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THE PREPARATION OF THE TEACHER OF MATHEMATICS IN SECONDARY SCHOOLS.

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THE RHYTHM OF HISTORY.

Of all the fascinating phases of the culture studies none has more to commend itself either to our æsthetic instincts or to our practical needs than the rhythm of history. The world's tastes of yesterday, its ideals, its methods of business, are discarded today to seeming oblivion, only to reappear tomorrow. History pulsates; the graphic arts flourish, they languish, they flourish again, and so on in endless rhythm; the tastes of the schools in one generation are not those of the next, but in due time reappear on the crest of the wave; religious zeal seems to have conquered the world in one century, only to be followed by a period of spiritual depression, and so throughout the range of human interests. But these pulsations, these waves of the human soul, are by no means alike; the great law of evolution decrees that each succeeding one shall have its own characteristics, even as those caused by a pebble in a pond are of increasing breadth and circuit.

Now the relation of mathematics to natural science is subject to this same law. With Thales it was close, with Pythagoras it was less so, and with the disciples of the latter it was still less; but with Plato it turns, and with Aristotle it becomes close again; with Euclid it relaxes, and with Archimedes it strengthens; and so on until modern times. See, for example, the calculus, devised as a tool for physical research; in the next generation more abstract; in the time of Laplace and Lagrange again chiefly an aid to natural science; in the next generation a subject of investigation abstract from the scientific application; and although in the hands of Lord Kelvin exactly what it was in the hands of Newton, in the average

American college course of today a dull, meaningless mass of theory with few applications, with little interest, and of value far less than it might easily possess. Hence it seems to me a very hopeful sign that, even as in the days of Archimedes, of Roger Bacon, and of Newton, the general sciences drew close to mathematics, so in this association an intimate relationship should be recognized.

THE UNION OF MATHEMATICS AND SCIENCE.

And yet, even with this hopeful sign comes the ever present danger, the thought that the crest of the wave will be followed by a depression. For when science gives herself completely to the embrace of cold and calculating mathematics she never fails to repent, and in her repentance she again seeks her own; and when mathematics yields too much to the warming influence of science he gives of his own virility for the time and ceases to progress. When, however, each supports the other in perfect union, neither forgetting its own proper mission, then we have an offspring of ideas that grow strong and help the world to progress. It is with the desire to see such a union that I welcome the consideration of this topic by an association of science teachers.

WHAT MAKES A MATHEMATICIAN.

This relation of science to mathematics (and be it understood that the word "science" is used for brevity in its popular sense, mathematics being even more of a science than the "sciences")—this relation of science to mathematics leads to the remark that the idea of what constitutes a mathematician, like all ideas, has had its own rhythmic progress. If you will take a small map of India and place your thumb on Bombay and your forefinger on Calcutta, and then will close them slightly and move them a little towards the top of the map, they will rest on or near the two greatest mathematical centers of ancient India, the finger on Patna where Aryabhata lived and labored fourteen centuries ago, and gave us the oldest complete Hindu mathematical treatise that has come down to us; and the thumb on Ujjein, where a century and a half later Brahmagupta made a mathematical Mecca for his countrymen. This sage of Ujjein has given us a definition of a mathematician, and one which we may well consider in thinking of the making of a teacher; for, primitive though it be, it is not without its suggestiveness. Brahmagupta says: "He who distinctly and severally knows addition and the rest of the twenty logistics, and the eight determinations, including measurement by shadow, is a

mathematician." (I might pause to inquire, in view of this definition, if there is a mathematician here present.) Now what does this mean? Brahmagupta's twenty "logistics" were: our four fundamental operations, squares and cubes, with their respective roots, five rules in fractions, the rules of 3, 5, 7, 9, 11, and barter; his eight "determinations" were mixture, progression, plane figure, excavation, stack, saw, mound, and shadow,—to use his own phraseology. Now the details of all this will not interest us particularly, but the quaint language may catch our attention sufficiently to have us remember that a mathematician is not merely a man who knows the theories of the subject, but one who is able to use those theories in a practical way. And this has generally been the case with the world's greatest leaders in this field. Archimedes, for example, wanted higher mathematics that he might use it, and so he laid the foundations of the calculus nearly two thousand years before Leibnitz and Newton, building immediately upon the works of Kepler, Cavalieri, Fermat, and Descartes, created the science as we know it. And so these other men just named, and Euler, Laplace, Lagrange, Legendre, Monge, Gauss, Cauchy not to speak of Klein in our own day, men whom we look up to as among the world's greatest mathematicians, all knew the larger uses as well as the mere theories of this oldest of all sciences.

