XXXIV.—*Connection between the Atomic Weight of Contained Metals, and the Crystallographical Characters of Isomorphous Salts. The Volume and Optical Relationships of the Potassium, Rubidium, and Cæsium Salts of the Monoclinic Series of Double Sulphates, $R_2M(SO_4)_2 \cdot 6H_2O$.*

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IN a previous communication (Trans., 1893, **63**, 337), the results of a detailed investigation of the morphological characters of 22 double sulphates of this series were laid before the Society. The present memoir deals with the volumes and the optical properties of these same 22 salts, and thus completes their crystallographical history. The interesting nature of the morphological relationships described in the former communication rendered it very desirable that an investigation, of as thorough a nature, should be carried out concerning the physical properties of the salts. This series of isomorphous salts is undoubtedly the most important yet known to us among those crystallising in the systems of lesser symmetry, and no effort has been spared to render the work now presented as complete

and accurate as possible. Each salt has been studied in the same detailed manner as the normal sulphates of potassium, rubidium, and cæsium described to the Society in 1894 (*Trans.*, 1894, **65**, 628), and results of equal interest to those resulting from the latter investigation have followed.

The data hitherto existing concerning these salts have been of a character totally inadequate for the purposes of making important deductions concerning their relationships. The potassium salts of the series were studied in the year 1859 by Murmann and Rotter (*Sitz. Ber. der Kais. Akad. Wiss.*, **34**, 135), but the character of the work can only now be regarded as affording very rough approximations as regards the optic axial angles of the salts, and as very crude indeed with respect to the refractive indices, the β indices alone being usually determined, and in several cases the rough results given will be shown to be very inaccurate. Two of the potassium salts, those containing magnesium and iron, were included by Topsøe and Christiansen in their well-known investigation in 1874 (*Ann. Chim. Phys.*, 1874, [5], **1**, 1), but even in this work the optic axial angles are only given for sodium light, and the β refractive indices alone were directly determined; the indirectly determined α and γ indices will be shown to deviate considerably from the values now given, all of which were directly determined. The most important work which has hitherto been carried out on this series is that of Perrot (*Archiv. des Sciences phys. et nat. Genève*, 1891, 26, and 669, and 1893, 3), who has investigated the eight rubidium salts, and the potassium and cæsium salts containing zinc as the second metal. The refractive indices of these salts were determined by Perrot by means of Soret's total-reflection method, thus avoiding the necessity of grinding prisms, and it is very satisfactory to find that the values of the indices thus determined, evidently with great care, coincide almost exactly with the values now given for the same salts, as determined by means of accurately orientated prisms. Perrot has not determined the other optical properties with sufficient completeness. The optic axial angles were usually determined indirectly, with the introduction of the intermediate index of refraction of the crystal in the calculation, a method open to considerable error, and usually only for sodium light. The error in the stauroscopic determinations of the orientation of the axes of the optical ellipsoid is also larger than in the determinations now given. The densities, moreover, were determined by means of the hydrostatic balance, and frequently differ considerably from the numbers now presented, determined with much larger quantities of the salts by a pycnometer method conducted with extreme precautions. The point of Perrot's work was the refraction observations, as exemplifying the utility of Soret's method, and the accord between

those observations and the work now published is all that could be desired.

Seven of the eight caesium salts have never hitherto been subjected to investigation.

From the foregoing statement of previous work on the series, it will be evident that an exhaustive investigation of the whole 22 salts, of a most careful character, and on strictly comparable lines, was very desirable in order that our knowledge of this important series might be sufficiently accurate to enable general deductions of a trustworthy nature to be drawn as to the effect of the replacement of one metallic constituent by another.

METHOD OF WORK.

Volume Investigation.

The Densities.—The method adopted for determining the relative densities was the “pycnometer method with cap,” fully described in the communication concerning the simple normal sulphates (*loc. cit.*, 650), and due to Chapman Jones. The determinations were in every case conducted with the most transparent crystals, ground to the finest powder in an agate mortar, and sifted through fine platinum gauze. Except in those cases requiring special treatment, owing to efflorescence, which will be described in connection with the particular salts concerned, the powder was dried for a short time at the ordinary temperature between hard-pressed filtering paper, free from fluff, and then resifted several times, in order to remove the small quantity of moisture due to the minute inclusions of mother liquor. When it no longer adhered to a glass rod it was transferred to the pycnometer, and the determination proceeded with. Carbon tetrachloride was employed throughout as the liquid, and its specific gravity was frequently verified during the determinations. Considerable quantities, from 2 to 5 grams, of the salts were employed in each determination, and they were taken from different crops, belonging to two distinct preparations. Four determinations were made for each salt. The rubidium and caesium sulphates employed in the preparations were largely derived from the stock obtained at the commencement of this work, analyses of which were given in the communication concerning the morphology of the series (p. 340); it was found necessary to supplement it, however, by an additional quantity supplied by Merck, of the same high order of purity as the last, and analyses and spectroscopic examination of which yielded equally satisfactory results.

All the densities were determined at 20°, and the results are compared with water at 4°. Hence they are all strictly comparable.

Distance Ratios (Topic Axes of Muthmann).—The formulæ employed in calculating these ratios are those referred to in the memoir concerning the simple alkaline sulphates (p. 658), as having been suggested to the author by Professor Becke, of Prague, immediately after the appearance of the memoir concerning the morphology of the double sulphates; they are Muthmann's more recent topic axes for rhombic symmetry, modified for monoclinic symmetry by the introduction of the sine of the monoclinic axial angle β . The actual meaning of the three ratios, χ , ψ , and w , is, that they represent the relative distances apart of the centres of the structural units, or groups of units if the structure consists of more than one kind of unit together corresponding to the constitutional formula, along the directions of the three morphological axes. The formulæ are as follows.

$$\chi = \sqrt[3]{\frac{a^2V}{c \sin \beta}}; \psi = \sqrt[3]{\frac{V}{ac \sin \beta}}; w = \sqrt[3]{\frac{c^2V}{a \sin \beta}};$$

where V is the molecular volume, and a , b , and c are the ordinary morphological axial ratios.

Optical Investigation.

The optical investigation of a large number of salts of a low order of symmetry, in accordance with the exhaustive scheme contemplated, involving the preparation of some hundreds of accurately orientated section-plates and prisms and their examination in monochromatic light, was a task of some magnitude, and it was the recognition of this fact which led the author to devise and cause to be constructed the instrument for grinding accurately orientated section-plates and prisms, and the monochromatic illuminator, which were described to the Royal Society two years ago (*Phil. Trans.*, 1894, A, 887 and 913). Without the former instrument, it is doubtful whether this work would ever have been completed, it would certainly never have been endowed with the same accuracy. About 200 section-plates have been prepared by means of it during the progress of this work, at least two for each salt parallel to the symmetry plane for use in the determination of the directions of the principal planes of the optical ellipsoid, three perpendicular to the first median line of the optic axes for the determinations of $2E$, the apparent optic axial angle in air, and $2Ha$, the apparent acute angle in an immersion liquid, and three perpendicular to the second median line for the measurement of $2Ho$, the apparent obtuse angle in the immersion liquid. In all cases, the highest accuracy of orientation was ascertained to have been attained, although frequently a complicated matter owing to awkward relations between the morphological faces and the direc-

tions of the median lines, due to the monoclinic nature of the symmetry. The plates perpendicular to the first median lines invariably exhibited the interference figures symmetrical to the centre of the field of the polariscope. For the determinations of refractive index about $150\ 60^\circ$ prisms, at least six from each salt, were also ground, each of which was formed by two faces symmetrically arranged, that is, equally inclined, to one of the principal planes of the optical ellipsoid, and whose intersecting edge was parallel to one of the three axes of the ellipsoid. Hence each prism directly furnished two of the three refractive indices. Usually two such prisms, in complementary directions, were ground upon the same crystal, the pair of prisms thus furnishing all three indices, and one of them in duplicate. This is probably the first time that this has been achieved, certainly on such a scale, with crystals of symmetry as low as monoclinic. The new grinding instrument has thus proved invaluable, and has more than fulfilled all that was expected of it. Moreover, barely half a dozen crystals among so large a number have been broken during the grinding, and the labour spent on them lost.

The section-plates and prisms were employed whenever possible without cemented cover glasses; the polish furnished by the two polishing discs of the instrument, of very finely ground glass and of polished glass respectively, lubricated with brick oil, has in many cases been ample to permit of excellent observations being obtained with them directly. In the cases of the strongly coloured, and hence absorbent nickel and cobalt salts, the rapidly efflorescing cadmium salts, and occasionally, owing to special difficulties, with some of the crystals of the copper salts, it has been necessary to employ thin miniature cover glasses, cemented with a solution of hard balsam in benzene. The large number of observations taken, however, will have eliminated any slight error due to the cover glasses, and in cases where there has been any doubt the cover glasses have been removed, re-set, and the observations repeated.

All the observations of optic axial angle and refractive index have been carried out for several wave-lengths, five in the case of the former and six in the case of the latter. The new monochromatic illuminator has been employed throughout, the source of light being the lime-light, in order that the receiving and exit slits might be as fine as possible and the monochromatism of a high order. The saving of time and the convenience generally afforded by this instrument has been immense, and the powerful illumination given by it has been a very great aid to accuracy in the cases of the highly coloured salts, an ordinary lithium flame, for instance, being almost incapable of penetrating a thick plate or a prism of any of the green salts containing nickel as the second metal, whereas with the monochromatic illumi-

nator a comparatively bright interference figure or image of the spectrometer slit is afforded.

Wülfing (*Min. und Petrogr. Mitth.*, 1895, **15**, 49) has since described a somewhat similar spectroscopic monochromatic illuminator, of cheaper construction, and criticises the author's use of a ground glass screen to diffuse the light sufficiently to fill the field of the observing instrument, on account of its weakening of the light. Instead, he arranges the observing instrument, the polariscopical goniometer, at such a distance from the exit slit that the bundle of rays, emerging from the latter, when it reaches the objective of the polariscope has attained the same diameter as that objective; he then introduces a lens whose focal length is equal to the distance from the slit, and thus fills the objective with parallel rays. The author of the present communication, long ago, before the instrument was described to the Royal Society, tried this method of diffusion, but discarded it for the ground glass diffuser, for the reason that although the illumination was slightly superior it was by no means so satisfactory. For unless the eye was maintained exactly in one position in front of the polariscope, it was found that the illumination became uneven, and dark patches were seen at the sides, very disturbing during the observation of interference figures, and the illumination even disappeared altogether. Doubtless a cap with a small central hole in front of the analyser might obviate this, but the evenness of the field is so perfect when ground glass is employed, and remains so, however much the eye moves about, that the author prefers it infinitely, since it renders the whole aperture of the nicol available, and the interference figures clear and constant. As regards the weakening of the intensity of the light, that depends largely on the texture of the glass diffuser.

As stated in the memoir (*loc. cit.*), describing the monochromatic illuminator, two screens are used. One is so finely ground that objects behind it can be seen, slightly blurred, through it; this is used for refractive index work with the Fuess spectrometers, Nos. 1a and 2a, and gives beautifully even images of the Websky slit. The other is only slightly coarser, being of the texture of the finest camera focussing screens; this is employed for optic axial angle work with the polariscopical goniometer, and the diffusion is just sufficient, and no more than is required, to fill the field of the polariscope. The intensity of the illumination is very little less than when the lens is employed, and this is not noticed when the lime-light or an electric arc is available, one or other of which the author always employs in work of the character of the present investigation, in order to be able to use slits of very fine opening, so as to obtain very nearly perfect monochromatism. The whole apparatus has worked admir-

ably during the course of this work, and in the cases of salts exhibiting extraordinary interference phenomena, such as the beautiful example of caesium magnesium sulphate, to be described in the course of this communication, and that of rubidium sulphate, previously described, it has proved simply invaluable.

Orientation of Axes of Optical Ellipsoid.—The determinations of the extinction angles in the symmetry plane were made with a new Fuess stauroscopec fitted with a Calderon double calcite plate. The extinctions could in most cases be determined to 5'. The sections parallel to the symmetry plane were always made from well-developed crystals, so that the portions of the faces left on the edges of the section afforded good images of the goniometer signal. The basal plane face $c(001)$ was usually taken as reference face in determining the position of the section when cemented upon one of the rectangular glass plates given with the Fuess instrument, and the crystal was arranged so that this face was as nearly as possible parallel to the marked reflecting edge of the glass plate. The small correction angle was measured on the excellent goniometer of the grinding instrument as soon as the cementing film of balsam solution had sufficiently set. Observations were made with the crystal above the plate, and then with the plate inverted and the crystal below it, in order to eliminate the error of coincidence between the direction of the nicols and the zero graduation of the circle. The dispersion of the median lines, that is, of the direction of the axes of the optical ellipsoid lying in the symmetry plane, is so small throughout the series as to be scarcely greater than the error in the determination of the extinction angles. Hence it is ample to conduct the observations for sodium light, and in general precisely the same results were arrived at by the use of white light.

The orientations are expressed with reference to the morphological axes a and c , and also, for the sake of comparison with the work of other observers, with respect to the normals to the basal plane $c\{001\}$ and the orthopinacoid $a\{100\}$.

Optic Axial Angles.—The section plates for the determination of the optic axial angles required to be 1 or 2 mm. thick, and thicker still for most of the caesium salts, on account of the very feeble double refraction which is characteristic of the series. The measurements were made with the delicate Fuess larger axial angle apparatus. Monobromonaphthalene was usually employed as the immersion liquid in the determinations of $2H_a$ and $2H_o$. The pairs of sections, perpendicular to the first and second median lines respectively, which were employed together to furnish the data for the calculation of the true angle $2V_a$ by means of the formula $\tan V_a = \frac{\sin H_a}{\sin H_o}$ are numbered in the tables of results 1 and $1a$, 2 and $2a$, or 3 and $3a$.

The two series of measurements with each pair were carried out immediately in succession in the same cell of liquid, in order that no change might in the interval occur in the refractive index of the latter. The order of dispersion exhibited by the calculated results was in every case confirmed by observation of the interference figures in white light, and actual measurement in monochromatic light, when the sections perpendicular to the first median line were immersed in a liquid of about the same refractive index as the mean of the crystal indices. The approximate true angle was then being observed, and the indications were of very considerable value, as the dispersion of the optic axes for different wave-lengths of light is usually very small in this series, so much so sometimes as to approach the few minutes of error in the calculation of the true angles.

Dispersion of the Median Lines.—The stauroscope affords no trustworthy method of determining such small dispersions, of the axes of the optical ellipsoid lying in the symmetry plane, as are presented in this series. They have been determined in every case by observations in white light, and measurements in monochromatic light, while the section was immersed in a liquid of almost exactly the same refractive index as the mean index of the crystal, or successively in two liquids slightly differing from that mean, but in opposite directions. The orientation of the crystal section was first carefully noted with reference to the existing faces left on the broad edges of the section; it may be here mentioned that the grinding instrument does its work so cleanly that these faces are rarely damaged. The circle readings are then carefully noted, while each axial brush in turn is brought between the pair of cross wires for each of the five wave-lengths in succession, or at least the two extreme ones, and thus the angular dispersion of each optic axis determined. From this, of course, the position of the median line for the two extreme wave-lengths can be deduced, and, knowing the position of the faces of the crystal, the actual dispersion within the crystal ascertained. Very great care, however, is requisite in the choice of liquids in the cases of crystals exhibiting such small dispersions, rarely more than 10—15' between red light of the wave-length of the lithium line and greenish-blue light of the wave-length of the F hydrogen line, as it frequently happens that the indications in two such liquids, differing not very greatly in refractive index, exhibit considerable deviations from each other. This occurs mainly, however, in connection with the dispersion of the two axes, the dispersion of the median lines is usually the same in both.

Refractive Indices.—The refractive indices were determined by means of a new No. 1a Fuess goniometer, fitted with all the most recent improvements. The author desires to express his sincere thanks to the Government Grant Committee of the Royal Society for

their considerable grant for the purchase of this beautiful instrument, the most perfect and accurate spectrometer for use with miniature prisms procurable. The monochromatic illuminator works admirably in connection with it. A side window in the screen lined with dark velvet which is employed to shield the spectrometer from extraneous light, fitted with an easily moving shutter, and with an incandescent gas lamp placed behind it, supplied with a metal cylindrical chimney pierced by a circular aperture, 1 inch in diameter, opposite the light, serves to illuminate the circle when the shutter is opened. It is convenient to have the lamp adjustable for height on a standard rod supported by a heavy base. It can then be used as source of light during the measurement of the angle of the prism, and the subsequent approximate adjustment of the prism for minimum deviation. It is convenient to have the spectrometer mounted on a special stand whose four legs terminate in ball castors; the instrument can then be readily moved so as to bring the illumination tube of the collimator in front either of the gas lamp or the monochromatic illuminator, and it is a particularly convenient arrangement in adjusting so as to get the maximum illumination of the image of the Websky slit refracted through the prism.

