Baku petroleum, $D_0 = 0.8278$, and $\frac{dD}{dt} = -0.000760$. For Ragosine machine oil, No. 1, $D_0 = 0.9131$, and $\frac{dD}{dt} = -0.000633$. For the products of distillation of Baku petroleum, the coefficients of alteration of specific gravity kD_0 , were published by me in the *Russ. Phys. Chem. Soc. Jour.*, 1883, p. 189—192.

† I will omit, that already after the year 1840, attempts were made to find a general law for the expansion of liquids, and to express it with relation to the coefficient of alteration of specific gravity. If I am not mistaken, the first attempt was made by Wilhelmi, but I have tried in vain to find out this forgotten investigation. Meanwhile it appears that Groshans, in 1853, Watterson Potter, in 1863, and

the expansion of liquids according to formula II. The applicability of this formula may be shown by a few examples.

The expansion of phosphorus tribromide, according to Thorpe's data, is represented empirically as follows:—

$t =$	40°	60°	80°	100°	120°	140°
$V =$	1.0348	1.0530	1.0720	1.0916	1.1123	1.1340

and, assuming $k = 0.000341$, we obtain, according to formula II:

$V =$	1.0348	1.0531	1.0721	1.0918	1.1123	1.1335
-------	--------	--------	--------	--------	--------	--------

Let us take another instance from a number of highly expansible liquids (Thorpe) like silicon tetrachloride. For this Thorpe gives:

$t =$	10°	20°	30°	40°	50°
$V =$	1.0136	1.0278	1.0425	1.0579	1.0738

Assuming in formula II $k = 0.001360$, we obtain:

$V =$	1.0138	1.0280	1.0425	1.0575	1.0730
-------	--------	--------	--------	--------	--------

This and other examples show that formula II is in accordance with experiments up to ten thousandths of the volume; but in determining the expansion itself errors may be assumed to have occurred, especially as regards very small fractions of volume, as indeed may be seen, not so much by comparing together the investigations of one and the same author, but rather from a comparison of the data of two different authors with reference to one and the same liquid. Thus for example Kopp gives at 100° for aniline the volume 1.0915, and Thorpe 1.0925; for silicon tetrachloride Pierre gives at 50° (of the mercury thermometer) 1.0753, and Thorpe for 50° (of the air thermometer) 1.0738 (this reduced to mercury thermometer = 1.0735), &c.

If we assume for k , in formula II, the real values below, we have the following volumes:—

	10°.	30°.	60°.	100°.	150°.
$k = 0.0009$	1.0091	1.0277	1.0571	1.0989	1.1561
$k = 0.0010$	1.0101	1.0309	1.0638	1.1111	1.1765
$k = 0.0011$	1.0111	1.0341	1.0707	1.1236	1.1976

For the upper part of the foregoing table these numbers correspond with the expansion of real liquids according to Thorpe's investigations. So $k = 0.0010$ nearly represents the expansion of AsCl_3 and

others (see *Fortschritte der Physik*), proposed to express the expansion of liquids by a formula similar to II. Evidently, then, the subject was at that time of interest to scientific men, but it has in the meantime been forgotten, and renewed investigations are required to call attention to it.

of PSCl_3 . In all the above bodies investigated by Thorpe, the value of k varies from 0.00080 to 0.00155.