THE TEACHER VS. THE MATHEMATICIAN.

Now a teacher of mathematics stands somewhat apart from the mathematician himself. I know very few men of recognized standing as investigators in mathematics whom I would dream of recommending as teachers of the subject; neither do I know many who are today moulding the teaching of mathematics whom I would recommend for positions requiring original investigation in mathematics. And this is merely a corollary to a psychological proposition; if our brain cells have become so formed or so interrelated that we, at adult age, have interests abnormally along one line, it is exceedingly difficult for us to concentrate attention on another line. We express this in a homely way by telling the cobbler to stick to his last. So it is only fair to say that an ideal teacher, an exceedingly difficult person to find, while possessed of an all-round knowledge of mathematics and interested in seeing it develop, can not be expected greatly to assist this development by his own original discoveries. On the other hand, a man who is

doing really great work in mathematical investigation, and such men are also exceedingly difficult to find, while interested in seeing the subject well taught, can not be expected greatly to assist in advancing the educational thought by his own efforts. The great teacher and the great mathematician are equally rare; they are equally to be honored; but they are never combined in one individual. And between these extremes the law still holds; as a man leans to one side, his interests fail in the other.

DOUBTS AS TO PROFESSIONAL TRAINING.

Now this law is often denied, both by original investigators in education and by original investigators in mathematics. We have in this country a good many pretty fair teachers of mathematics who think that they are great mathematicians, and it may be well that they should think so, since it increases that all-round interest in the science which it is so necessary for them to possess; but in reality they are mere dust floating about in the gigantic machine. On the other hand we have a good many men who are contributing more or less to the progress of mathematics, and who think they are great teachers, when they are ignorant of even the primary principles of education, usually even doubting that such principles exist. The one who is interested primarily in teaching, sometimes displays his ignorance by openly expressing non-appreciation of the work that is being done in higher mathematics, and thus he shows a narrow mind that excites the pity of all who look fairly at the progress of the world. On the other hand, the one who is interested primarily in mathematical research sometimes displays his ignorance by openly expressing his disbelief of the work that is done in the professional training of a teacher, a training which would at one time, when his mind was receptive of something besides mere mathematical truth, have been of service to him. This antagonism to matters educational arises from the fact that there was for a long time a tradition among those who ranked themselves as scholars that the guild of teachers burned incense before some fetich called Method, without worshiping at the altar of true knowledge; that its members bowed before form without content, shadow without substance.

In the early days of normal schools, and long before the professional college was thought of, there may have been some justification for the charge. But with the rise of the standard normal school of today, with at least two years beyond a four-years' high-school course, usually combining advanced academic work

with the professional, and with the birth of higher professional colleges, and with the growth of special training in other professions, the tradition has lost its earlier dogmatic form. Two generations ago physicians often asserted that the knowledge required for their fraternity was best obtained in the office of a good practitioner, but today we require not only graduation from a good medical college, but even postgraduate work. The lawyer, the preacher, the nurse, the bookkeeper, and the teacher as well, all demand knowledge peculiar to their several professions, and in advance of that of their immediate predecessors.

GROWTH OF THE DEMAND FOR PROFESSIONALLY TRAINED TEACHERS.

These facts have been so generally recognized that there is a rapidly increasing demand for professionally trained teachers of mathematics, as of other subjects, in secondary schools and colleges. The time was when part of this demand was satisfied by graduates of academies and normal schools, but this time has passed. The smaller villages may still have to employ such teachers, and unless they will pay enough to get a good class of college graduates it is better that they should; but all of the more responsible positions now demand the college degree, and rightly so. In addition to this, a graduate professional year is coming to be required wherever it is possible to find candidates who have taken it, and the future will see the doctor's degree demanded, plus or together with this advanced professional training. Last year half a dozen of my graduate students secured positions in college faculties, a fair index of the value placed upon the professional work by the very institutions that recently were doubtful of the advantages of a course in education.