Two series of observations of the angles of minimum deviation were always taken, one on each side of the incident light, that is, with the light incident on each of the prism faces in turn, and the mean considered as the true deviation. The readings rarely differed by more than two minutes.

It will be understood that the symbols under the headings "Nature of light" in the tables, mean as follows:—Li, light from the monochromatic illuminator of the same wave-length as the red lithium line; C of the wave-length of the red hydrogen line C; Na of the wave-length of the yellow D sodium lines; Tl of the thallium green line; F of the greenish-blue hydrogen line F; and G of the violet hydrogen line near G.

A general expression for any wave-length has been calculated for the intermediate index of refraction β of each salt, corrected for a vacuum; the mean of the observed values of this index for C, Na, and Tl light, corrected for a vacuum by the addition of 0.0004, have been employed in each case in calculating the formula. These formulæ reproduce the observed values (+ 0.0004) perfectly as far as F; another term would be required to yield the G values accurately, but its use would hardly compensate for its complexity. The α and γ indices are also very closely expressed by the formulæ when the first constants are diminished and increased respectively by amounts which are given for each salt. The relations of these differences from the β values determine the sign of the double refraction.

The Axial Ratios of the Optical Ellipsoid.—The relative velocities of light vibration along the three axes of the optical ellipsoid have been calculated for each salt, as in the cases of the simple sulphates previously described, by taking the ratio of the reciprocals of the three indices for any one and the same wave-length (usually sodium light). In the cases of the simple sulphates of rhombic symmetry the order of the morphological axes was taken, as these axes were coincident with the axes of the optical ellipsoid; in the present cases of the monoclinic double sulphates only one of the axes of the optical ellipsoid is coincident with a morphological axis, namely, the intermediate axis $\mathfrak{b}$ with the symmetry axis b . The other two axes of the optical ellipsoid are only conditioned by the fact that they must lie in the symmetry plane and be at right angles to each other; their relations to the two morphological axes lying in this plane differ for each salt. It will be of just as much interest, however, to compare their ratios for the different salts; hence they have been calculated, and are arranged in the order $\mathfrak{a} : \mathfrak{b} : \mathfrak{c}$; the first being the maximum, and the third the minimum axis. Vibrations parallel to $\mathfrak{a}$ yield the minimum refractive index α , and those parallel to $\mathfrak{c}$ give γ . The value of $\mathfrak{b}$ is taken as unity.

$$\text{Hence} \quad \frac{\mathfrak{a}}{\mathfrak{b}} = \frac{\beta}{\alpha} \quad \text{and} \quad \frac{\mathfrak{c}}{\mathfrak{b}} = \frac{\beta}{\gamma}.$$

It will be observed that the term optical elasticity is not employed in this communication. The important generalisation of Fletcher (*Mineral. Mag.*, 1891, 278) that the whole of the optical characters of crystals may be completely expressed with reference to the geometrical characters of a surface which is in general an ellipsoid, the optical indicatrix, renders it no longer necessary to employ terms which refer to particular versions of the undulatory theory. The term "axes of optical elasticity" employed in the communication concerning the simple sulphates is now replaced by the expression "axes of the optical ellipsoid."

The fundamental fact of Fletcher's generalisation, which is now adopted as the basis of the treatment of the optical properties of crystals by Groth (1895 edition of *Physikalische Krystallographie*), is expressed in the original memoir as follows:—

"The characters of a ray of plane-polarised homogeneous light transmitted within a medium are indicated by geometrical characters at a corresponding point on an ellipsoid; the direction of the ray is that of a diameter intersecting perpendicularly the normal drawn to the ellipsoid at the corresponding point; the velocity is inversely proportional to the length of the normal intercepted by the ray; the plane of polarisation is perpendicular to the normal."

The optical velocity ellipsoid referred to in this communication,

whose axes are the reciprocals of the refractive indices, is the polar reciprocal of the optical indicatrix, the magnitude of the axes of the two ellipsoids being inverted, the maximum axis of the velocity ellipsoid being the minimum axis of the indicatrix, and *vice versa*. As regards the axes of the velocity ellipsoid, their respective velocities refer to the vibrations, not to the transmission, of light along those directions.

Molecular Optical Constants.—These constants, afforded by the formulæ of Lorenz and of Gladstone and Dale, have been calculated for each salt from the data afforded by the determinations of density and of refractive index. It must be remembered that the same remark applies to these constants as to the axial ratios of the optical ellipsoid; the directions within the crystals are only identical throughout the series in the case of the values for *b*, which is coincident with the symmetry axis; the directions of *a* and *c* differ somewhat in the various members of the series.

Observations of Optic Axial Angle and of Refraction at Higher Temperatures.—The salts of this series, containing as they do water of crystallisation, cannot naturally be heated to 100° or more without decomposition. Most of them, however, have been found to withstand a temperature of 70° sufficiently long before becoming opaque to enable measurements of the apparent optic axial angle in air and of refractive index to be carried out. The apparatus employed for the observation of the interference figures at 70° was the ordinary air bath arrangement, furnished with two thermometers, supplied by Fuess with the larger optic axial angle goniometer. For the determinations of refractive index at higher temperatures, the latest form of heating apparatus supplied with the No. 1a goniometer was employed; this admirable arrangement is described by Fuess in the *Neues Jahrbuch für Mineralogie*, 1890, **1**, 161. The whole of the salts could not be investigated as regards their refraction at higher temperatures by means of uncovered prisms, on account of deterioration of the ground surfaces. In most cases this difficulty was overcome by using prisms fitted with cemented cover-glasses, and when equilibrium had been established at the particular higher temperature, taking a series of alternate observations for sodium light of minimum deviation and the prism angle; it was generally found possible to obtain at least two pairs of constant readings before opacity supervened.

Analyses of all the salts employed in this investigation, of a satisfactory character, were given in the former communication concerning the morphology of the series, so that there can be no question as to their purity or identity.

SALTS CONTAINING MAGNESIUM.

Potassium Magnesium Sulphate, $K_2Mg(SO_4)_2 \cdot 6H_2O$.

The crystals of this salt *cleave* readily parallel to the orthodome $r'\{\bar{2}01\}$, as stated by Topsøe and Christiansen. The cleaved surfaces yield sharp single images of the goniometer slit, and the circle reading corresponding was exactly 180° from that of the image afforded by the remaining r' face, in each of the cases investigated.

Volume.

Relative Density.—Four determinations were made with four separate crops of clear, well-formed crystals, two being taken from each of two distinct preparations. Every individual crystal used to make up the quantity taken for each determination was verified by careful inspection of its faces with a lens, in order to avoid the possibility of any admixture with the sparingly soluble sulphate of potassium, which is apt to crystallise out if it is in the slightest excess. One preparation was that used for the goniometrical work described in the previous communication, and the other was an independent one similarly prepared by quantitative admixture of the pure component sulphates. The following results were obtained.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
3.3397	2.0274
2.8918	2.0272
4.2998	2.0279
1.8775	2.0282
Mean....	<u>2.0277</u>

Joule and Playfair obtained 2.053. Perrot obtained numbers varying from 2.018 to 2.075, affording as mean 2.04.

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{402}{2.028} = 198.23.$$

Distance Ratios (Topic Axes).—It was shown in the former communication concerning the morphology that the axial angle $\beta = 75^\circ 12'$ and

$$a : b : c = 0.7413 : 1 : 0.4993.$$

Combining these with the molecular volume, as described in the introduction, we obtain for the distance ratios—

$$\chi : \psi : \omega = 6.0881 : 8.2128 : 4.1006.$$

Optics.

The optical properties of potassium magnesium sulphate were partially studied by Murmann and Rotter (*loc. cit.*). The plane of

the optic axes is the symmetry plane $b\{010\}$, as is general throughout the series.

Orientation of Axes of Optical Ellipsoid.—The directions of extinction in the symmetry plane were determined on two polished section-plates ground parallel to $b\{010\}$. Extinction occurred in the two cases at $1^\circ 5'$ and $1^\circ 15'$ respectively from the direction of the normal to $c\{001\}$, and on that side of the latter nearest the vertical axis c of the crystal as indicated in the accompanying figure (Fig. 1). The mean value, $1^\circ 10'$, is taken to represent the direction of extinction. Murmann and Rotter give $1^\circ 0'$.

FIG. 1.

Examination of section-plates ground perpendicular respectively to this direction, and to the direction at right angles to it in the symmetry plane, show that this is the direction of the second median line, the direction at right angles to it in the symmetry plane being the first median line.

The *double refraction*, determined by means of a quarter undulation mica plate when examining in convergent polarised light the section-plate perpendicular to the first median line, and confirmed later by the relation of the three indices of refraction, is of positive sign, as stated by other observers. Hence the first median line is the minimum axis of the optical ellipsoid c , and the second median line the maximum a . The intermediate axis b is coincident with the symmetry axis b .

It was shown in the previous communication that the morphological axial angle ac is $75^\circ 12'$. Hence—

The first median line c makes an angle of $1^\circ 10'$ with the crystallographical axis a , and the second median line a makes an angle of $13^\circ 38'$ with the vertical axis c , both median lines lying in the obtuse angle of the axes a and c .

The first median line is also inclined $88^\circ 50'$ to the normal to $c\{001\}$, and $13^\circ 38'$ to the normal to $a\{100\}$. Murmann and Rotter, using their value of ac , gave $1^\circ 0'$, $13^\circ 55'$, and $89^\circ 0'$ for the corre-

sponding angles. Topsøe and Christiansen gave $0^{\circ} 30'$, $14^{\circ} 25'$, and $89^{\circ} 30'$.

In Fig. 1, and all the subsequent figures representing the orientations, the positions of the optic axes are indicated by small rings.

Optic Axial Angle.—Three pairs of section-plates were ground and polished accurately perpendicular to the first and second median lines. The plates obtained of the thickness stated in the introduction gave excellent definition. The dispersion is very small, almost nil for the apparent angle in air as seen in white light, one of the brushes being colourless and the other very nearly so, being faintly bordered with red outside and blue inside. The small amount of dispersion was clearly visible, however, when the monochromatic light apparatus was employed and the prism rotated so as to obtain light first from one end of the spectrum and then from the other. The accompanying tables give the results of the measurements of $2E$, the apparent angle in air, of $2Ha$, the apparent acute, and $2Ho$, the apparent obtuse angle in monobromonaphthalene, and of the calculations of the true angle $2Va$ therefrom.

Determination of apparent Angle in Air of Potassium Magnesium Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean $2E$.
Li.....	$72^{\circ} 51'$	$72^{\circ} 35'$	$72^{\circ} 43'$	$72^{\circ} 43'$
C.....	$72 52$	$72 36$	$72 44$	$72 44$
Na.....	$72 54$	$72 40$	$72 47$	$72 47$
Tl.....	$72 56$	$72 43$	$72 51$	$72 50$
F.....	$72 59$	$72 47$	$72 56$	$72 54$

Determination of true Optic Axial Angle of Potassium Magnesium Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of $2Ha$.	No. of section perp. 2nd median line.	Observed values of $2Ho$.	Calculated value of $2Va$.	Mean value of $2Va$.
Li.....	{ 1	$42^{\circ} 16'$	1a	$107^{\circ} 52'$	$48^{\circ} 5'$	$48^{\circ} 0'$
	{ 2	$42 13$	2a	$107 53$	$48 2$	
	{ 3	$42 10$	3a	$108 10$	$47 54$	
C.....	{ 1	$42 15$	1a	$107 50$	$48 4$	$47 59$
	{ 2	$42 9$	2a	$107 40$	$48 1$	
	{ 3	$42 5$	3a	$108 2$	$47 52$	
Na.....	{ 1	$42 2$	1a	$107 20$	$48 0$	$47 54$
	{ 2	$41 54$	2a	$107 4$	$47 56$	
	{ 3	$41 51$	3a	$107 34$	$47 46$	
Tl.....	{ 1	$41 46$	1a	$106 42$	$47 55$	$47 48$
	{ 2	$41 38$	2a	$106 30$	$47 50$	
	{ 3	$41 35$	3a	$107 0$	$47 39$	
F.....	{ 1	$41 29$	1a	$106 0$	$47 50$	$47 40$
	{ 2	$41 19$	2a	$105 50$	$47 43$	
	{ 3	$41 15$	3a	$106 39$	$47 26$	

The true angle was not determined directly by Murmann and Rotter, but was calculated from the acute angle in oil and the medium refractive index β ; the value thus obtained was $48^\circ 21'$, presumably for sodium light. As pointed out in the communication concerning the simple normal sulphates (*loc. cit.*, 671), such calculated values cannot have the value of direct observations with a complementary pair of sections. Murmann and Rotter state that the apparent angle in air is smaller for red than for blue, although no measurements for different colours were carried out, and that the true angle, judged from the observation in oil, is larger for red than for blue; this agrees completely with the above results. Topsøe and Christiansen give $47^\circ 59'$ as the angle for sodium light.

The order of dispersion of the optic axes was confirmed by an observation in turpentine, whose index of refraction is only very slightly greater than the mean index of the crystals of this salt. The angle observed for C light was $47^\circ 41'$, for sodium light $47^\circ 33'$, and for thallium light $47^\circ 26'$.

Dispersion of the Median Lines.—The observation of the interference figure exhibited in turpentine by a section perpendicular to the first median line affords a means of ascertaining the dispersion of the median lines. This dispersion is so slight that the stauroscope affords no certain evidence of it. The figure in turpentine exhibits the inclined dispersion very clearly. One of the hyperbolic brushes is almost free from colour when white light is employed, while the other is fringed with blue outside and red within, indicating the greater inclination of the axes for red, and that one of the axes remains practically stationary for all colours. A careful series of determinations with the aid of the purest monochromatic light showed a dispersion of this axis of only $3'$, whilst the other was dispersed $18'$, between C and Tl light. The median lines must therefore be dispersed between Li and F, in accordance with the mean values derived above for the true angle ($48^\circ 0'$ to $47^\circ 40'$), about $10'$. On observing carefully the orientation of the section, which retained the natural faces on its edges unimpaired by the grinding, it was found that the axis which showed coincidence for all colours was the one which passed between the normals to $a\{100\}$ and $r\{201\}$ (nearer to the former), while the axis which showed dispersion was the one emerging approximately normal to $r'\{201\}$.

Hence the first median line lies nearer to the crystallographic axis a by about $10'$ for red than it does for blue. This agrees with the observation of Murmann and Rotter, who record no measurement, however, of the amount of the dispersion.

Effect of Rise of Temperature on the Optic Axial Angle.—The apparent angle in air was measured at 80° on Section 2. It was found to

be $75^{\circ} 10'$ for sodium light, the dispersion remaining about the same. Hence the apparent optic axial angle in air increases about $2\frac{1}{2}^{\circ}$ for 60° rise of temperature. The section remained unimpaired at this temperature during the time occupied by the observation.

Refractive Indices.—The indices of refraction were determined by means of six prisms, two being ground upon each of three clear and well-formed crystals, the directions being such that each prism yielded two indices, as described in the foregoing description of the method of work. The results are as follows.

Determinations of Refractive Indices of Potassium Magnesium Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Mean refractive index.	Values of Topsøe and Chris- tiansen.
α Vibrations parallel 2nd median line α .	Li	1·4581	1·4581	1·4580	1·4581	—
	C	1·4585	1·4586	1·4585	1·4585	1·4582
	Na	1·4605	1·4607	1·4608	1·4607	1·4602
	Tl	1·4630	1·4631	1·4631	1·4631	—
	F	1·4655	1·4659	1·4659	1·4658	1·4649
	G	1·4698	—	1·4700	1·4699	—
β Vibrations parallel sym- metry axis β β .	Li	1·4602	1·4603	1·4604	1·4603	—
	C	1·4606	1·4609	1·4607	1·4607	1·4610
	Na	1·4628	1·4628	1·4631	1·4629	1·4633
	Tl	1·4652	1·4651	1·4653	1·4652	—
	F	1·4677	1·4679	1·4679	1·4678	1·4682
	G	1·4720	—	1·4720	1·4720	—
γ Vibrations parallel 1st median line γ .	Li	1·4729	1·4725	1·4728	1·4727	—
	C	1·4733	1·4729	1·4731	1·4731	1·4743
	Na	1·4759	1·4752	1·4754	1·4755	1·4768
	Tl	1·4780	1·4775	1·4778	1·4778	—
	F	1·4810	1·4808	1·4811	1·4810	1·4827
	G	1·4853	—	1·4852	1·4853	—

The intermediate index of refraction β (in a vacuum) is accurately expressed for any wave-length λ as far as F in the spectrum by the formula

$$\beta = 1.4508 + \frac{483 \ 699}{\lambda^2} - \frac{1 \ 727 \ 800 \ 000 \ 000}{\lambda^4} + \dots$$

The α indices are very closely reproduced if the constant 1·4508 is diminished by 0·0022, and the γ indices if the constant is increased by 0·0126, the closer approximation of the β values to those of α determining the positive nature of the double refraction.