But just as for gases, the expression $V = 1 + kt$ applies only to a first approximation, or to a so-called ideal gas, so does the expression $V = (1 - kt)^{-1}$ apply for liquids only to a first approximation, namely, ideal liquids.* Real gases and liquids show deviations to the one side or to the other from the above-mentioned expressions of expansion. These deviations are not large in either case (they are of the same order as regards k and V), and, as may be expected, they are especially remarkable near temperatures at which the state of the bodies changes. Thus vapours, as is well known, exhibit at temperatures near the boiling points of their respective liquids rates of expansion different from those which belong to them at greater distances from the temperatures at which they are converted into liquids. In like manner liquids near their boiling temperatures show an apparent change of the coefficient of expansion.† Phosphorus tribromide, for example, for which $k = 0.000841$, exhibits near its boiling point (173°) a rate of expansion greater than that which is calculated from formula II. At 160° , for example, the volume 1.1562 is found, instead of 1.555, as calculated from II. The experimental data for liquids near their temperatures of solidification, are not numerous enough to enable us to say with confidence that near the temperatures at which they pass into the solid state, their coefficient of expansion is always changed; still cases are known in which such a change is insignificant. Thus it has recently been shown by Beilby (*Chem. Soc. J.*, 1883, Trans., 338), for paraffin melting at 38° , that liquid paraffin shows, from 38° to 60° , a perfectly constant change of specific gravity, namely, $\frac{dD}{dt} = -0.000727$. This constancy proves that in this case the expansion of paraffin already follows formula III.

In the case of actual liquids, the deviation from the ideal form of expansion, as far as can be judged from known cases—other conditions remaining constant—increases not only as the liquid approaches

* According to my opinion, an ideal liquid, as a basis of kinetic theory of liquids, is characterised in reference to the action of heat by two signs: firstly, for such a liquid, $V = (1 - kt)^{-1}$, or from this, $\frac{dV}{dt} = kV^2$, or $\frac{dD}{dt} = -kD_0$; and secondly, for it $a^2 = a_0^2 (1 - k_1 t)$ in which a^2 is the capillary constant at t° , a_0^2 at 0° , from which follows $\frac{da^2}{dt} = -a_0^2 k_1$. This last expression is founded upon experiments of Gay-Lussac, Brunner, Frankenheim, my own, De Heen, and of others.

† A subject for further experimental investigation would be the solution of the question, in what manner and how much the expansion of volatile liquids is changed at different pressures. I should like to submit this theoretically important question to an experimental solution (see my paper in *Liebig's Annalen*, 109, 1).

the point at which its state of aggregation is changed, but also with diminishing density, increasing cohesion (the value a^3), and diminishing molecular weight, or, what is the same, with diminishing vapour-density.* So in the case of two isomeric liquids (*e.g.*, aniline and picoline, or $\text{CH}_2\text{ClCH}_2\text{Cl}$ and CH_3CHCl_2 , from Thorpe's data), the molecular weight is the same, and the deviations (as well as the numeric value of k) increase with diminishing specific gravity. Exactly as water, amongst common liquids, possesses the smallest molecular weight, its freezing and boiling temperatures being at the same time very near each other, whereas the capillary constant which expresses its cohesion is very great, so, it seems to me, that by these circumstances the well-known deviations in the expansion of water are satisfactorily accounted for. About 50° the value k for water approaches 0.00047; about $25\text{--}30^\circ$ it is one and a half times smaller, whereas at 100° it is one and a half times greater than this value. No other liquid hitherto well studied shows an equally rapid change of expansion. Besides, I think, that the study of the expansion of aqueous solutions, and the development of Grassi's investigations (*Ann. Chim. Phys.* [3], **31**, 437) on the compressibility of water (it diminishes with increase of k , whereas in the case of other liquids it increases) and of aqueous solutions, will be able to throw some light upon the exceptional and singular deviations (from II and III) in expansions shown by water. As regards the deviations of real liquids from the ideal form of expansion, I would observe in the first place, that the value of the deviation is, as a rule, numerically small, and that it has a different sign for different liquids, similar for example to the deviations of real gases from the Boyle-Mariotte law.

So for mercury near 0° , $k = \frac{dV}{V^2 dt} = 0.000180$, at $100^\circ = 0.000178$, at $200^\circ = 0.000175$;† *i.e.*, the numerical value of k decreases with

* Such a dependence of the coefficient of expansion of gases from their molecular weight has been shown by me (*Compt. rend.*, February 14, 1876, and in my *Principles of Chemistry*, 4th edition, p. 156). As the coefficient of expansion of gases increases with increasing molecular weight, chlorine must have a large coefficient of expansion, and this should be taken into account in regard to the so-called dissociation of the chlorine molecule.