THE NATURE OF THE WORK OF MAKING A TEACHER OF MATHEMATICS.

This too long preliminary discussion brings me to the question as to the preparation demanded today for a satisfactory teacher of secondary mathematics. I say "today," because tomorrow will demand more, and I say "secondary," meaning to include high schools, normal schools, and the freshman year in college.

In answering the question, it is unnecessary to discuss the matter of personality. That is largely beyond the powers of the professional school. All that we who are training teachers can expect to do is this: other things being equal we can make what we sometimes hear called a "born teacher" much better able to do

good work, saving him from a multitude of blunders, and showing him the large questions which he should not allow to be obscured by a multitude of unimportant details.

First, what academic work does he need in mathematics? As a minimum he must have two years of college work, carrying him through the elements of the calculus. Any less than this would unfit him intelligently to handle either geometry or algebra. In addition to this my own students are practically required to take projective geometry because of its bearing upon the Euclidean science, and the advanced theory of equations because of the light which it throws on algebra. But most of my graduate students take much more than this, the theory of functions, differential equations, solid analytics, the theory of surfaces, and modern ideas in geometry being the courses from which selections are most frequently made. My recommendation is that every student shall take as much work as possible in pure mathematics, consistent with the other important demands upon his time. Of these demands one is that of applied mathematics. I am urging more and more strongly that our students should take courses in general applications (and we have recently established such a special course for teachers), in mechanics, and in astronomy. Any other plan would fail to give that balance between pure and applied mathematics which is necessary to the making of a teacher who would show the practical uses of the subjects which he treats.

Secondly, what professional courses does he need? In the first place he should know how mathematics has grown up. Ignorance of this fact would tend to make him unprogressive; he would look upon mathematics as a fixed science, undeveloping and undevelopable. This is the attitude of mind of the majority of teachers to-day. Against any proposed change they invoke the fetich of "mental discipline," believing, with an ignorance fostered by indolence of mind, that what has been good enough for the Past should be sufficient for the Present. Hence at least one year's work in the history of mathematics is demanded of all of my students, and a second year's work is advised, and I know of no better stimulus than this for the appreciation of a purely professional course. To know how elementary geometry has been crystallized in its present form, is almost a *sine qua non* to intelligently considering any question of the reform of the subject; to know the early geometric methods of solving equations is necessary if one would appreciate several improvements in teaching

which are now being advocated; and to know the early story of the calculus is equally important if we would rescue the subject from its present status. These are but a few illustrations of the innumerable lessons which history has for the teaching (not to speak of the developing) of the subject.

And finally, after the student knows the science, its history and its applications, at least one year of professional training in the teaching of mathematics is necessary. In this the successes and failures of the masters should be made known to him, encouraging him to do better work, and warning him against well-recognized dangers. He should get, what so few teachers have, and what mere academic instruction does not bring, a knowledge of a teacher's library, of the world's best textbooks, and of the literature which appeals to the pupil. There should also be brought before him the best methods of treating typical chapters, the possibility of improving the sequence, and the important question of the interrelationship of the branches of mathematics and of their contact with the other sciences and with the life about us. If we are to effect the reformation in algebra that has been effected in arithmetic, if we are to vitalize our applications of equations, if we are to modernize our geometry along the recent French lines, if we are to put trigonometry where it belongs and teach it as something to be used, and if we are to work out the various other problems of similar nature that are at this time pressing upon us for solution, it will be possible largely through the efforts and co-operation of that earnest body of workers which is constantly increasing by the advent of college graduates with a broad professional training. For such a body of workers the school has great need. Never has there been such a seething period in the educational world. The questions of the improvement of the teaching of mathematics are no longer local, no longer national; a reform in Italy is known at once in America, in New Zealand, and in that eastern Island Empire whose recent success in the field of battle and on the seas is paralleled by its advancement in education as well. To keep pace with other countries, to take what we can use and reject what is not suited to our needs, and to keep pace with the demands of science, of engineering, of commerce, of labor, and of finance—all this lays heavy demands upon the guild of mathematics teachers, and the proper training of apprentices who would enter that ancient and honorable order may well command the interest and the thoughtful co-operation of an association with the ideals this one so carefully guards.