The only indices of refraction given by Murmann and Rotter are four values of β , namely, for red (no specific wave-length) 1·468, for

yellow 1·470, for green 1·474, and for blue 1·476. These numbers are much too high, they are even higher than those of the rubidium salt, as determined both by the author and by Perrot, and it will be shown that throughout the series the indices of the rubidium salt are always higher than those of the potassium salt. The values given by Topsøe and Christiansen are shown in the last column in the table; only the β indices were determined directly, and the indirectly determined values of α and γ are seen to be considerably out of agreement with the directly determined indices now given.

Alteration of Refraction by Increase of Temperature.—A number of determinations were made with three of the prisms at 70°. Owing to the rapidity with which efflorescence occurs when the crystals of this series are warmed the refracted images of the slit rapidly deteriorate, so that the same reliance cannot be placed on the observations as at the ordinary temperature. Hence the complete series of numbers obtained for the various wave-lengths could not fairly be taken as evidence of minute changes of dispersion. But the whole series agree in indicating that each index of refraction is lowered about 0·0019 (the limits being 0·0015—0·0022) by a rise of 50° (from 18° to 70°) of temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—These ratios calculated from the reciprocals of the refractive indices for sodium light are as follows.

$$a : b : c = 1.0015 : 1 : 0.9915.$$

The ratios calculated from the values for other wave-lengths are practically identical with these.

Molecular Optical Constants.—The following are the refraction constants, calculated by means of the Lorenz formula, for the rays C and G.

Axis of optical ellipsoid	α .	β .	γ .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n$ {	C. 0·1348	0·1353	0·1385
	G. 0·1377	0·1382	0·1415
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$ {	C. 54·18	54·40	55·66
	G. 55·34	55·55	56·88

The dispersion constants are as under—

Specific dispersion, $n_g - n_c$	0·0029	0·0029	0·0030
Molecular dispersion, $m_g - m_c$	1·16	1·15	1·22

The following are the values of the molecular refraction calculated from the formula of Gladstone and Dale for the wave-length of the red hydrogen line C.

Molecular refraction, $\frac{n - 1}{d} M$	90·97	91·40	93·86
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As the refraction diminishes with rise of temperature, that is similarly to the density, all these values will remain sensibly constant for all temperatures at which the salt is capable of existing.

Rubidium Magnesium Sulphate, $\text{Rb}_2\text{Mg}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

There is an excellent *cleavage* parallel, $r'\{201\}$. In the cases examined the cleaved surfaces yielded single sharp images of the goniometer signal, at exactly 180° from the remaining r' face.

Volume.

Relative Density.—The following are the results of four determinations made with separate crops of well-formed crystals belonging to two different preparations. One of these preparations was that made for the goniometrical work previously published; the other was a later preparation, the rubidium sulphate employed for which was supplied by Merck, as stated in the introduction, and ascertained to be of equal purity to the earlier stock. It was, moreover, several times recrystallised.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
3.5051	2.3773
3.4360	2.3841
4.2037	2.3818
4.3690	2.3856

Mean 2.3822

Perrot obtained a mean of 2.41 at 15° , the individual determinations varying from 2.377 to 2.438 .

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{494.4}{2.382} = 207.56.$$

Distance Ratios.—It was previously shown that the axial angle β was $74^\circ 1'$, and that

$$a : b : c = 0.7400 : 1 : 0.4975.$$

Combining these with the molecular volume, the distance ratios obtained are

$$\chi : \psi : \omega = 6.1941 : 8.3705 : 4.1643.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The directions of extinction in the symmetry plane were determined staurosopically with three section-plates ground accurately parallel to the symmetry plane $b\{010\}$. The first two gave results differing so much from the direction found by Perrot that a third was ground upon an excellent crystal derived from the later preparation containing rubidium

sulphate from a different source. Perrot states in his 1890 memoir that the acute bisectrix makes an angle of about 92° , in front, with the normal to the face (001), and in the 1893 memoir he gives $91^\circ 30'$. Instead of extinction occurring at $1\frac{1}{2}^\circ$ or 2° from the basal plane, the author's crystals all indicated about 5° , as shown in Fig 2. The results were

Section 1	$5^\circ 5'$
Section 2	$5 \ 20$
Section 3	$5 \ 20$
Mean	$5 \ 15$

FIG. 2.

As in the case of the potassium salt, the axis nearly perpendicular to the basal plane is the second median line, and the one at right angles in the symmetry plane is the first median line. The *sign of double refraction* is positive, as in the potassium salt. Hence the first median line is again c , the second median line a , and the intermediate axis b is coincident with the morphological axis b . The axial angle ac , as given in the last communication, is $74^\circ 1'$. The orientation of the axes may therefore be stated as follows.

The first median line c lies in the acute angle of the morphological axes a and c , and makes an angle of $5^\circ 15'$ with the axis a .

The second median line a lies in the obtuse angle ac , and makes an angle of $21^\circ 14'$ with the vertical axis c .

The first median line is also inclined $95^\circ 15'$ to the normal to $c\{001\}$, and $21^\circ 14'$ to the normal to $a\{100\}$.

Optic Axial Angle.—The following measurements of the apparent angle in air were obtained with three section-plates ground accurately perpendicular to the first median line.

Determination of apparent Angle in Air of Rubidium Magnesium Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li.....	75° 5'	75° 15'	75° 25'	75° 15'
C.....	75 0	75 10	75 20	75 10
Na	74 35	74 35	74 50	74 40
Tl	74 10	74 0	74 20	74 10
F.....	73 30	73 0	73 30	73 20

Perrot gives 76° 40' for the apparent angle in red light.

The following are the measurements of the apparent acute angle in monobromonaphthalene, carried out with the same three sections, and of the apparent obtuse angle in the same liquid, carried out with three section-plates ground perpendicular to the second median line, together with the values of the true internal angle calculated therefrom.

Determination of true Optic Axial Angle of Rubidium Magnesium Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean value of 2Va.
Li.....	{ 1	43° 20'	1a	108° 5'	49° 2'	49° 6'
	{ 2	43 20	2a	107 35	49 10	
	{ 3	43 15	3a	107 35	49 5	
C.....	{ 1	43 15	1a	108 0	48 58	49 2
	{ 2	43 15	2a	107 30	49 6	
	{ 3	43 10	3a	107 30	49 2	
Na.....	{ 1	42 50	1a	107 30	48 44	48 46
	{ 2	42 45	2a	107 10	48 43	
	{ 3	42 50	3a	107 5	48 50	
Tl.....	{ 1	42 24	1a	106 50	48 30	48 29
	{ 2	42 25	2a	106 40	48 32	
	{ 3	42 15	3a	106 35	48 24	
F.....	{ 1	41 50	1a	106 0	48 10	48 10
	{ 2	41 50	2a	106 0	48 10	
	{ 3	41 50	3a	106 0	48 10	

Sections 1 and 1a were ground upon the same crystal, this being possible because of the very feeble double refraction, which necessitates thick sections in order to afford interference figures with small rings and well-defined brushes.

Perrot obtained 50° 2' for orange-red with one section, and 49° 30' for yellow light with another, with the aid of the knowledge of the mean refractive index of the substance.

Dispersion of the Median Lines.—The dispersion of the median lines is again too small to be measured with certainty by the stauroscope. It was determined by observation of the interference figures afforded by the sections perpendicular to the first median line in turpentine,

whose refractive index is very near the mean of the indices of the crystal. The fact that the optic axial angle for red is greater than that for blue was confirmed; one of the brushes was practically devoid of colour when observed in white light, whilst the other was coloured red on the inner side nearest the stationary axis, and blue outside. On measuring the angles for C and Tl light, the readings for the axis whose brush is colourless in white light were found to be identical, whilst those for the other axis differed by $16'$. The constant axis was, as in the case of the potassium salt, the one nearly normal to $a\{100\}$, the movable axis being approximately normal to $r'\{\bar{2}01\}$, and further removed from the median line for red than for blue.

Hence the first median line lies nearer to the crystallographic axis a for blue than it does for red, and the amount of dispersion between Li and F would appear from the table to be about $20'$.

Effect of Rise of Temperature on the Optic Axial Angle.—Several measurements of the apparent optical axial angle in air were made at 80° for sodium light, and all the values obtained were within a third of a degree of $74^\circ 40'$, the angle at the ordinary temperature. Hence any change of angle for 60° rise of temperature falls within the limits of error of observation.

Refractive Indices.—Six prisms were employed, three being ground on one and the same crystal, yielding one value of α , two of β , and

Determination of Refractive Indices of Rubidium Magnesium Sulphate.

Index.	Nature of light.	Crystal 1, 3 prisms.	Crystal 2, 1 prism.	Crystal 3, 1 prism.	Crystal 4, 1 prism.	Mean refrac- tive index.	Perrot's values.
α Vibs. par. 2nd me- dian line	Li	1·4648	—	1·4646	1·4645	1·4646	—
	C	1·4651	—	1·4650	1·4649	1·4650	1·4648
	Na	1·4672	—	1·4672	1·4671	1·4672	1·4670
	Tl	1·4696	—	1·4695	1·4695	1·4695	—
	F	1·4723	—	1·4724	1·4725	1·4724	1·4721
	G	1·4760	—	—	1·4764	1·4762	—
β Vibs. par. symmetry axis b ($\bar{b}$)	Li	1·4659	1·4666	1·4665	1·4664	1·4664	—
	C	1·4664	1·4670	1·4669	1·4668	1·4668	1·4667
	Na	1·4686	1·4689	1·4691	1·4688	1·4689	1·4690
	Tl	1·4708	1·4716	1·4714	1·4712	1·4713	—
	F	1·4739	1·4747	1·4743	1·4743	1·4743	1·4739
	G	1·4780	1·4785	—	1·4781	1·4782	—
γ Vibs. par. 1st me- dian line	Li	1·4753	1·4758	—	—	1·4755	—
	C	1·4757	1·4762	—	—	1·4759	1·4756
	Na	1·4779	1·4780	—	—	1·4779	1·4782
	Tl	1·4802	1·4808	—	—	1·4805	—
	F	1·4833	1·4837	—	—	1·4835	1·4833
	G	1·4874	1·4878	—	—	1·4876	—

three of γ , and the other three being ground upon three other separate crystals. The results are as follows.

The intermediate index β , reduced to a vacuum, is exactly expressed, for any wave-length λ as far as F, by the formula

$$\beta = 1.4592 + \frac{318\ 735}{\lambda^2} + \frac{1\ 105\ 200\ 000\ 000}{\lambda^4} + \dots$$

The α indices are also very closely reproduced by the formula if the constant 1.4592 is diminished by 0.0018, and the γ indices if it is increased by 0.0091, the β indices being thus nearest to the α values, in accordance with the positive sign of double refraction.

It will be observed that the indices given by Perrot agree very closely with the numbers now given.

Alteration of Refraction by Increase of Temperature.—Measurements with two prisms at 70° indicated that the indices are lowered by about 0.0015 for an increase of 50° of temperature, the dispersion remaining sensibly the same.

Ratio of Axes of Optical Velocity Ellipsoid.—The calculated values of these ratios are

$$a : b : c = 1.0012 : 1 : 0.9939.$$

Molecular Optical Constants.—These are as follows.

Axis of optical ellipsoid.....	α .	b .	c .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n$	{ C. 0.1161	0.1165	0.1185
	{ G. 0.1185	0.1190	0.1210
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$	{ C. 57.42	57.61	58.57
	{ G. 58.60	58.82	59.80
Specific dispersion, $n_G - n_C$	0.0024	0.0025	0.0025
Molecular dispersion, $m_G - m_C$	1.18	1.21	1.23
Molecular refraction, $\frac{n - 1}{d} M$	C 96.60	96.97	98.86

These values will hold good for all temperatures at which the salt is capable of existing, as the refraction and density are similarly affected by change of temperature.

Cæsium Magnesium Sulphate, Cs₂Mg(SO₄)₂·6H₂O.

Among the large number of crops of cæsium magnesium sulphate obtained for the purposes of the volume and optical work, several were observed to consist of crystals exhibiting well-developed faces of the orthodome $r(201) - 2P_{\infty}$, a form not included in the list given for this salt in the previous communication concerning the morphology of the series (*loc. cit.*, 350). There is a good *cleavage* parallel $r\{201\}$, the fractures yielding well-defined

single images of the goniometer signal at exactly 180° to the r' face remaining on the part of the crystal examined.

Volume.

Relative Density.—The following are the results of four determinations made with different crops of excellent clear crystals of this salt.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
4.4357	2.6694
4.1772	2.6728
4.5400	2.6688
3.0734	2.6706

Mean **2.6704**

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{589.4}{2.67} = 220.75.$$

Distance Ratios.—The axial angle β was previously shown to be $72^\circ 54'$, and

$$a : b : c = 0.7279 : 1 : 0.4946.$$

Combining these with the molecular volume, we obtain

$$\chi : \psi : \omega = 6.2778 : 8.6246 : 4.2657.$$

Optics.

Cæsium magnesium sulphate exhibits most interesting and exceptional optical properties, including the phenomenon of crossed axial plane dispersion, accompanied by great sensitiveness of the optic axial angle to change of wave-length in the illuminating light and to temperature, together with remarkable refraction phenomena. It is very similar indeed to the case of normal rubidium sulphate described in the communication on the normal sulphates (*loc. cit.*, 680), and the reason for these exceptional properties will be subsequently shown to be equally interesting and important when the three salts containing magnesium are compared.

Orientation of Axes of Optical Ellipsoid.—The directions of extinction in the symmetry plane were determined on two sections ground parallel to the symmetry plane $b\{010\}$. Extinction occurs much further removed, forwards and downwards in the upper front octant, from the normal to the basal plane than in the rubidium and potassium salts, as represented in Fig. 3. The two sections indicated respectively extinction angles of $29^\circ 40'$ and $29^\circ 35'$ with the normal to $c\{001\}$. The mean value $29^\circ 37'$ may be taken as representing the true extinction angle.

On proceeding to grind section-plates perpendicular to the directions of the two axes of the optical ellipsoid indicated by the extinc-

parallel to the lower tangent screw of the graduated adjusting apparatus. The zone of faces was then adjusted by use of the goniometrical telescope and collimator, exactly perpendicular to the grinding plane, and the face $c\{001\}$ exactly parallel to the lower tangent screw by adjustment of the rotatable cylinder of the crystal holder. Both these adjustments being perfected, the upper tangent screw, which was parallel to the normal to $c\{001\}$, was rotated for the extinction angle $29^{\circ} 37'$ in the direction required by the foregoing data to bring the first median line perpendicular to the grinding table. The first surface of the section-plate was then ground, and the second surface subsequently, either by use of the arrangement for grinding a parallel surface or in a like manner to the first.

The interference figure afforded by such a section of caesium magnesium sulphate in convergent polarised white light is quite different to that of the corresponding potassium and rubidium salts, and very similar to that of normal rubidium sulphate. It exhibits evidence of large dispersion of the optic axes; the hyperbolic brushes are broad spectrum bands with dark centres. When the monochromatic illuminator is used, it is observed that the brushes are separated about 25° for the extreme red, and approach each other more and more as the wave-length is diminished, until for a colour in the blue between F and G they unite in the centre, and the uniaxial cross and rings are produced; further reduction of the wave-length then causes the brushes to separate in the direction at right angles to their former one, the second median line changing from the other axis of the ellipsoid lying in the symmetry plane to the morphological symmetry axis normal to the symmetry plane.

In the photographic reproduction of the interference figures of normal rubidium sulphate, given opposite p. 691 of the communication concerning the three normal sulphates (*loc. cit.*), Nos. 1, 2, 3, 4, 5, 6, and 7 represent almost exactly what was observed. No. 1 resembles the figure for Li light when the small-angle polariscopical goniometer used for exact measurements is employed, No. 2 that for C light, No. 3 for Na, No. 4 for Tl, No. 5 for F, No. 6 for the wave-length in the blue for which the uniaxial cross is produced, and No. 7 the figure for G light, with the axes separated in the vertical plane.