† It is known that the expansion of mercury determined from Regnault's experiments, shows many experimental inaccuracies, and this important point requires fresh investigation. If the expansion of mercury be calculated according to formula II, taking $k = 0.0001775$, then at $100^\circ \text{ V} = 1.0181$, instead of 1.0182, which number follows from the experiments, and for $300^\circ \text{ V} = 1.0562$, instead of 1.0559. I think, therefore, that by applying the method of hydrostatic weighing of platinum with a known coefficient of expansion, determined according to Fizeau's method, more exact data for the real expansion of mercury may be obtained than by using the method of two corresponding tubes, or of two barometers. A platinum areometer will, possibly, best answer the purpose.

increasing temperature. The same would be the case with arsenic trifluoride, according to Thorpe's researches upon that rare substance.

If k decreases, the difference of specific gravities $\frac{dD}{dt}$ will evidently decrease as regards its absolute value. The specific gravity of mercury is in reality at $0^\circ = 13.596$, at $100^\circ = 13.353$, at $200^\circ = 13.113$, at $300^\circ = 12.878$, and this gives kD_0 at $50^\circ = 0.00243$, at $150^\circ = 0.00240$, at $250^\circ = 0.00235$. The reverse kind of deviation (positive), namely, the increase of k with increasing temperature, is exhibited by the majority of other liquids, especially by liquid hydrocarbons rich in hydrogen and liquids of low boiling point and large coefficient k . As an instance I take the mean of Kopp's and Pierre's data for ether.

t	=	0°	10°	20°	30°
V	=	1.0000	1.0155	1.0317	1.0489
D	=	0.7362	0.7250	0.7136	0.7019
$\frac{dD}{dt} *$	=	-0.00111	-0.00113	-0.00115	-0.00118
k	=	0.00150	0.00153	0.00156	0.00159
$V=(1-0.00154t)^{-1}$	=	1.0000	1.0156	1.0318	1.0485

The last line, calculated on the assumption of the ideal state, *i.e.*, with a constant value of $k = 0.00154$, shows that the difference between calculation and experiment is small also in this case; for the experiments of Kopp differ much more widely from those of Pierre. Thus at 10° (reduced to the air thermometer) the volume is 1.0153 according to Kopp, and 1.0156 according to Pierre.

As another instance I take diisobutyl, C_8H_{18} . Thorpe's experiments give—

t	=	0°	20°	40°	60°	80°	100°
V	=	1.0000	1.0243	1.0498	1.0771	1.1070	1.1401

Here the value of k increases rapidly with increasing temperature, as is apparent from the mean value of k from 0° to t° , viz. :—

$$k = \frac{V-1}{t.V_2} = 0.001186 \quad 0.001186 \quad 0.001193 \quad 0.001208 \quad 0.001229$$

If in this case, in which the deviations are greater than in the majority of cases, the expression II is applied, we obtain—

$$V = (1-0.0012t)^{-1} = 1.0246 \quad 1.0504 \quad 1.0776 \quad 1.1062 \quad 1.1364$$

From this it is seen that here, especially near the boiling tempera-

* Limit of ratio between the rates of increase of D and of t .

ture, the deviations exceed the possible experimental errors,* which proves undoubtedly that small positive deviations from II exist, just as the expansion of real gases shows unmistakeable deviations from the expression $V = 1 + kt$.

