22. On Simpson's Rule

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But $OP = OC \cos \frac{1}{2}(B - C),$
 $\therefore HP = \frac{a}{2} \cos \frac{1}{2}(B - C),$
 $\therefore \frac{1}{2}(b + c) \sin \frac{A}{2} = \frac{a}{2} \cos \frac{1}{2}(B - C).$

Also $\frac{\frac{1}{2}(b - c)}{\sin \frac{1}{2}(B - C)} = OC$
 $= \frac{\frac{1}{2}a}{\sin \frac{1}{2}(B + C)}.$

The figure is of wide and remarkable use in dealing with the various conic sections, and the trigonometrical use of it was suggested by one of Dr. Taylor's demonstrations. I hope to make it the subject of another note.

E. M. LANGLEY.

(ii.) *Proof.*— $\tan \overline{A + B} = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}.$

Draw angles A, B above and below FG.
 Make FG = 1, draw HGK perpendicular to it,

$$\therefore HG = \tan A, \quad GK = \tan B.$$

Draw HLM perpendicular to FK,

$$\begin{aligned} \therefore M = G = \text{rt. } L, \quad \therefore FMGH \text{ concyclic,} \\ \therefore LHG = B, \quad \therefore LG = \tan A \cdot \tan B, \\ \therefore FL = 1 - \tan A \cdot \tan B. \end{aligned}$$

Also right-angled triangles FML, HMK similar,

$$\therefore MH/FM = HK/FL,$$

$$\therefore \tan \overline{A + B} = (\tan A + \tan B)/(1 - \tan A \cdot \tan B).$$

J. H. HOOKER.

22. On Simpson's Rule.

I wish to call attention to the importance of Simpson's Rule in finding the volume of a solid. All the simple solids, such as the sphere, cone, pyramid, wedge, paraboloid of revolution, and their frusta, and also spheroids and ellipsoids, obey the rule accurately.

The rule is: If a solid is bounded by two parallel planes, the average cross-section parallel to these planes is obtained (either accurately or approximately) by adding together the two end sections and four times the parallel section mid-way between them, and dividing by 6. The volume is this average section multiplied by the distance between the end sections.

Take, for example, the sphere. The end sections are both zero, and the mid-section = πr^2 , therefore the average section = $\frac{1}{6}(0 + 0 + 4\pi r^2) = \frac{2}{3}\pi r^2$. Multiply this by $2r$, which is the height of the solid, and we obtain the volume $\frac{4}{3}\pi r^3$.

Similarly in all the other cases.

The volumes of all prismoids are found by the rule.

If the solid is an irregular one, as in the case of earth-work, the rule gives a good approximate value for the average section. In case of great irregularity, the solid must be divided up into several lengths by parallel planes, and the rule applied to each portion.

Another point is the importance of teaching students to verify their formulæ by testing their accuracy in simple cases.

For example, the average cross-section of a frustum of a

circular cone, whether right or oblique, is $\frac{\pi}{3}(a^2 + ab + b^2)$, where a, b are the radii of the two circular ends. This is obtained at once by Simpson's rule. It can be tested by applying it (1) to a whole cone, (2) to a thin slice, or, which is the same thing, by letting the frustum become a cylinder.

Again, the mid-section of a spherical frustum is

$$\frac{1}{2}(\pi a^2 + \pi b^2) + \frac{1}{4}\pi h^2.$$

The accuracy of this can be tested by taking (1) a whole sphere, (2) a thin slice. If desired also, a third test would be by taking a symmetrical frustum.

Similarly, the average section of a spherical frustum, viz.,

$$\frac{1}{2}(\pi a^2 + \pi b^2) + \frac{1}{6}\pi h^2$$

can be tested by the tests (1) and (2), and also by taking a hemisphere, whose average section is necessarily equal to that of a whole sphere.

In fact, the values of the coefficients in the expression

$$A(a^2 + b^2) + Bab + Ch^2$$

can be found, according as the expression is to represent the average section of a conical or pyramidal frustum, or of a spherical frustum, or the mid-section of either, by applying the above tests.

A third important point is that the students should, as far as possible, find the volumes of actual solids, by handling them and making the necessary measurements. In fact, mensuration should be taught, not only as a branch of mathematics but as an experimental science. In these days there is perhaps the less need to dwell on the importance of this point, as teachers are in all departments recognising the importance of experiments conducted by the pupils themselves. At the same time I wonder how many boys who know, for instance, that the area of a rectangle is obtained by multiplying its length and breadth, have actually measured the length and breadth of any rectangle, such as a table or desk lid, and then calculated its area. And perhaps still fewer have been set to work to find the volume of a solid. A cask is a very good example to which to apply Simpson's Rule.

In connection with the gauging of casks, the following rules are useful.