The following measurements of the apparent angle in air were obtained (p. 369).

The measurements for G were carried out with the plate rotated 90° in its own plane from the position occupied while making the measurements as far as F. The determination of the wave-length for which the uniaxial cross was produced was made by taking the reading of the prism circle of the monochromatic light apparatus for the

Determination of apparent Angle in Air of Cæsium Magnesium Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.	
Li.....	25° 40'	25° 30'	25° 50'	25° 40'	} In sym- metry plane.
C.....	25 5	25 10	25 30	25 15	
Na.....	22 55	22 45	22 50	22 50	
Tl.....	19 0	19 20	19 10	19 10	
F.....	13 30	13 15	13 15	13 20	} In plane at right angles.
Wave-length 450.	Coincident.	Coincident.	Coincident.	Coincident.	
G.....	12 0	13 0	14 30	13 10	

coincident position of the optic axes, and ascertaining the corresponding wave-length from the curve referred to in the memoir describing the monochromatic light apparatus. The readiness with which this determination may be made is one of the great advantages of that apparatus. The prism circle reading was taken in each case first while the plate was arranged for the measurements in the symmetry plane, and again when arranged in the position at right angles, after the measurements for G light had been carried out. The mean of each pair was then taken, and as the three means were all very close together, the limiting wave-lengths being 446 to 454, the mean 450 is given as the wave-length for coincidence.

The optic axial angle is too small to enable the obtuse angle to be seen through a plate perpendicular to the second median line, even in monobromonaphthalene. Hence the same plan has been adopted as was used in the case of rubidium sulphate of determining the true angle by observations of the apparent angle when immersed in two liquids whose refractive indices are very nearly, and the mean exactly equal to the mean refractive index of the crystal. Such liquids are turpentine ($\mu_D = 1.48$) and toluene ($\mu_D = 1.491$).

The interference figures as seen in white light while immersed in these liquids are very beautiful. When the monochromatic illuminator is employed, the brushes are sharp and clearly separated for red light, with a section 5 mm. thick, and surrounded by several rings; as the light is changed through orange and yellow to green, they approach each other, and eventually coalesce for a shade of blue, forming a sharp uniaxial cross surrounded by several circles, and as violet is approached they again separate in the plane at right angles, as in case of the apparent angle in air.

The measured angles are given in the following table. Sections 2 and 3 were employed, Section 1 not being so thick, and the figures, therefore, not so clearly defined.

Determination of the true Optic Axial Angle of Cæsium Magnesium Sulphate by Immersion in Turpentine and Toluene.

Nature of light.	No. of section.	In turpentine.	In toluene.	Mean.	Mean value of $2V_a$.	
Li.....	$\begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$	$\begin{matrix} 18^\circ 20' \\ 18 20 \end{matrix}$	$\begin{matrix} 18^\circ 10' \\ 17 50 \end{matrix}$	$\begin{matrix} 18^\circ 15' \\ 18 5 \end{matrix}$	$18^\circ 10'$	In symmetry plane.
C	$\begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$	$\begin{matrix} 18 5 \\ 18 10 \end{matrix}$	$\begin{matrix} 18 5 \\ 17 40 \end{matrix}$	$\begin{matrix} 18 5 \\ 17 55 \end{matrix}$	$18 0$	
Na	$\begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$	$\begin{matrix} 16 50 \\ 16 20 \end{matrix}$	$\begin{matrix} 16 30 \\ 16 10 \end{matrix}$	$\begin{matrix} 16 40 \\ 16 15 \end{matrix}$	$16 25$	
Tl.....	$\begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$	$\begin{matrix} 14 50 \\ 14 20 \end{matrix}$	$\begin{matrix} 14 30 \\ 13 50 \end{matrix}$	$\begin{matrix} 14 40 \\ 14 5 \end{matrix}$	$14 20$	
F	$\begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$	$\begin{matrix} 12 10 \\ 10 40 \end{matrix}$	$\begin{matrix} 12 0 \\ 10 20 \end{matrix}$	$\begin{matrix} 12 5 \\ 10 30 \end{matrix}$	$11 15$	
Wave-length in blue.	2 and 3. Coincident. Coincident.					
G.....	$\begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$	$\begin{matrix} 6^\circ 20' \\ 7 20 \end{matrix}$	$\begin{matrix} 6^\circ 40' \\ 7 40 \end{matrix}$	$\begin{matrix} 6^\circ 30' \\ 7 30 \end{matrix}$	$7^\circ 0'$	In plane at right angles.

The readings of the prism circle for the wave-length in the blue for which the axes coincided with the first median line were as follows.

	In turpentine.	In toluene.	Mean.	Mean prism circle reading for coincidence.
Section 2	6° 35'	6° 45'	6° 40' }	6° 22'
Section 3	6 10	6 0	6 5 }	

The reading 6° 22' of the prism circle corresponds to the passage of light of the wave-length 450 through the exit slit of the monochromatic light apparatus, the same as was found for the production of the uniaxial figure in air, and for this wave-length the crystals of cæsium magnesium sulphate simulate the optical properties of uniaxial crystals.

Dispersion of the Median Lines.—As is the case with the optic axial angle, the order of dispersion of the median lines is exceptional for this salt. The differences between the readings for each axis indicate that the first median line is nearer to the crystallographic axis *a* for lithium light by about 20' than it is for F light. The dispersion of each axis is large, and is largest for the one lying nearest to the crystallographic axis *a*.

Effect of Rise of Temperature on the Optic Axial Angle.—The optical behaviour of the crystals of this salt when their temperature is raised is particularly interesting. Both plates 2 and 3 were heated, and the phenomena carefully followed. As soon as the temperature begins to rise, the axes commence to approach each other. At 50°, with both

plates, the axes coalesced and the uniaxial cross was produced for F light. At 70° , the cross was produced for Tl light, the axes being dispersed for F light in the vertical plane at right angles, while for Na light they were still separated in the horizontal plane by $12^\circ 40'$, for C light by $17^\circ 5'$, and for Li light by $18^\circ 0'$. At 85° , the axes were united to form the cross for Na light. The valuable section 3 of 5 mm. thickness was not heated higher, but section 2 was further studied. At 100° , the axes were crossed for C light, and finally at 103° the uniaxial cross was produced for Li light, the dispersion of the axes occurring now entirely in the vertical plane. For the short time at which the temperature was maintained so high the crystal was not perceptibly injured. On cooling, the phenomena recurred in the reverse order, but it was found, on again measuring the angles at the ordinary temperature some days afterwards, that the separation of the axes had been permanently diminished by about $40'$, and on heating again the cross was produced for the various colours at slightly lower temperatures than on the occasion of the first heating.

Refractive Indices.—The exceptional optic axial angle phenomena of cæsium magnesium sulphate, resembling the phenomena presented by rubidium sulphate, although a more extreme case, is due to a similar cause to that obtaining in the case of the latter salt; namely, double refraction so feeble that one of the extreme axes of the optical ellipsoid falls nearly coincident with the intermediate axis *b*. The double refraction being positive, and the first median line consequently being *c*, the minimum axis of the optical ellipsoid, it is the maximum axis *a* which is nearly coincident with *b*. The similarity of the cases of Rb_2SO_4 and the salt now under discussion indicated the probability of this, and that it is the fact was at once revealed on examination of a prism ground so as to furnish the two refractive indices α and β , corresponding to the two axes *a* and *b*. Such a prism, as in the case of Rb_2SO_4 , exhibited only one refracted image of the slit instead of two as usual. This image, however, had the peculiarity shown by the analogous one of rubidium sulphate that it remained visible in all positions of the nicol prism carried in front of the eyepiece of the spectrometer. It was, in fact, a case of two overlapping images, one extinguishing at 0° and the other at 90° . On replacing the ordinary eyepiece by the higher power eyepiece and illuminating with C light, two images were distinctly seen, differing in position by one minute of arc, corresponding to a difference of 0.0002 in refractive index. This difference diminished with the wave-length of the illuminating light, and for F light the two images were absolutely coincident as far as observation could be carried. This is in accordance with the optic axial angle phenomena, for beyond F, for wave-length 450, *a* and *b* are coincident and the

crystal is apparently uniaxial. Beyond this the directions of a and b are reversed, the symmetry axis b is no longer b but a , and gives the least refractive index α . Unfortunately, G light is relatively so weak in luminosity that an observer cannot be quite sure of a difference of $1'$ in position of the G images, but there can be no doubt whatever that the two images will be again separated, and that the one which was formerly right will now be left, and *vice versa*.

Three such prisms were ground, and the phenomena were identical with each. They were ground so that the refracting edge in each case was parallel to the symmetry axis b , and the faces equally inclined to the plane containing this axis b (optical ellipsoidal axis b) and the axis a of the optical ellipsoid. Hence the image which extinguished at 0° of the nicol was that affording α , and that extinguishing at 90° gave β . Three other prisms were likewise ground affording β and γ .

In order that there should be no doubt whatever about the relative values of α and β , calculations have been made of the value of α , assuming the accuracy of β which was afforded six times by the prisms and the mean value of which is therefore indubitable, from the measured values of β , γ , and the optic axial angle $2Va$ by means of the formula

$$\cos Va = \sqrt{\frac{1 - \frac{1}{\beta^2}}{1 - \frac{1}{\alpha^2}} - \frac{1}{\gamma^2}}.$$

Such a calculation, as shown in the case of rubidium sulphate, affords accurate information.

The results of the calculations, assuming $\beta_{Li} = 1.4830$ and $\beta_F = 1.4912$, are

$$\alpha_{Li} = 1.48286 \quad \text{and} \quad \alpha_F = 1.49122.$$

Hence α and β differ by between 0.0001 and 0.0002 for lithium light, and the amount of difference diminishes until the two values become identical for F light. For G light α and β in the reverse positions differ by about 0.0001 . This is in accordance with what was actually observed for C, Na, Tl, and F light.

The following (p. 373) is the table of refractive indices embodying the results of the measurements, confirmed as regards α by the above calculations.

The intermediate index β , reduced to a vacuum, may be expressed for any wave-length as far as F by the following general formula.

$$\beta = 1.4706 + \frac{679\,587}{\lambda^2} - \frac{4\,793\,400\,000\,600}{\lambda^4} + \dots$$

The α index is given by diminishing the β value by 0.0002 as far as C, 0.0001 between C and Tl, and is identical with β beyond this up to

F. The γ index is given by increasing the constant 1.4706 by 0.0058. This is thus an extreme case of positive double refraction, the α and β indices being so close together as to actually coincide for certain wave-lengths.

Determination of Refractive Indices of Cæsium Magnesium Sulphate.

Index.	Direction of vibrations.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 1 prism.	Crystal 3, 2 prisms.	Crystal 4, 1 prism.	Mean refractive index.
α	Parallel 2nd median line lying in symmetry plane	Li	1.4833	1.4827	1.4823	—	1.4828
		C	1.4837	1.4831	1.4827	—	1.4832
		Na	1.4861	1.4856	1.4853	—	1.4857
		Tl	1.4883	1.4879	1.4877	—	1.4880
		F	1.4916	1.4913	1.4906	—	1.4912
	Parallel symmetry axis, b , now 2nd median line	G	1.4964	—	1.4950	1.4953	1.4956
β	Parallel symmetry axis, b	Li	1.4835	1.4829	1.4825	1.4830	1.4830
		C	1.4839	1.4833	1.4830	1.4834	1.4834
		Na	1.4862	1.4857	1.4854	1.4857	1.4858
		Tl	1.4885	1.4881	1.4879	1.4880	1.4881
		F	1.4917	1.4913	1.4906	1.4911	1.4912
	In symmetry plane at right angles to 1st median line	G	1.4962	—	1.4951	—	1.4957
γ	Parallel 1st median line c .	Li	1.4889	—	1.4887	1.4888	1.4888
		C	1.4893	—	1.4892	1.4892	1.4892
		Na	1.4916	—	1.4916	1.4915	1.4916
		Tl	1.4942	—	1.4940	1.4938	1.4940
		F	1.4972	—	1.4970	1.4969	1.4970
		G	1.5020	—	—	1.5011	1.5015

Alteration of Refraction by Increase of Temperature.—Measurements at 70° indicated that the indices are lowered by about 0.0016 by an increase of 50° in temperature. The prisms affording the values of α and β exhibit coincident images at 70° , even when examined with the high power eye-piece, as would be expected from the optic axial angle phenomena at this temperature. For wave-lengths beyond the green thallium line the α vibrations are parallel to the symmetry axis b and the β vibrations occur in the symmetry plane.

Ratios of Axes of Optical Velocity Ellipsoid.—These ratios calculated for sodium light are

$$a : b : c = 1.0001 : 1 : 0.9961.$$

The closeness of the relative values of the first two axes is very clearly manifested by these ratios.

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Molecular Optical Constants.—The following are the calculated values of these constants.

Axis of optical ellipsoid.....	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n$..	{ C. 0.1071	0.1071	0.1082
	{ G. 0.1094	0.1094	0.1105
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$ {	C. 63.11	63.15	63.78
	G. 64.49	64.50	65.14
Specific dispersion, $n_g - n_c$	0.0023	0.0023	0.0023
Molecular dispersion, $m_g - m_c$	1.38	1.35	1.36
Molecular refraction, $\frac{n - 1}{d} M$	C. 106.76	106.80	108.08

These values will be independent of temperature as the refraction diminishes with the density.

SALTS CONTAINING ZINC.

Potassium Zinc Sulphate, $K_2Zn(SO_4)_2 \cdot 6H_2O$.

There is a perfect *cleavage* parallel $r'\{201\}$, the cleaved surfaces affording sharp single images of the goniometer slit, at precisely 180° to the r' face present on each fragment.

Volume.

Relative Density.—The following four determinations were made.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
4.1195	2.2399
3.7180	2.2426
4.6208	2.2402
4.3405	2.2426

Mean **2.2413**

Perrot obtained 2.245 at 16° ; Joule and Playfair gave 2.24.

Molecular Volume.— $\frac{M}{d} = \frac{443}{2.241} = 197.68$.

Distance Ratios.—The axial angle β was previously shown to be $75^\circ 12'$, and

$$a : b : c = 0.7413 : 1 : 0.5044.$$

Combining these with the molecular volume, the calculated distance ratios are

$$\chi : \psi : \omega = 6.0618 : 8.1773 : 4.1247.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two section-plates, ground parallel to the symmetry plane, exhibited the following ex-

tion angles with respect to the normal to the basal plane $c\{001\}$, the direction being behind the normal nearer to the vertical axis c of the crystal, as indicated in Fig. 4.

Section 1	4° 40'
Section 2	4 20
Mean.. ...	<hr/> 4 30

FIG. 4.

Murmann and Rotter gave $4^{\circ} 43'$.

The axis of the optical ellipsoid nearly normal to the basal plane is again the second median line, and as the *sign of double refraction* is positive, this axis is a ; the direction of extinction at right angles to this in the symmetry plane is the minimum axis of the ellipsoid c , and is the first median line. The symmetry axis b is again the intermediate axis b . As the axial angle ac was shown in the former communication to be $75^{\circ} 12'$, the orientation of the axes is as follows.

The first median line c lies in the obtuse angle of the axes ac , and is inclined $4^{\circ} 30'$ to the axis a .

The second median line a lies also in the obtuse angle ac , and is inclined $10^{\circ} 18'$ to the vertical axis c .

The first median line is also inclined $85^{\circ} 30'$ to the normal to $c\{001\}$ and $10^{\circ} 18'$ to the normal to $a\{100\}$.

Optic Axial Angle.—The following measurements of the apparent angle in air were made with three excellent sections ground perpendicular to the first median line.

Determination of apparent Angle in Air of Potassium Zinc Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li.....	$112^{\circ} 14'$	$112^{\circ} 24$	$112^{\circ} 14'$	$112^{\circ} 17'$
C.....	112 16	112 26	112 16	112 19
Na.....	112 24	112 36	112 26	112 29
Tl.....	112 33	112 46	112 36	112 38
F.....	112 45	112 58	112 48	112 50

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The angle given by Murmann and Rotter is $114^{\circ} 14'$, and by Perrot 114° .

