XII. *Elementary Demonstration of the Composition of Pressures.* By THOMAS JACKSON, LL. D. F. R. S. EDIN.
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IT is well-known as a fundamental principle in statics, that
“ Two pressures, represented in direction and quantity by
“ two adjoining sides of a parallelogram, are equivalent to one
“ represented in direction and quantity by the diagonal
“ which passes through the point at which these sides meet.”
A demonstration of this proposition, that shall be at once sufficiently concise, and sufficiently elementary, to admit of its being with propriety introduced into a course of academical instruction, has been hitherto, so far as I know, a *desideratum*. The following may perhaps be found to possess that advantage.

LEMMA.

“ If the equivalent of two pressures, represented by the adjoining sides of a rectangle, given in species, be always represented in direction by the corresponding diagonal, it shall
“ be represented by the same in quantity.”

Let

Let $ABCD$ be a rectangle, and AC the diagonal passing through A ; draw EAF perpendicular to AC , and let fall the perpendiculars BF , BH , DG , DE : then shall $ABCD$, $AFBH$ and

$AEDG$ be similar rectangles; and if, in each of these, the equivalent of the pressures represented in direction and quantity by the sides, be represented in direction by the diagonal, it shall be represented by the same in quantity also. For if the forces AH and AF be equivalent to $m AB$; AE and AG shall be equivalent to $m AD$; and AB and AD to $m AC$; or $m AB$ and $m AD$ to $m^2 AC$: that is, the forces AH , AF , AE , and AG , will be equivalent to $m^2 AC$: But AE and AF are equal and opposite: hence the forces AH and AG are equivalent to $m^2 AC$. But AH and AG are equivalent to AC ; therefore $m = 1$.

FIG. 2

1. Now let $ABDC$ be any square; and let the sides AB and CD be produced indefinitely towards B and D ; draw the diagonal AD ; in CD produced, take DF equal to AD ; join AF ; take FH equal to AF ; join AH , and so on. And complete the rectangles $ACFE$, $ACHG$, &c.

It is obvious, that AD , AF , AH , &c. bisect the angles BAC , BAD , BAF , &c. respectively. Hence the resultant of
AB

AB and AC, which are equal, must be represented in direction by AD; and therefore by the same in quantity also. The equivalent of AE and AC being the same with that of BE and AD, which are equal, will be represented in direction, and therefore in quantity by AF, (*vid.* Lemma). Thus may the proposition be proved of any rectangle whose diagonal makes with one of the sides any angle found by the continued bisection of a right angle.

2. Let (a) be any angle in the series above mentioned; the proposition shall be true in relation to any rectangle whose diagonal forms with one of the sides an angle that is any multiple of (a) .

Let AB and BC be two sides of a rectangle whose diagonal AC makes with AB an angle in relation to which the proposition has been already proved; and let CAG be equal to (a) ; the proposition shall be true in relation to the angle BAG; for let GED be parallel

to AC, and draw the perpendiculars AD, EF, GH. It is already proved, that two forces represented by AD and AF are equivalent to the single force represented by AE; for $\angle DAE = \angle BAC$. AE may therefore be resolved into AD and AF; that is, the forces AE and AC are equivalent to the forces AD and AH, or to the single force AG. Since, then, AB and BC are equivalent to AC; and AC and CG equivalent to AG; AB and BG must be equivalent to AG.

3. Let

3. Let $\angle BAD$ be an angle incommensurable with a right angle. The proposition is true in relation to $\angle BAC$, the multiple of (a) next less than $\angle BAD$, and of $\angle BAE$, the multiple of (a) next higher, the difference between which $(= a)$ may be less than any

FIG. 4

assigned angle. But the equivalent of AB and BD must, in respect of direction, be always intermediate between the equivalent of AB , BC , and that of AB , BE . It must, therefore, pass through D ; and this is evidently true in the case of any similar rectangle; that is, so long as the angle $\angle BAD$ remains the same. Hence it must be represented by AD , also in quantity (*vid.* Lemma).

From what is said above in § 2. it is manifest that what has now been proved universally of the rectangle, may be extended to the oblique-angled parallelogram.