As experiments prove the existence of some deviations from the ideal expansion on both sides, it is apparent that all other values for n , except $n = -1$ in the general formula $V = \left(1 + \frac{kt}{n}\right)^n$, will be less in accordance with reality and with the uniformity in the expansion of liquids than the first proposed approximation:—

$$V = \frac{1}{1-kt}.$$

The preceding results may be summed up as follows:—In the

* The determination of the absolute expansion of liquids involves, besides apparent and accidental experimental errors, especially the following ones: (1) the variation of the coefficient of expansion of the vessel with varying temperature is not taken into account (as seen distinctly from the experiments of Regnault, Recknagel, and others); (2) the determinations are, as a rule, carried on in uncovered long thermometer-like apparatus, and by this not only the possibility of diminution and change of part of the liquid occurs (this could still be taken into account), but a large column of the liquid reaches above the level of the bath, the correction for the temperature of which is very doubtful; even the low temperature of the upper parts of the vessel gives rise to currents within the liquid, and this does not permit to assume a definite volume at the conditions of full equilibrium of temperature; (3) no correction is made for pressure, and yet all liquids are compressible, and the pressure is different according to the different height of the column in the dilatometer and different stand of the barometer (all data should refer either to normal pressure or to pressure = 0); for this reason all data (especially for compressed gases which yield strongly compressible liquids) obtained in sealed tubes, in which the pressure varies greatly, prove not to correspond to reality; (4) according to a generally accepted custom all investigators take the mean of experiments, in order to derive from it the probable absolute expansion, though they may apparently involve single occasionally erroneous determinations. Even among the data of such an excellent investigator as Thorpe, one is taken into the calculation, namely, that of arsenic fluoride (*loc. cit.*, p. 352) for 12.5°, though it apparently involves an occasional error; (5) the indications of the mercurial thermometer are only rarely (Regnault, Thorpe) reduced to air thermometer; (6) little trouble is taken in order to keep away moisture, but its presence will lessen the expansion of the majority of the liquids, &c.

P. de Heen (*Bulletin de l'Acad. Royale des Sciences Bruxelles*, 1882, p. 528) assumes, on the basis of different hypotheses, another law of dilatation, namely, $V \left(\frac{1}{1-1.333 \dots at} \right)^{\frac{3}{2}}$, and explains all deviations from this law only by dissociation in the physical or in the chemical sense. But as the deviations for mercury from De Heen's formula are very appreciable, and tend to another side (the expansion is smaller than it should be according to the formula) than that required by dissociation, and as the majority of liquids of high boiling temperature exhibit a smaller expansion than that which is required by De Heen's formula, it cannot be regarded at all as an expression of reality.

expansion of liquids a peculiar regularity and a qualitative uniformity is observed, and the first approximation to reality is expressed by the equation:—

$$V = (1 - kt)^{-1}, \text{ or } D = D_0(1 - kt),$$

in which k is a constant coefficient, which characterises each separate liquid, like the specific gravity, boiling point, capillary constant (a^3), and other fundamental data of that kind. Now as isomeric bodies often show remarkable differences in the value of this coefficient k , as it changes in general with the composition of the body, as the compressibility depends upon pressure, and, as the cohesion, specific gravity, and boiling point (the most volatile bodies have a large k , the liquefied gases an especially large one), as well as molecular weight, appear to be connected with the value of this coefficient, so an investigation, directed to the determination of its value and its change under different conditions (with change of temperature, pressure, composition, and electrical state) promises to become very important for the mechanics of liquids, as it may be expected to clear up the complicated dependence of volume upon temperature, find a true general law of expansion of liquids, and throw light upon the apparent deviations from the first approximation shown above. This, however, is by itself sufficient in the majority of physico-chemical investigations of liquid bodies, just as Gay-Lussac's law, though not absolutely exact, is sufficient in the majority of physico-chemical work with gases.

This coefficient k may be called the *determinator* of expansion. If the densities D and D_1 at the temperatures t and t_1 are known, then

$$k = \frac{D - D_1}{Dt_1 - D_1t}. \text{ If experiment yields volumes } V \text{ and } V_1 \text{ at } t \text{ and } t_1,$$

$$\text{then } k = \frac{V_1 - V}{t_1V_1 - tV}.$$