Determination of true Optic Axial Angle of Potassium Zinc Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of $2H\alpha$.	No. of section perp. 2nd median line.	Observed values of $2Ho$.	Calculated value of $2Va$.	Mean value of $2Va$.
Li.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 60^{\circ} 11' \\ 60 \quad 25 \\ 60 \quad 26 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 95^{\circ} 36' \\ 95 \quad 46 \\ 95 \quad 40 \end{array} \right.$	$\left\{ \begin{array}{l} 68^{\circ} 11' \\ 68 \quad 18 \\ 68 \quad 21 \end{array} \right.$	$68^{\circ} 17'$
C.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 60 \quad 9 \\ 60 \quad 22 \\ 60 \quad 24 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 95 \quad 34 \\ 95 \quad 42 \\ 95 \quad 39 \end{array} \right.$	$\left\{ \begin{array}{l} 68 \quad 10 \\ 68 \quad 17 \\ 68 \quad 20 \end{array} \right.$	$68 \quad 16$
Na.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 59 \quad 54 \\ 60 \quad 7 \\ 60 \quad 10 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 95 \quad 12 \\ 95 \quad 18 \\ 95 \quad 18 \end{array} \right.$	$\left\{ \begin{array}{l} 68 \quad 8 \\ 68 \quad 15 \\ 68 \quad 18 \end{array} \right.$	$68 \quad 14$
Tl.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 59 \quad 39 \\ 59 \quad 51 \\ 59 \quad 51 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 94 \quad 47 \\ 94 \quad 53 \\ 94 \quad 46 \end{array} \right.$	$\left\{ \begin{array}{l} 68 \quad 6 \\ 68 \quad 13 \\ 68 \quad 16 \end{array} \right.$	$68 \quad 12$
F.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 59 \quad 21 \\ 59 \quad 30 \\ 59 \quad 31 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 94 \quad 20 \\ 94 \quad 20 \\ 94 \quad 16 \end{array} \right.$	$\left\{ \begin{array}{l} 68 \quad 3 \\ 68 \quad 10 \\ 68 \quad 13 \end{array} \right.$	$68 \quad 9$

Monobromonaphthalene was the immersion liquid employed.

Very accurate measurement is required, using sections of sufficient thickness (2 or 3 mm.) to afford very small rings and fine brushes, in order to be certain of such a small amount of dispersion. A most valuable indication of the true order of dispersion of this salt is afforded by observations of the angle in turpentine and in toluene, whose refractive indices are only slightly lower and higher respectively than the mean refractive index of the crystal. As measured in toluene, the angle for red is indubitably larger than for blue, and by about 20° . In turpentine scarcely any difference is perceptible. Hence the true angle is certainly largest for red light, and the order of dispersion given in the table is confirmed.

Murmann and Rotter give $69^{\circ} 5'$, without stating the order of dispersion. Perrot gives $69^{\circ} 3'$ for C, $68^{\circ} 20'$ for Na, and $68^{\circ} 37'$ for F, the tendency being for red to be greatest.

The angles calculated from the three refractive indices of the crystals are not trustworthy unless the indices are absolutely correct to the fourth decimal place, as shown in the communication concerning the simple sulphates. As this cannot be guaranteed, and in all probability the possible error amounts to 0.0002 after the most elaborate precautions, such calculations are useless for the purpose in view.

Dispersion of the Median Lines.—In turpentine, the optic axes both exhibit dispersion in the same direction, the hypobolic brushes being

fringed with colour in the following order, assuming the obtuse angle of the morphological axes, *ac*, to be on the left hand.

red | blue 1st M.L. red | blue.

The axes being dispersed inversely as the order of the fringes, the first median line is evidently slightly on the left of the central line for blue and on the right for red. As regards the relative amount of dispersion of each axis, there is no appreciable difference. In toluene, the axis on the left exhibits no coloured fringes, the other being as in turpentine; thus

no colour | no colour 1st M.L. red | blue.

Here again, however, the first median line for blue is evidently left of its position for red. Hence there can be no doubt that this represents the order of dispersion of the median lines. The true dispersion at each axis will be intermediate between the two cases, and nearer to the arrangement for toluene, whose refractive index is nearer the mean index of the crystal.

The first median line is consequently dispersed so as to lie a few minutes nearer to the crystallographic axes *a* for red than it does for blue, and the optic axis lying in the obtuse angle of the morphological axes *ac* is nearly coincident for all colours.

It is worth noticing the considerable difference between the indications as regards the latter optic axis in turpentine and in toluene, showing, as it does, how misleading the observations in either alone might be, and the care required in the choice of immersion liquids for this purpose.

Effect of rise of Temperature on the Optic Axial Angle.—Section 3 was heated to 90° and measurements of 2E were made at this temperature. The angle for sodium light was 115° 12'. Hence the optic axial angle increases nearly 3° for 70° rise of temperature.

Refractive Indices.—The results of the determinations of refractive index with six prisms ground upon five different crystals are given on p. 378; in the last column the values obtained by Perrot are given.

It is satisfactory to note the close agreement of Perrot's values with those now given. Murmann and Rotter give somewhat different values, for C they are α 1.476, β 1.480, and γ = 1.489; the γ value is particularly incorrect.

The mean refractive index of potassium zinc sulphate, corrected to a vacuum, is accurately expressed by the following formula for any wave-length λ as far as F.

$$\mu = 1.4690 + \frac{609\ 571}{\lambda^2} - \frac{3\ 444\ 700\ 000\ 000}{\lambda^4} + \dots$$

Determination of Refractive Indices of Potassium Zinc Sulphate.

Index.	Nature of light.	Crystal 1, 1 prism.	Crystal 2, 1 prism.	Crystal 3, 2 prisms.	Crystal 4, 1 prism.	Crystal 5, 1 prism.	Mean refrac- tive index.	Perrot's values.
α vibrations parallel 2nd median line <i>a</i> .	Li	1.4748	—	1.4750	1.4745	—	1.4748	—
	C	1.4752	—	1.4754	1.4749	—	1.4752	1.4749
	Na	1.4776	—	1.4777	1.4772	—	1.4775	1.4775
	Tl	1.4796	—	1.4801	1.4793	—	1.4797	—
	F	1.4826	—	1.4834	1.4819	—	1.4826	1.4826
	G	—	—	1.4870	1.4862	—	1.4866	—
β vibrations parallel symmetry axis <i>b</i> (<i>b</i>).	Li	1.4804	1.4805	1.4810	1.4802	1.4806	1.4805	—
	C	1.4808	1.4809	1.4814	1.4806	1.4810	1.4809	1.4811
	Na	1.4834	1.4832	1.4837	1.4829	1.4832	1.4833	1.4836
	Tl	1.4858	1.4857	1.4861	1.4853	1.4858	1.4857	—
	F	1.4892	1.4890	1.4894	1.4882	1.4886	1.4889	1.4888
	G	—	—	1.4936	1.4924	1.4926	1.4929	—
γ vibrations parallel 1st median line <i>c</i> .	Li	—	1.4938	1.4939	—	1.4936	1.4938	—
	C	—	1.4942	1.4944	—	1.4940	1.4942	1.4940
	Na	—	1.4969	1.4972	—	1.4966	1.4969	1.4967
	Tl	—	1.4993	1.4996	—	1.4994	1.4994	—
	F	—	1.5026	1.5031	—	1.5023	1.5027	1.5024
	G	—	—	1.5072	—	1.5063	1.5067	—

As the dispersion differs appreciably for the three indices, the α and γ values are not so closely reproduced as usual by the same formula by diminishing and increasing the constant 1.4690 by fixed amounts; they are, however, on the average less and greater respectively than the β values by 0.0058 and 0.0136, the β indices being nearer to the α in accordance with the positive nature of the double refraction.

Alteration of Refraction by Rise of Temperature.—From measurements carried out at 70° with a pair of prisms, it appears that the indices are diminished by about 0.0018 for 50° rise of temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The calculated values of these ratios are

$$a : b : c = 1.0039 : 1 : 0.9909.$$

Molecular Optical Constants.—These are as follows—

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1258	0.1271	0.1301
	{ G. 0.1283	0.1298	0.1328
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 55.72	56.29	57.61
	{ G. 56.86	57.48	58.84
Specific dispersion, $n_g - n_c \dots$	0.0025	0.0027	0.0027
Molecular dispersion, $m_g - m_c \dots$	1.14	1.19	1.23
Molecular refraction, $\frac{n - 1}{d} M \dots$	C. 94.02	95.14	97.77

As rise of temperature affects density and refraction in like manner, these values will be independent of temperature.

Rubidium Zinc Sulphate, $Rb_2Zn(SO_4)_2 \cdot 6H_2O$.

There is a good *cleavage* parallel, $r'\{201\}$. The fractures were found to yield clear reflections of the goniometer signal at exactly 180° in each case to the remaining r' face.

Volume.

Relative Density.—The following are the results of four determinations of the density with different crops belonging to two entirely independent preparations, containing rubidium sulphate from the two different sources previously alluded to.

Weight of salt employed.	Sp. gr. at 20°/4°.
4.7593	2.5817
4.5051	2.5839
5.1428	2.5817
5.1740	2.5888
Mean	2.5840

Perrot obtained values ranging from 2.573 to 2.610, and gives the mean as 2.595.

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{535.4}{2.584} = 207.2.$$

Distance Ratios.—It was shown in the former communication that the axial angle β is $74^\circ 7'$, and

$$a : b : c = 0.7373 : 1 : 0.5011.$$

Combining these with the molecular volume we obtain

$$\chi : \psi : \omega = 6.1596 : 8.3543 : 4.1863.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The extinctions on two sections parallel to the symmetry plane with respect to the normal to the basal plane were

Section 1	$0^\circ 40'$
Section 2	1 0
Mean	<hr/> 0 50

FIG. 5.

The direction is forwards as shown in Fig. 5.

The axis of the optical ellipsoid which is nearly normal to $c\{001\}$ is again the second median line, and the axis at right angles to this in the symmetry plane is the first median line.

The *sign of double refraction* is positive, as usual for the series, a fact confirmed by the quarter undulation mica plate, and by the relations of the refractive indices. Hence the orientations may be stated as follows, having regard to the fact previously shown that the axial angle ac is $74^\circ 7'$.

The first median line ϵ lies in the acute angle of the morphological axes ac , and makes an angle of $0^\circ 50'$ with the axis a .

The second median line a lies in the obtuse angle ac , and makes an angle of $16^\circ 43'$ with the vertical axis c .

The first median line is also inclined $90^{\circ} 50'$ to the normal to $c\{001\}$ and $16^{\circ} 43'$ to the normal to $a\{100\}$.

Perrot gives $90^{\circ} 30'$ as the inclination of the first median line to $\{001\}$.

Optic Axial Angle.—Three admirable pairs of plates were obtained by grinding, perpendicular to the first and second median lines. The angle in air is very large, and accurate measurements are only to be obtained with large and perfectly transparent sections. The following are the results.

Determination of the apparent Angle in Air of Rubidium Zinc Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	$125^{\circ} 44'$	$125^{\circ} 34'$	$125^{\circ} 36'$	$125^{\circ} 38'$
C	125 45	125 35	125 38	125 39
Na	125 50	125 42	125 45	125 46
Tl	125 55	125 50	125 53	125 53
F	126 5	126 0	126 5	126 3

Perrot gives 128° for the angle in white light.

Determination of true Optic Axial Angle of Rubidium Zinc Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of $2H\alpha$.	No. of section perp. 2nd median line.	Observed values of $2H\alpha$.	Calculated values of $2V\alpha$.	Mean $2V\alpha$.
Li.....	1	$65^{\circ} 12'$	1a	$91^{\circ} 52'$	$73^{\circ} 44'$	$73^{\circ} 42'$
	2	65 15	2a	92 1	73 42	
	3	65 8	3a	91 56	73 39	
C	1	65 7	1a	91 46	73 42	73 40
	2	65 11	2a	91 57	73 40	
	3	65 4	3a	91 50	73 38	
Na	1	64 38	1a	91 24	73 32	73 33
	2	64 46	2a	91 33	73 33	
	3	64 45	3a	91 29	73 34	
Tl.....	1	64 16	1a	91 1	73 24	73 27
	2	64 20	2a	91 5	73 26	
	3	64 25	3a	91 4	73 30	
F	1	63 42	1a	90 32	73 12	73 18
	2	63 45	2a	90 27	73 18	
	3	63 55	3a	90 30	73 24	

Monobromonaphthalene was employed as the immersion liquid.

Perrot indicates an optic axial angle of 74° — 75° .

Dispersion of the Median Lines.—The interference figure in toluene, whose index of refraction is identical with the mean index of the crystal, indicates that the optic axis which emerges between $a\{100\}$ and $r\{201\}$, is nearly coincident for all colours, whilst the other axis not far from normal to $r'\{201\}$ is dispersed about half a degree between Li and F, and for red is further removed from the crystal-

lographic axis a than for blue, which agrees with the amount of dispersion indicated in the above table of measurements. Hence the first median line is dispersed about $15'$ between Li and F, and lies for blue nearer to the crystallographic axis a than it does for red.

Effect of Rise of Temperature on Optic Axial Angle.—Section 1 was heated to 70° , and the angle for sodium light found to be $127^\circ 0'$. Hence the apparent angle in air increases about $1\frac{1}{4}^\circ$ for 50° rise of temperature.

Refractive Indices.—The following are the refractive indices, determined by means of six prisms ground upon four crystals.

Determination of Refractive Indices of Rubidium Zinc Sulphate.

Index.	Nature of light.	Crystal 1, 1 prism.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Crystal 4, 1 prism.	Mean refractive index.	Perrot's values.
α Vibs. par. 2nd median line	Li	—	1·4805	1·4809	1·4808	1·4807	—
	C	—	1·4809	1·4813	1·4812	1·4811	1·4806
	Na	—	1·4831	1·4835	1·4834	1·4833	1·4833
	Tl	—	1·4856	1·4859	1·4857	1·4857	—
	F	—	1·4884	1·4888	1·4887	1·4886	1·4883
	G	—	1·4929	—	—	1·4929	—
β Vibs. par. symmetry axis b (h)	Li	1·4855	1·4855	1·4858	1·4857	1·4856	—
	C	1·4859	1·4860	1·4862	1·4859	1·4860	1·4859
	Na	1·4882	1·4883	1·4888	1·4884	1·4884	1·4882
	Tl	1·4906	1·4907	1·4910	1·4908	1·4908	—
	F	1·4938	1·4936	1·4938	1·4940	1·4938	1·4942
	G	1·4981	1·4978	—	—	1·4980	—
γ Vibs. par. 1st median line c	Li	1·4945	1·4948	1·4949	—	1·4947	—
	C	1·4949	1·4952	1·4953	—	1·4951	1·4945
	Na	1·4975	1·4976	1·4975	—	1·4975	1·4975
	Tl	1·5000	1·5002	1·5002	—	1·5001	—
	F	1·5031	1·5034	1·5033	—	1·5033	1·5030
	G	1·5078	1·5078	—	—	1·5078	—

The intermediate index β , corrected to a vacuum, is expressed for any wave-length λ as far as F by the formula

$$\beta = 1.4742 + \frac{602\ 511}{\lambda^2} - \frac{3\ 323\ 600\ 000\ 000}{\lambda^4} + \dots$$

The α indices are reproduced by the formula to within one or two units in the fourth decimal place if the constant 1·4742 is diminished by 0·0051, and the γ indices if it is increased by 0·0092. The smaller amount of the former difference determines the positive nature of the double refraction.

Alteration of Refraction by Increase of Temperature.—The indices

were determined upon two prisms at 70°, and found to be about 0·0012 lower at this temperature than the values above given.

Ratio of Axes of Optical Velocity Ellipsoid.—The values calculated from the refractive indices are as follows.

$$a : b : c = 1\cdot0034 : 1 : 0\cdot9939.$$

Molecular Optical Constants.—The calculated values of these constants are as under.

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0·1102	0·1112	0·1130
	{ G. 0·1125	0·1135	0·1154
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$	{ C. 59·02	59·53	60·48
	{ G. 60·25	60·78	61·79
Specific dispersion, $n_G - n_C$	0·0023	0·0023	0·0024
Molecular dispersion, $m_G - m_C$	1·23	1·25	1·31
Molecular refraction, $\frac{n - 1}{d} M$	C 99·77	100·78	102·67

These values will be independent of temperature, as the refraction has been shown to diminish as the temperature rises, and the density is similarly affected.

Cæsium Zinc Sulphate, Cs₂Zn(SO₄)₂·6H₂O.

There is a well-defined *cleavage* parallel to the orthodome $r'\{\bar{2}01\}$. The cleaved surfaces afford sharp images of the goniometer signal at 180° in each case from the residual r' face.

Volume.

Relative Density.—The following four determinations were made with crystals belonging to different crops of the two independent preparations.