To find the number of gallons of water in an upright cylinder, multiply the depth of the water in yards by the square of the diameter of the cylinder in inches, and divide by 10. The result is the number of gallons to an accuracy of 2 per cent.

Further:—

If the operation is performed without dividing by 10, the result is the weight of water in lbs., since a gallon of water weighs 10 lbs.

Proof of 1st theorem:—Let depth = h yards, and diameter = d inches.

Then quantity of water in the cylinder

$$= \frac{\pi}{4} \left(\frac{d}{12} \right)^2 (3h) \text{ cubic feet}$$

$$= \frac{\pi}{192}(d^2h) \text{ cubic feet}$$

$$= \frac{\pi}{192} \times 6 \cdot 23(d^2h) \text{ gallons,}$$

which is about equal to $\frac{1}{10}\left(1 + \frac{1}{50}\right)(d^2h)$ gallons,

$\therefore$ quantity of water = $\frac{1}{10}\left(1 + \frac{1}{50}\right)d^2h$ gallons,

and weight of water = $\left(1 + \frac{1}{50}\right)d^2h$ lbs.

If the vessel is not a cylinder, the value of d^2 for the equivalent cylinder is given by Simpson's Rule, viz.

$$d^2 = \frac{1}{6}(d_1^2 + 4d_2^2 + d_3^2),$$

if d_1, d_3 are the end diameters, and d_2 the mid-diameter (measured in inches) of the vessel.

I hope in a subsequent number to give some applications of Simpson's Rule to plane areas.

ALFRED LODGE.

23. On (i.) division into classes, and (ii.) homogeneous products.

The formation of homogeneous products may be connected with the division of a number of given things into classes by the following method, in which dividing lines are regarded as things to be permuted with the rest.

(1) To find the number of ways of dividing n like things into r different classes, blank classes being admissible.

Place the n things in a line and then divide into r classes by $r - 1$ dividing lines. These lines may be placed in intervals between consecutive things, any number in one interval, or one or more of them may be placed at either end of the row. Thus, representing like things by dots, any order of eight dots and five dividing lines, as the following,

|| | . | | . . .

indicates one way of dividing eight like things into six different classes, the numbers in the classes in this case being respectively 0, 0, 4, 1, 0, 3.

In the general case, therefore, each possible division into classes is indicated by an order amongst the n like things and the $r - 1$ like dividing lines; and hence the number required is the same as the number of permutations of $n + r - 1$ things of which n are alike and also the remaining $r - 1$ alike: that is,

$$\frac{|n + r - 1|}{|n| |r - 1|}$$

(2) If blank classes are inadmissible, or more generally if there must be at least q in each class, then, after placing q in each class, this method can be applied to the remaining $n - rq$ things that must now be divided amongst the r classes. The number of ways is therefore

$$\frac{|n - rq + r - 1|}{|n - rq| |r - 1|}$$

(3) If some or all of the n things are different the method of section (1) can be used, but it must be noted that the result takes into account the order in each class.

Thus the number of ways of arranging n different things in r different classes is

$$\frac{|n + r - 1|}{|r - 1|}$$

and the number of ways of arranging a a a b b c in three classes is the same as the number of permutations of a a a b b c |, viz.

$$\frac{|8|}{|3| |2| |2|}$$

(4) The number of homogeneous products of r dimensions formed from n quantities is the same as the number of combinations that can be formed from n things when there are r things in each group, but repetitions *ad libitum* are allowed. Whichever view be taken, since each group is only counted once, whatever may be the order of its component things, we may suppose that there is a definite order according to which the things forming each group are to be arranged, or in other words a recognised order of precedence amongst the n given things. This being agreed upon, we may next suppose that we form any one of the groups required by taking a row of r vacant places and then filling each vacant place by one of the n things, repetitions being allowed. The order of precedence being followed in filling the vacant places, it will be sufficient to mark off by a dividing line those places into each of which the first thing is to be put, then by a second dividing line those places into each of which the second thing is to be put, and so on. The number of dividing lines required will be $n - 1$. Thus if we have to form groups of eight letters from the six a b c d e f, allowing repetitions, and if the order of precedence is alphabetical, then representing vacant places by dots, any order of eight dots and five lines indicates one of the groups required: e.g., taking the homogeneous product view in this case,

. . | . | . | . | . | . indicates $a^2 b c d e^2 f$,
 . . . | | | | indicates $a^3 f^5$,
 | | . . | | | indicates $c^2 e^6$,
 | | | | indicates a^8 .

In the general case therefore each possible group is indicated by an order amongst the r dots and the $n - 1$ dividing lines; and hence the number of groups or of homogeneous products is

$$\frac{|n + r - 1|}{|r| |n - 1|}$$

PERCY J. HARDING.

QUESTIONS FOR SOLUTION BY STUDENTS.

58. Construct the points whose pedal lines with respect to a given triangle pass through a given point, and show that there are in general three solutions.

W. C. FLETCHER.