Weight of salt employed.	Sp. gr. at 20°/4°.
4·4625	2·8668
5·1008	2·8707
3·6943	2·8652
3·5480	2·8652

Mean **2·8670**

Perrot obtained numbers varying from 2·826 to 2·888, and ascribes 2·88 as the true density.

$$\text{Molecular Volume.} \quad \frac{M}{d} = \frac{630\cdot4}{2\cdot867} = 219\cdot88.$$

Distance Ratios.—The axial angle ac was shown in the former communication to be 72° 59', and the axial ratios

$$a : b : c = 0\cdot7274 : 1 : 0\cdot4960.$$

Combining these with the molecular volume, we find

$$\chi : \psi : w = 6.2598 : 8.6058 : 4.2684.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The extinctions on two section-plates ground parallel to the symmetry plane with respect to the normal to the basal plane were

Section 1	12° 50'
Section 2	13 40
Mean	<hr/> 13 15

FIG. 6.

The direction is forwards, as clearly represented in Fig. 6.

The axis near the normal to $c\{001\}$ is, as usual in the series, the second median line, the other axis of the optical ellipsoid which lies in the symmetry plane being the first median line.

A quarter undulation mica, and the relations of the three refractive indices, indicate a positive *sign of double refraction*, and with the knowledge of the axial angle ac , previously shown to be $72^\circ 59'$, the orientations work out as follows.

The first median line r lies in the acute angle of the morphological axes ac , and is inclined $13^\circ 15'$ to the axis a .

The second median line a lies in the obtuse angle ac , and is inclined $30^\circ 16'$ to the vertical axis c .

The first median line is also inclined $103^\circ 15'$ to the normal to $c\{001\}$, and $30^\circ 16'$ to the normal to $a\{100\}$.

Perrot gives 112° as the inclination of first median line to $\{001\}$, a much larger angle than that exhibited by the author's crystals.

Optic Axial Angle.—The following measurements were obtained with three pairs of sections ground perpendicular to the first and second median lines. The angle in air is so large that only two of the sections exhibited it sufficiently clearly.

Determination of apparent Angle in Air of Cæsium Zinc Sulphate.

Nature of light.	Section 1.	Section 2.	Mean 2E.
Li	131° 20'	131° 16'	131° 18'
C	131 10	131 10	131 10
Na	130 30	130 40	130 35
Tl	129 52	130 8	130 0
F	129 10	129 20	129 15

Determination of true Optic Axial Angle of Cæsium Zinc Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean 2Va.
Li.....	1	66° 59'	1a	92° 45'	74° 38'	74° 30'
	2	66 42	2a	92 55	74 22	
	3	74 57	3a	106 14	74 31	
C.....	1	66 55	1a	92 43	74 36	74 27
	2	66 38	2a	92 52	74 20	
	3	74 51	3a	106 18	74 26	
Na.....	1	66 15	1a	92 24	74 16	74 11
	2	66 1	2a	92 32	74 2	
	3	74 37	3a	106 25	74 14	
Tl.....	1	65 37	1a	92 12	73 53	73 52
	2	65 33	2a	92 17	73 48	
	3	74 11	3a	106 38	73 54	
F.....	1	64 53	1a	91 55	73 28	73 31
	2	64 55	2a	91 50	73 32	
	3	73 50	3a	106 54	73 34	

The two first pairs of sections were measured in monobromonaphthalene, and the third pair in toluene, the results confirming each other admirably. Perrot gives for the angle 78° for white light, but it was not determined directly.

Dispersion of the Median Lines.—Additional observations of the interference figures were made in cedar oil ($\mu_D = 1.520$) and benzene ($\mu_D = 1.507$). The mean of the indices of toluene and cedar oil is approximately equal to the mean refractive index of the crystal, and the index of benzene is also about the same mean value. The phenomena in the latter liquid are consequently, and were actually observed to be, the mean of the phenomena exhibited in the two former liquids, and may be taken as representing the true character of the dispersion of the two optic axes, and hence of the first median line.

In benzene the two axial brushes are observed to be about equally fringed with colour in the following symmetrical order.

blue | red 1st M.L. red | blue

Moreover, the differences of the readings for red and blue light at

each axis were found to be each about 25'. Hence the median lines in this salt exhibit no appreciable dispersion.

Effect of Rise of Temperature on the Optic Axial Angle.—The apparent optic axial angle in air was determined at 70° with Section 1 in sodium light, and found to be 128°. Hence 2E decreases about 2½° for 50° rise of temperature.

Refractive Indices.—The following are the results of the measurements with six prisms ground upon four different crystals.

Determination of Refractive Indices of Cæsium Zinc Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 1 prism.	Crystal 4, 1 prism.	Mean refractive index.	Perrot's values.
α Vibs. par. 2nd median line π	Li	1.4993	1.4989	1.4999	—	1.4994	—
	C	1.4997	1.4993	1.5003	—	1.4998	1.4997
	Na	1.5022	1.5019	1.5026	—	1.5022	1.5020
	Tl	1.5046	1.5042	1.5053	—	1.5047	—
	F	1.5079	1.5076	1.5083	—	1.5079	1.5080
	G	1.5127	1.5123	1.5124	—	1.5125	—
β Vibs. par. symmetry axis b (b)	Li	1.5020	1.5016	—	1.5023	1.5020	—
	C	1.5024	1.5020	—	1.5027	1.5024	1.5025
	Na	1.5049	1.5045	—	1.5050	1.5048	1.5049
	Tl	1.5073	1.5069	—	1.5076	1.5073	—
	F	1.5105	1.5100	—	1.5107	1.5104	1.5107
	G	1.5154	1.5147	—	1.5153	1.5151	—
γ Vibs. par. 1st median line ϵ	Li	1.5063	1.5060	1.5068	1.5066	1.5064	—
	C	1.5067	1.5064	1.5073	1.5070	1.5068	1.5070
	Na	1.5092	1.5089	1.5098	1.5093	1.5093	1.5095
	Tl	1.5118	1.5114	1.5124	1.5120	1.5119	—
	F	1.5150	1.5147	1.5158	1.5153	1.5152	1.5154
	G	1.5199	1.5195	1.5203	1.5198	1.5199	—

The intermediate axis β , reduced to a vacuum, is accurately represented as far as F, for any wave-length λ , by the formula

$$\beta = 1.4914 + \frac{539\ 594}{\lambda^2} - \frac{2\ 103\ 000\ 000\ 000}{\lambda^4} + \dots$$

The α indices are reproduced if the constant 1.4914 is diminished by 0.0026, and the γ values if it is increased by 0.0045. The former difference being the smaller, determines the positive nature of the double refraction.

Alteration of Refraction by Increase of Temperature.—Measurements with two of the prisms at 70° indicated that the indices are diminished about 0.0016 for 50° rise of temperature, the limits being 0.0013 to 0.0018

Ratio of Axes of Optical Velocity Ellipsoid.—The refractive indices lead to the following ratios.

$$a : b : c = 1.0017 : 1 : 0.9970.$$

Molecular Optical Constants.—These are as follows.

Axis of optical ellipsoid	α .	β .	γ .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n_{\infty}$ {	C. 0.1026	0.1031	0.1038
	G. 0.1048	0.1053	0.1061
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$ {	C. 64.69	64.98	65.46
	G. 66.08	66.36	66.88
Specific dispersion, $n_G - n_C$	0.0022	0.0022	0.0023
Molecular dispersion, $m_G - m_C$	1.39	1.38	1.42
Molecular refraction $\frac{n - 1}{d}$ M	C 109.98	110.56	111.52

These values are independent of temperature, refraction diminishing with density by rise of temperature.

SALTS CONTAINING IRON.

Potassium Ferrous Sulphate, $K_2Fe(SO_4)_2 \cdot 6H_2O$.

There is an excellent cleavage parallel to the orthodome $r'\{\bar{2}01\}$, as stated by Topsøe and Christiansen.

Volume.

Relative Density.—The following four determinations were made with different crops of clear crystals, freshly recrystallised and quite free from traces either of oxidation or efflorescence.

Weight of salt employed.	Sp. gr. at 20°/4°.
3.3358	2.1707
2.8228	2.1724
3.7412	2.1664
3.2737	2.1680

Mean **2.1694**

Schiff gave 2.18 as the density of this salt.

$$\text{Molecular Volume. } \frac{M}{d} = \frac{434}{2.169} = 200.09.$$

Distance Ratios.—The axial angle ac was previously shown to be $75^\circ 28'$, and the axial ratios

$$a : b : c = 0.7377 : 1 : 0.5020.$$

Combination of these with the molecular volume yields the following values for the distance ratios.

$$\chi : \psi : \omega = 6.0739 : 8.2336 : 4.1332.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The direction of extinction in the symmetry plane was given by each of two section-plates ground parallel to the symmetry plane $b\{010\}$ as $2^\circ 35'$ behind the normal to the basal plane $c\{001\}$, as indicated in Fig. 7.

FIG. 7.

Murmann and Rotter found $2^\circ 46'$. Topsøe and Christiansen give $3^\circ 44'$.

As usual in the series, this direction is the second median line, the first median line being the direction at right angles in the symmetry plane.

The *sign of double refraction* being again positive, determined as in the cases of the salts previously described, the orientations may be stated as follows, knowing the axial angle ac to be $75^\circ 23'$.

The first median line t lies in the obtuse morphological axial angle ac , and is inclined $2^\circ 35'$ to the axis a .

The second median line a also lies in the obtuse angle ac , and is inclined $11^\circ 57'$ to the vertical axis c .

The first median line is also inclined $87^\circ 25'$ to the normal to $c\{001\}$ and $11^\circ 57'$ to the normal to $a\{100\}$.

Optic Axial Angle.—Three excellent pairs of sections exhibiting small rings and clearly defined brushes were ground perpendicular to the first and second median lines, and the following measurements obtained with them.

Determination of Apparent Angle in Air of Potassium Ferrous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	109° 45'	109° 35'	109° 30'	109° 37'
C	109 50	109 40	109 34	109 41
Na	110 20	110 9	110 0	110 10
Tl	110 50	110 30	110 25	110 35
F	111 20	111 0	110 55	111 5

The apparent angle in air is given by Murmann and Rotter as $111^{\circ} 56'$ for sodium light.

Determinations of true Optic Axial Angle of Potassium Ferrous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean 2Va.
Li.....	{ 1	66° 34'	1a	111° 36'	67° 8'	67° 1'
	{ 2	67 45	2a	114 32	67 3	
	{ 3	66 16	3a	111 46	66 52	
C.....	{ 1	66 34	1a	111 34	67 9	67 2
	{ 2	67 45	2a	114 30	67 4	
	{ 3	66 16	3a	111 44	66 53	
Na.....	{ 1	66 37	1a	111 23	67 14	67 7
	{ 2	67 48	2a	114 24	67 8	
	{ 3	66 19	3a	111 33	66 58	
Tl.....	{ 1	66 40	1a	111 12	67 20	67 12
	{ 2	67 51	2a	114 17	67 12	
	{ 3	66 22	3a	111 23	67 4	
F.....	{ 1	66 44	1a	110 54	67 28	67 19
	{ 2	67 55	2a	114 7	67 18	
	{ 3	66 26	3a	111 6	67 12	

The immersion liquid employed with the pairs 1 and 1a, 3 and 3a was toluene; in the case of 2 and 2a turpentine was used. One pair of sections (3 and 3a) were remeasured in monobromonaphthalene, and the values of 2Va obtained found to be practically identical with those obtained in toluene.

Murmann and Rotter found $68^{\circ} 4'$ for the true angle for sodium light, and Topsøe and Christiansen $67^{\circ} 18'$.

Dispersion of the Median Lines.—The observations of the interference figures in white light, in toluene and turpentine, whose indices of refraction are respectively higher and lower than the mean index of the crystals, together with a knowledge of the orientation of the sections during measurement of the optic axial angle in monochromatic light, and observations of the circle readings for each axis, indicate that the first median line is dispersed in such a manner that for Li light it lies nearer to the crystallographic axis *a* by about $10'$ than it does for F light. Both axes show dispersion, the one lying in the acute angle of the morphological axes *ac* only very slightly, as stated by Murmann and Rotter.

Effect of Rise of Temperature on the Optic Axial Angle.—The apparent angle in air was measured with Section 2 at 70° . The angle was found to be increased by about 2° for this rise of 50° of temperature. The section became opaque immediately after the reading was obtained, owing to decomposition.

Refractive Indices.—The following determinations were obtained with six prisms ground upon four different crystals.

Determination of Refractive Indices of Potassium Ferrous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 1 prism.	Crystal 4, 1 prism.	Mean refractive index.	Values of Topsøe and Christiansen.
α Vibs. par. 2nd median line	Li	1·4729	1·4733	1·4732	—	1·4731	—
	C	1·4732	1·4737	1·4736	—	1·4735	1·4751
	Na	1·4757	1·4760	1·4760	—	1·4759	1·4775
	Tl	1·4780	1·4782	1·4784	—	1·4782	—
	F	1·4810	1·4811	1·4812	—	1·4811	1·4833
	G	1·4852	—	—	—	1·4852	—
β Vibs. par. symmetry axis b (h)	Li	1·4792	1·4797	1·4796	1·4796	1·4795	—
	C	1·4796	1·4800	1·4800	1·4800	1·4799	1·4806
	Na	1·4820	1·4822	1·4823	1·4821	1·4821	1·4832
	Tl	1·4845	1·4847	1·4851	1·4846	1·4847	—
	F	1·4876	1·4878	1·4879	1·4877	1·4877	1·4890
	G	1·4917	—	—	1·4923	1·4920	—
γ Vibs. par. 1st median line	Li	1·4938	1·4943	—	1·4942	1·4941	—
	C	1·4942	1·4946	—	1·4946	1·4945	1·4947
	Na	1·4968	1·4970	—	1·4969	1·4969	1·4973
	Tl	1·4993	1·4997	—	1·4996	1·4995	—
	F	1·5025	1·5030	—	1·5030	1·5028	1·5041
	G	1·5067	—	—	1·5075	1·5071	—

Murmann and Rotter give the following values for β : for red 1·478, for yellow 1·480, for green 1·484, and for violet 1·489. The values obtained by Topsøe and Christiansen, of which only those for β were directly determined, are given in the last column, and differ considerably from the values now given.

The intermediate index β (in a vacuum) is represented, for any wave-length λ as far as F, by the following formula.

$$\beta = 1\cdot4728 + \frac{266\ 439}{\lambda^2} + \frac{2\ 437\ 900\ 000\ 000}{\lambda^4} + \dots$$

The α values are closely reproduced by the formula if the constant 1·4728 is diminished by 0·0064, and the γ values if it is increased by 0·0147. The closer approximation of the α and β values than the β and γ values determines the positive nature of the double refraction.

Alteration of Refraction by Increase of Temperature.—The prisms were found incapable of withstanding a heat of 70° unimpaired, so the observations were limited to 55°. At this temperature the indices were about 0·0010 lower than at the ordinary temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The above indices of refraction furnish the following ratios.

$$a : b : c = 1.0042 : 1 : 0.9901.$$

Molecular Optical Constants.—Following are the calculated values of these constants.

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1296 G. 0.1323	0.1310 0.1339	0.1344 0.1373
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 56.22 G. 57.41	56.87 58.09	58.34 59.60
Specific dispersion, $n_G - n_C$	0.0027	0.0029	0.0029
Molecular dispersion, $m_G - m_C$	1.19	1.22	1.26
Molecular refraction, $\frac{n - 1}{d} M$	C. 94.82	96.10	99.03

As refraction and density diminish together for rise of temperature, the above values are practically constant for all temperatures at which the salt is capable of existing.

Rubidium Ferrous Sulphate, $Rb_2Fe(SO_4)_2 \cdot 6H_2O$.

There is a perfect cleavage parallel $r'\{201\}$. The cleaved surfaces yield single images of the collimator slit of the goniometer, at 180° to the r' face left on each fragment.

Volume.

Relative Density.—The following four determinations were made with different crops.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
4.3513	2.5150
3.9436	2.5202
3.4670	2.5123
3.4734	2.5146

Mean 2.5155

Perrot gives the density at 15° as 2.51.

$$\text{Molecular Volume. } \frac{M}{d} = \frac{526.4}{2.516} = 209.22.$$

Distance Ratios.—The axial angle ac was given in the previous memoir as $74^\circ 16'$, and the ratio of the axes

$$a : b : c = 0.7377 : 1 : 0.5004.$$

Combination with the molecular volume affords the ratios

$$\chi : \psi : \omega = 6.1832 : 8.3817 : 4.1942.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Extinction in the symmetry plane occurs nearly normal and parallel to the basal plane $c\{001\}$. Two sections, ground parallel to the symmetry plane $b\{010\}$, afforded the following extinction angles with respect to the normal to $c\{001\}$.

Section 1	1° 55'
Section 2	0 55
Mean	<u>1 25</u>

FIG. 8.

The direction is forwards, as indicated in Fig. 8.

This axis is again the second median line, the direction at right angles in the symmetry plane being the first median line.

The *sign of double refraction* is, as usual in the series, positive, a fact demonstrated by use of the quarter undulation mica, and by the relations of the refractive indices.

As the axial angle ac is $74^\circ 16'$, the orientations are as under.

The first median line ϵ lies in the acute angle of the morphological axes ac , and is inclined $1^\circ 25'$ to the axis a .

The second median line α lies in the obtuse angle ac , and is inclined $17^\circ 9'$ to the vertical axis c .

The first median line is also inclined $91^\circ 25'$ to the normal to $c\{001\}$, and $17^\circ 9'$ to the normal to $a\{100\}$.

Perrot states that the first median line is inclined $91^\circ 30'$ to the normal to $\{001\}$, practically agreeing with the value now given.

Optic Axial Angle.—The following measurements were obtained with three pairs of section-plates ground perpendicular to the first and second median lines.

Determination of apparent Angle in Air of Rubidium Ferrous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	125° 9'	124° 5'	124° 51'	124° 42'
C	125 16	124 12	124 58	124 49
Na	125 39	124 47	125 9	125 12
Tl	126 8	125 28	125 26	125 41
F	126 40	126 20	125 48	126 16

Determination of true Optic Axial Angle of Rubidium Ferrous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean 2Va.
Li	{ 1	73° 5'	1a	105° 54'	73° 27'	73° 24'
	{ 2	73 2	2a	105 58	73 24	
	{ 3	64 48	3a	92 2	73 21	
C	{ 1	73 3	1a	105° 50'	73 27	73 23
	{ 2	73 0	2a	105 56	73 23	
	{ 3	64 44	3a	91 58	73 20	
Na	{ 1	72 56	1a	105 41	73 26	73 21
	{ 2	72 53	2a	105 47	73 21	
	{ 3	64 30	3a	91 44	73 16	
Tl	{ 1	72 48	1a	105 32	73 24	73 18
	{ 2	72 45	2a	105 40	73 19	
	{ 3	64 15	3a	91 30	73 11	
F	{ 1	72 35	1a	105 21	73 20	73 13
	{ 2	72 32	2a	105 29	73 14	
	{ 3	64 0	3a	91 16	73 6	

The immersion liquid was toluene for the first two pairs of sections, and monobromonaphthalene for the third pair.

The order of the colours fringing the hyperbolic brushes of the interference figures in toluene, using white light, confirms the order of dispersion; one brush was nearly colourless, that corresponding to the optic axis emerging between $a\{100\}$ and $r\{201\}$, whilst the other was clearly fringed with red on the side nearest the colourless brush and blue on the outer side, proving that the true angle for red is greater than the angle for blue. The fringes, when the observation is made in air, confirm the measured values, one brush being only very faintly coloured, while the other was brightly tinted red outside and blue within, proving that the apparent angle in air is greatest for blue.

Perrot estimated the true angle to be $73^\circ 2'$ for sodium light by observation with one plate in oil and utilisation of the medium index of refraction β of the crystal.

Dispersion of the Median Lines.—The phenomena and measurements in toluene prove that the first median line lies nearer to the morphological axis a for blue light than for red, the amount of dispersion between Li and F being about $10'$.

Effect of Rise of Temperature on the Optic Axial Angle.—One of the section-plates perpendicular to the first median line was examined at 70°, and the angle for sodium light was found to be 126° 0'. Hence the angle appears to increase about 1° for 50° rise of temperature.

Refractive Indices.—Six prisms were ground upon five different crystals. The results are given in the table, p. 395.

The β index in a vacuum is accurately expressed, for any wavelength λ as far as F, by the formula

$$\beta = 1.4747 + \frac{508\,396}{\lambda^2} - \frac{1\,863\,100\,000\,000}{\lambda^4} + \dots$$

The α indices are reproduced if the constant 1.4747 is diminished by 0.0059 and the γ values if it is increased by 0.0103. The smaller amount of the former difference again determines positive double refraction.

Alteration of Refraction by Rise of Temperature.—Determinations of refractive index at 60°, beyond which the observations cannot satisfactorily be carried on account of decomposition, indicate that at this temperature the indices are lower by about 0.0014 than they are at the ordinary temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The indices of refraction lead to the following ratios.

$$a : b : c = 1.0040 : 1 : 0.9931.$$

Molecular Optical Constants.—These are as under.

Axis of optical ellipsoid.....	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1128	0.1140	0.1160
	{ G. 0.1153	0.1164	0.1186
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 59.40	60.02	61.09
	{ G. 60.70	61.30	62.41
Specific dispersion, $n_g - n_c \dots \dots \dots$	0.0025	0.0024	0.0026
Molecular dispersion, $m_g - m_c \dots \dots \dots$	1.30	1.28	1.32
Molecular refraction, $\frac{n - 1}{d} M \dots \dots \dots$	C. 100.36	101.58	103.71

These values will be substantially permanent for all temperatures at which the salt is capable of existence, as refraction and density vary together.

Cesium Ferrous Sulphate, Cs₂Fe(SO₄)₂.6H₂O.

There is a good cleavage parallel $r'\{\bar{2}01\}$. The cleaved surfaces were found to yield excellent images of the goniometer signal precisely at 180° to the r' face on the fragment examined.

Determination of Refractive Indices of Rubidium Ferrous Sulphate.

Index.	Nature of light.	Crystal 1, 1 prism.	Crystal 2, 1 prism.	Crystal 3, 2 prisms.	Crystal 4, 1 prism.	Crystal 5, 1 prism.	Mean refrac- tive index.	Perrot's values.
α vibrations parallel 2nd median line α .	Li	1.4792	1.4783	1.4795	1.4787	—	1.4789	—
	C	1.4795	1.4786	1.4799	1.4790	—	1.4783	1.4791
	Na	1.4818	1.4809	1.4821	1.4812	—	1.4815	1.4812
	Tl	1.4843	1.4832	1.4844	1.4837	—	1.4839	—
	F	1.4872	1.4865	1.4876	1.4867	—	1.4870	1.4868
	G	1.4916	—	—	—	—	1.4916	1.4916
β vibrations parallel symmetry axis b (h).	Li	1.4849	—	1.4852	1.4845	1.4841	1.4847	—
	C	1.4854	—	1.4856	1.4848	1.4846	1.4851	1.4847
	Na	1.4877	—	1.4878	1.4870	1.4870	1.4874	1.4870
	Tl	1.4899	—	1.4903	1.4895	1.4893	1.4898	—
	F	1.4932	—	1.4932	1.4926	1.4925	1.4929	1.4926
	G	1.4975	—	—	—	1.4971	1.4973	—
γ vibrations parallel 1st median line ϵ .	Li	—	1.4947	1.4954	—	1.4947	1.4949	—
	C	—	1.4950	1.4958	—	1.4950	1.4953	1.4953
	Na	—	1.4974	1.4981	—	1.4976	1.4977	1.4978
	Tl	—	1.5000	1.5008	—	1.5002	1.5003	—
	F	—	1.5033	1.5037	—	1.5033	1.5034	1.5036
	G	—	—	—	—	1.5080	1.5080	—

Volume.

Relative Density.—Four determinations with excellent clear crystals of this salt from two preparations gave the accompanying numbers.

Weight of salt employed.	Sp. gr. at 20°/4°.
4·0813	2·7886
3·4243	2·7918
3·8732	2·7902
3·5673	2·7931

Mean **2·7909**

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{621·4}{2·791} = 222·64.$$

Distance Ratios.—The axial angle ac was previously shown to be 73° 8' and the morphological axial ratio

$$a : b : c = 0·7261 : 1 : 0·4953.$$

The distance ratios obtained by combining these with the molecular volume are as under.

$$\chi : \psi : \omega = 6·2799 : 8·6487 : 4·2837.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The extinction angle in the symmetry plane with respect to the normal to the basal plane was determined on two section-plates ground parallel to the symmetry plane $b\{010\}$, the results being as follows.

Section 1	11° 10'
Section 2	11 40
Mean	11 25

FIG. 9.

The direction is forwards, as represented in Fig. 9.

This axis is, as usual in the series, the second median line, and the other axis at right angles to this in the symmetry plane is the first median line.

The *sign of double refraction* is again positive, as determined by the relations of the three refractive indices, and demonstrated by means of the quarter undulation mica.

The monoclinic angle being $73^{\circ} 8'$, the orientations are :

The first median line ϵ lies in the acute angle ac of the morphological axes, and is inclined $11^{\circ} 25'$ to the axis a .

The second median line a lies in the obtuse angle ac , and is inclined $28^{\circ} 17'$ to the vertical axis c .

The first median line is also inclined $101^{\circ} 25'$ to the normal to $c\{001\}$ and $28^{\circ} 17'$ to the normal to $a\{100\}$.

Optic Axial Angle.—The apparent angle in air is very large and difficult to measure. Only one of the three section-plates, which were ground perpendicular to the first median line, afforded trustworthy readings. The mean angle obtained for sodium light was $132^{\circ} 47'$. The dispersion is very small, the angle being slightly larger for blue than for red, one of the brushes being almost free from colour when examined in white light, whilst the other is tinted blue on the side nearest the first median line and red outside.

Determination of the true Optic Axial Angle of Cæsium Ferrous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean value of 2Va.
Li.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 67^{\circ} 10' \\ 67 \quad 8 \\ 67 \quad 12 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 92^{\circ} 5' \\ 92 \quad 10 \\ 92 \quad 20 \end{array} \right.$	$\left\{ \begin{array}{l} 75^{\circ} 4' \\ 75 \quad 1 \\ 75 \quad 0 \end{array} \right.$	$\left. \begin{array}{l} 75^{\circ} \\ 2' \end{array} \right\}$
C.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 67 \quad 4 \\ 67 \quad 5 \\ 67 \quad 7 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 92 \quad 0 \\ 92 \quad 7 \\ 92 \quad 15 \end{array} \right.$	$\left\{ \begin{array}{l} 75 \quad 2 \\ 75 \quad 0 \\ 74 \quad 58 \end{array} \right.$	$\left. \begin{array}{l} 75^{\circ} \\ 0 \end{array} \right\}$
Na.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 66 \quad 41 \\ 66 \quad 45 \\ 66 \quad 42 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 91 \quad 45 \\ 91 \quad 44 \\ 92 \quad 0 \end{array} \right.$	$\left\{ \begin{array}{l} 74 \quad 52 \\ 74 \quad 56 \\ 74 \quad 46 \end{array} \right.$	$\left. \begin{array}{l} 74^{\circ} \\ 51 \end{array} \right\}$
Tl.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 66 \quad 11 \\ 66 \quad 20 \\ 66 \quad 17 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 91 \quad 25 \\ 91 \quad 20 \\ 91 \quad 45 \end{array} \right.$	$\left\{ \begin{array}{l} 74 \quad 40 \\ 74 \quad 50 \\ 74 \quad 35 \end{array} \right.$	$\left. \begin{array}{l} 74^{\circ} \\ 42 \end{array} \right\}$
F.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 65 \quad 40 \\ 65 \quad 50 \\ 65 \quad 40 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 90 \quad 58 \\ 90 \quad 50 \\ 91 \quad 20 \end{array} \right.$	$\left\{ \begin{array}{l} 74 \quad 30 \\ 74 \quad 42 \\ 74 \quad 20 \end{array} \right.$	$\left. \begin{array}{l} 74^{\circ} \\ 31 \end{array} \right\}$

The three pairs of plates ground perpendicular to the first and second median lines were immersed in monobromonaphthalene for the determinations of 2Ha and 2Ho. The order of dispersion was confirmed by observations of the interference figures afforded by the plates perpendicular to the first median line when immersed in benzene, whose refractive index is almost identical with the mean index of the crystal. The brushes were observed in white light to be

each fringed with red on the side nearest the first median line and blue outside, indicating a larger angle for red than for blue. Actual measurement showed a difference of $17'$ between C and Tl, confirming the above values.

Dispersion of the Median Lines.—The observations in benzene further showed that the axis emerging between $a\{100\}$ and $r\{201\}$ was slightly less dispersed than that on the other side of the median line, hence the first median line is nearer to the morphological axis a for blue than it is for red, the amount of difference being only a very few minutes.

Effect of Rise of Temperature on the Optic Axial Angle.—The section-plate which afforded the determination of 2E was heated to 70° , when the angle for sodium light was found to be $131^\circ 20'$. Hence the apparent angle in air diminishes about $1\frac{1}{2}^\circ$ for 50° rise of temperature.

Refractive Indices.—These were determined by means of six prisms ground upon four different crystals. The results are as follows.

Determination of Refractive Indices of Cesium Ferrous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 1 prism.	Crystal 4, 1 prism.	Mean refractive index.
α Vibrations parallel 2nd median line α .	Li	1.4977	1.4973	1.4978	1.4976	1.4976
	C	1.4981	1.4977	1.4981	1.4981	1.4980
	Na	1.5004	1.5001	1.5005	1.5002	1.5003
	Tl	1.5031	1.5026	1.5028	1.5026	1.5028
	F	1.5062	1.5057	1.5063	1.5061	1.5061
	G	1.5107	1.5103	—	1.5104	1.5105
β Vibrations parallel symmetry axis b (β).	Li	1.5009	1.5005	1.5008	—	1.5007
	C	1.5013	1.5009	1.5011	—	1.5011
	Na	1.5035	1.5035	1.5036	—	1.5035
	Tl	1.5061	1.5060	1.5063	—	1.5061
	F	1.5092	1.5093	1.5094	—	1.5093
	G	1.5135	1.5140	—	—	1.5137
γ Vibrations parallel 1st median line γ .	Li	1.5070	1.5061	—	1.5065	1.5065
	C	1.5074	1.5065	—	1.5069	1.5069
	Na	1.5098	1.5090	—	1.5093	1.5094
	Tl	1.5125	1.5117	—	1.5120	1.5121
	F	1.5159	1.5148	—	1.5153	1.5153
	G	1.5203	1.5193	—	1.5198	1.5198

The β index reduced to a vacuum is accurately represented, for any wave-length λ as far as F, by the formula

$$\beta = 1.4911 + \frac{462\ 915}{\lambda^2} - \frac{645\ 800\ 000\ 000}{\lambda^4} + \dots$$

The α indices are closely reproduced if the constant 1.4911 is diminished by 0.0032 and the γ values if it is increased by 0.0059. The smaller amount of the former difference determines the positive nature of the double refraction.

Alteration of Refraction by Rise of Temperature.—Determinations of refractive index were made at 70°, and the mean results indicate that the indices are lower at this temperature by about 0.0017 than they are at the ordinary temperature. The prisms appeared capable of withstanding this temperature for some little time before opacity ensued, affording another indication of the greater stability of the cæsium salts than those of rubidium and potassium.

Ratio of Axes of Optical Velocity Ellipsoid.—The foregoing refractive indices afford the following ratios.

$$a : b : c = 1.0021 : 1 : 0.9961.$$

Molecular Optical Constants.—The calculated values of these constants are as under.

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1051	0.1057	0.1067
	{ G. 0.1073	0.1079	0.1090
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$	{ C. 65.31	65.65	66.29
	{ G. 66.69	67.04	67.71
Specific dispersion, $n_G - n_C$	0.0022	0.0022	0.0023
Molecular dispersion, $m_G - m_C$	1.38	1.39	1.42
Molecular refraction, $\frac{n - 1}{d} M$	C. 111.01	111.70	112.99

These values will be sensibly constant for all temperatures at which the salt is able to exist, as refraction and density are altered in the same direction by change of temperature.

SALTS CONTAINING MANGANESE.

Further efforts to prepare the potassium salt of this group have been unavailing.

Rubidium Manganous Sulphate, $Rb_2Mn(SO_4)_2 \cdot 6H_2O$.

An excellent *cleavage* is developed parallel $r'\{201\}$; the cleaved surfaces exhibit sharply defined images of the goniometer signal at 180° to the r' face left on each fragment.

Volume.

Relative Density.—The following four determinations were carried out with different crops of clear crystals.

400 TUTTON: CRYSTALLOGRAPHY OF SULPHATES OF

Weight of salt employed.	Sp. gr. at 20°/4°.
5·6872	2·4542
4·5172	2·4561
4·3798	2·4637
4·2636	2·4618

Mean **2·4590**

Perrot gives the density at 15° as 2·49.

$$\text{Molecular Volume.} - \frac{M}{d} = \frac{525\cdot4}{2\cdot459} = 213\cdot66.$$

Distance Ratios.—The axial angle having been shown to be 74° 3', and the axial ratio

$$a : b : c = 0\cdot7382 : 1 : 0\cdot4950,$$

the distance ratios obtained by combination of these constants with the molecular volume are

$$\chi : \psi : w = 6\cdot2542 : 8\cdot4723 : 4\cdot1938.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Some difficulty was experienced in determining the extinction angle in the symmetry plane by means of sections ground parallel to that plane, on account of slight surface alteration which interfered with the sharpness of the extinction. Four section-plates were ground as rapidly as possible consistent with due care, but only approximate determinations of extinction could be carried out with each. The mean, however, is probably very near the truth, and was 1° 0' forward from the normal to $c\{001\}$ as shown in Fig. 10, the limits being about a degree each side of this position.

FIG. 10.

This direction is again the second median line, the other axis of the optical ellipsoid at right angles to this in the symmetry plane being the first median line.

The *sign of double refraction* is likewise positive, a fact which was

proved by means of the quarter undulation mica, and by the mutual relations of the three refractive indices.

The axial angle being $74^{\circ} 3'$, the orientations may be stated as follows.

The first median line ϵ lies in the acute angle of the morphological axes ac , and is inclined $1^{\circ} 0'$ to the axis a .

The second median line a lies in the obtuse angle ac , and is inclined $16^{\circ} 57'$ to the vertical axis c .

The first median line is also inclined $91^{\circ} 0'$ to the normal to $c\{001\}$ and $16^{\circ} 57'$ to the normal to $a\{100\}$.

Perrot gives the inclination to $\{001\}$ as $91^{\circ} 20'$.

Optic Axial Angle.—The following determinations were carried out with three pairs of section-plates ground perpendicular to the first and second median lines. They afforded clear interference figures, the surface alteration so inconvenient for the determinations of extinction being too slight to materially affect the interference figures when the sections were freshly prepared.

Determination of apparent Angle in Air of Rubidium Manganous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	$109^{\circ} 40'$	$109^{\circ} 50'$	$109^{\circ} 48'$	$109^{\circ} 46'$
C	$109 42$	$109 52$	$109 52$	$109 49$
Na	$109 52$	$110 0$	$110 0$	$109 57$
Th	$110 3$	$110 10$	$110 8$	$110 7$
F	$110 22$	$110 18$	$110 17$	$110 19$

Determination of true Optic Axial Angle of Rubidium Manganous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of $2H_a$.	No. of section perp. 2nd median line.	Observed values of $2H_o$.	Calculated values of $2V_a$.	Mean value of $2V_a$.
Li	1	$59^{\circ} 12'$	1a	$96^{\circ} 21'$	$67^{\circ} 4'$	$67^{\circ} 10'$
	2	$59 27$	2a	$96 20$	$67 17$	
	3	$59 15$	3a	$96 20$	$67 8$	
C	1	$59 8$	1a	$96 18$	$67 3$	$67 8$
	2	$59 21$	2a	$96 13$	$67 15$	
	3	$59 10$	3a	$96 15$	$67 6$	
Na	1	$58 54$	1a	$95 52$	$67 2$	$67 5$
	2	$59 0$	2a	$95 53$	$67 8$	
	3	$58 58$	3a	$95 54$	$67 4$	
Th	1	$58 40$	1a	$95 27$	$67 1$	$67 1$
	2	$58 38$	2a	$95 26$	$67 0$	
	3	$58 40$	3a	$95 27$	$67 1$	
F	1	$58 20$	1a	$94 55$	$66 58$	$66 55$
	2	$58 10$	2a	$94 55$	$66 50$	
	3	$58 15$	3a	$94 50$	$66 56$	

Monobromonaphthalene was the immersion liquid employed in the

measurements of 2Ha and 2Ho. The order of dispersion was confirmed by observation in white light of 2Ha in turpentine. Whilst one optic axial brush was almost free from a trace of colour, the other was brightly tinted red inside nearest the first median line, and blue outside, thus indicating a larger angle for red than for blue. An observation in toluene afforded confirmatory evidence. Perrot gives $67^{\circ} 38'$ for the angle in yellow light, deduced from an observation with one plate in oil.

Dispersion of the Median Lines.—In conducting the observation in turpentine the position of the section was carefully noted with reference to the crystal faces upon its broad edges, and it was found that the immovable optic axis was the one which emerges between $\alpha\{100\}$ and $r\{201\}$, hence the first median line is dispersed in such a manner that for blue light it lies nearer by about $10'$ to the morphological axis a than it does for red.

Effect of Rise of Temperature on the Optic Axial Angle.—One of the section-plates perpendicular to the first median line was heated to 70° , at which temperature the angle for sodium light was found to be $110^{\circ} 50'$. Hence the apparent angle in air increases about a degree for 50° rise of temperature.

Refractive Indices.—Six prisms were employed, ground upon four different crystals. The following table represents the results.

Determination of Refractive Indices of Rubidium Manganous Sulphate.

Index.	Nature of light.	Crystal 1, 1 prism.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Crystal 4, 1 prism.	Mean refractive index.	Perrot's values.
α Vibs. par. 2nd median line	Li	—	1.4737	1.4741	1.4746	1.4741	—
	C	—	1.4741	1.4745	1.4749	1.4745	1.4741
	Na	—	1.4764	1.4767	1.4770	1.4767	1.4764
	Tl	—	1.4787	1.4791	1.4796	1.4791	—
	F	—	1.4814	1.4823	1.4826	1.4821	1.4818
	G	—	1.4859	—	1.4868	1.4864	—
β Vibs. par. symmetry axis b (h)	Li	1.4770	1.4782	1.4784	1.4788	1.4781	—
	C	1.4775	1.4787	1.4787	1.4792	1.4785	1.4785
	Na	1.4797	1.4807	1.4809	1.4816	1.4807	1.4809
	Tl	1.4818	1.4833	1.4835	1.4839	1.4831	—
	F	1.4850	1.4860	1.4863	1.4869	1.4860	1.4864
	G	—	1.4901	1.4910	1.4910	1.4907	—
γ Vibs. par. 1st median line c	Li	1.4872	1.4882	1.4886	—	1.4880	—
	C	1.4877	1.4887	1.4889	—	1.4884	1.4886
	Na	1.4899	1.4910	1.4911	—	1.4907	1.4910
	Tl	1.4926	1.4935	1.4939	—	1.4933	—
	F	1.4958	1.4969	1.4967	—	1.4965	1.4970
	G	—	1.5012	1.5019	—	1.5015	—

The β index reduced to a vacuum is accurately represented, for any wave-length λ as far as F, by the formula

$$\beta = 1.4694 + \frac{420\,367}{\lambda^2} - \frac{491\,200\,000\,000}{\lambda^4} + \dots$$

The α indices are closely reproduced by the formula if the constant 1.4694 is diminished by 0.0040 and the γ values if it is increased by 0.0100. The positive nature of the double refraction is evident from the smaller amount of the former difference from β .

Effect of Rise of Temperature on Refraction.—The indices were determined at 70°, and found to be about 0.0018 less at this temperature than the numbers given in the preceding table.

Ratio of Axes of Optical Velocity Ellipsoid.—The foregoing refractive indices lead to the following ratios.

$$a : b : c = 1.0027 : 1 : 0.9933.$$

Molecular Optical Constants.—The indices also afford the following constants when combined with the density and molecular weight.

Axis of optical ellipsoid	α .	b .	c .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1145	0.1153	0.1173
	{ G. 0.1169	0.1178	0.1200
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 60.14	60.58	61.65
	{ G. 61.43	61.89	63.04
Specific dispersion, $n_G - n_C \dots$	0.0024	0.0025	0.0027
Molecular dispersion, $m_G - m_C \dots$	1.29	1.31	1.39
Molecular refraction, $\frac{n}{d} \cdot \frac{1}{M} \dots C.$	101.47	102.32	104.44

As in the case of the salts previously described, refraction and density vary together for change of temperature, and therefore the above constants will remain sensibly constant for all variations of temperature within whose limits the crystallised substance is capable of existing.

Cæsium Manganous Sulphate, $\text{Cs}_2\text{Mn}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

In addition to the faces described in the previous memoir as being observed upon the crystals of this salt, many of the crystals specially prepared for the optical work exhibited well-defined small faces of the primary hemipyramid $\sigma\{111\} - P$.

There is a perfect *cleavage* parallel $r'\{\bar{2}01\}$. The fractured surfaces afford sharp single images of the goniometer signal, and at precisely 180° to the r' face left on each fragment.

Volume.

Relative Density.—Four determinations with different crops of perfectly clear, transparent crystals yielded the following results.

Weight of salt employed.	Sp. gr. at 20°/4°.
4.4329	2.7365
4.4677	2.7393
3.2177	2.7385
2.4249	2.7356

Mean 2.7375

$$\text{Molecular Volume.} - \frac{M}{d} = \frac{620.4}{2.738} = 226.59.$$

Distance Ratios.—Combining the above molecular volume with the morphological constants given in the previous memoir, $\beta = 72^\circ 53'$ and

$$a : b : c = 0.7274 : 1 : 0.4913,$$

we obtain for the distance ratios the following values.

$$\chi : \psi : \omega = 6.3441 : 8.7217 : 4.2850.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two excellent sections were obtained by grinding, parallel to the symmetry plane $b\{010\}$, exhibiting no tendency to deterioration as in case of rubidium manganous sulphate. The extinction angles afforded by them with respect to the normal to $c\{001\}$ were in each case precisely $8^\circ 20'$, the direction being forwards, as indicated in Fig. 11.

FIG. 11.

This direction is the second median line, and the first median line is, as usual in the series, perpendicular to it in the symmetry plane.

The *sign of double refraction* is positive, determined as in the cases of the salts previously described. As the morphological axial angle ac is $72^\circ 53'$, the orientations are as under.

The first median line ϵ lies in the acute angle of the morphological axes ac , and is inclined $8^\circ 20'$ to the axis a .

The second median line a lies in the obtuse angle ac , and is inclined $25^\circ 27'$ to the vertical axis c .

The first median line is likewise inclined $98^{\circ} 20'$ to the normal to $c\{001\}$, and $25^{\circ} 27'$ to the normal to $a\{100\}$.

Optic Axial Angle.—Three pairs of sections afforded the following values.

Determination of apparent Angle in Air of Cæsium Manganous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	$97^{\circ} 26'$	$97^{\circ} 20'$	$97^{\circ} 21'$	$97^{\circ} 22'$
C	$97 \ 21$	$97 \ 15$	$97 \ 17$	$97 \ 18$
Na	$97 \ 1$	$96 \ 54$	$96 \ 55$	$96 \ 57$
Tl	$96 \ 41$	$96 \ 28$	$96 \ 30$	$96 \ 33$
F	$96 \ 10$	$95 \ 54$	$95 \ 55$	$96 \ 0$

Determination of true Optic Axial Angle of Cæsium Manganous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean value of 2Va.
Li.....	1	$53^{\circ} 48'$	1a	$102^{\circ} 55'$	$60^{\circ} 6'$	$60^{\circ} 10'$
	2	$53 \ 49$	2a	$102 \ 46$	$60 \ 10$	
	3	$54 \ 0$	3a	$102 \ 59$	$60 \ 15$	
C	1	$53 \ 43$	1a	$102 \ 50$	$60 \ 3$	$60 \ 7$
	2	$53 \ 42$	2a	$102 \ 42$	$60 \ 5$	
	3	$53 \ 56$	3a	$102 \ 55$	$60 \ 12$	
Na.....	1	$53 \ 26$	1a	$102 \ 33$	$59 \ 54$	$59 \ 57$
	2	$53 \ 21$	2a	$102 \ 21$	$59 \ 54$	
	3	$53 \ 38$	3a	$102 \ 36$	$60 \ 4$	
Tl.....	1	$53 \ 5$	1a	$102 \ 13$	$59 \ 44$	$59 \ 46$
	2	$52 \ 57$	2a	$102 \ 0$	$59 \ 41$	
	3	$53 \ 16$	3a	$102 \ 12$	$59 \ 53$	
F	1	$52 \ 30$	1a	$101 \ 50$	$59 \ 21$	$59 \ 28$
	2	$52 \ 30$	2a	$101 \ 35$	$59 \ 26$	
	3	$52 \ 45$	3a	$101 \ 40$	$59 \ 38$	

The immersion liquid was monobromonaphthalene in each determination of 2Ha and 2Ho. The order of dispersion was confirmed by observation of the interference figures in white light afforded by the sections perpendicular to the first median line, both in toluene and in cedar oil, the mean of whose refractive indices is near the mean index of the crystal.

Dispersion of the Median Lines.—The observations in toluene and cedar oil in white and in monochromatic light indicate that the optic axis which emerges not far from normal to $a\{100\}$ is only slightly dispersed, for blue being nearest to the first median line, whilst the other axis exhibits considerable dispersion, being again for blue nearest the median line. Hence the first median line is dispersed so that for blue it lies nearer to the morphological axis a than for red,

the amount indicated by the observations in monochromatic light being about 15' between Li and F.

Effect of Rise of Temperature on the Optic Axial Angle.—Section 1 was heated to 70°, and at this temperature the apparent angle in air measured 94° 35' for sodium light. Hence 2E diminishes about 2½° for 50° rise of temperature.

Refractive Indices.—The following values were obtained from six prisms ground upon three different crystals.

Determination of Refractive Indices of Cesium Manganous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Mean refractive index.
α Vibrations parallel 2nd median line π	Li	1·4619	1·4920	1·4916	1·4918
	C	1·4923	1·4923	1·4919	1·4922
	Na	1·4948	1·4947	1·4944	1·4946
	Tl	1·4974	1·4972	1·4970	1·4972
	F	1·5006	1·5003	1·5000	1·5003
	G	1·5050	1·5046	1·5041	1·5046
β Vibrations parallel symmetry axis b ($\bar{b}$)	Li	1·4933	1·4933	1·4937	1·4936
	C	1·4938	1·4942	1·4941	1·4940
	Na	1·4962	1·4968	1·4968	1·4966
	Tl	1·4988	1·4991	1·4993	1·4991
	F	1·5020	1·5022	1·5023	1·5022
	G	1·5064	1·5066	1·5068	1·5066
γ Vibrations parallel 1st median line ϵ .	Li	1·4994	1·4997	1·4995	1·4995
	C	1·4998	1·5001	1·4999	1·4999
	Na	1·5025	1·5027	1·5024	1·5025
	Tl	1·5050	1·5052	1·5051	1·5051
	F	1·5082	1·5084	1·5083	1·5083
	G	1·5128	1·5130	1·5128	1·5129

The intermediate axis β in a vacuum is expressed, for any wavelength λ as far as F, by the formula

$$\beta = 1\cdot4803 + \frac{722\ 115}{\lambda^2} - \frac{4\ 944\ 600\ 000\ 000}{\lambda^4} + \dots$$

The α indices are reproduced very closely if the constant 1·4803 is diminished by 0·0019, and the γ indices if the constant is increased by 0·0059. The former being the smaller difference, determines the positive nature of the double refraction.

Effect of Rise of Temperature on Refraction.—The indices afforded by determinations at 70° indicate a lowering of about 0·0018 by 50° rise of temperature.

Ratio of Ares of Optical Velocity Ellipsoid.—The foregoing refractive indices yield the following ratios.

$$a : b : c = 1\cdot0013 : 1 : 0\cdot9961.$$

Molecular Optical Constants.—Beneath are given the calculated values of these constants.

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1061	0.1064	0.1075
	{ G. 0.1083	0.1087	0.1098
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 65.81	66.01	66.68
	{ G. 67.21	67.43	68.14
Specific dispersion, $n_G - n_C$	0.0022	0.0023	0.0023
Molecular dispersion, $m_G - m_C$	1.40	1.42	1.45
Molecular refraction, $\frac{n - 1}{d} M$	C. 111.62	112.03	113.36

These values are again practically independent of temperature, as the refraction diminishes along with the density on increase of temperature.

SALTS CONTAINING NICKEL.

Potassium Nickelous Sulphate, $K_2Ni(SO_4)_2 \cdot 6H_2O$.

The crystals exhibit a perfect *cleavage* parallel to $r'\{201\}$. The fractures yield clearly defined single images of the goniometer signal at exactly 180° in each case from the r' face left on the fragment.

Volume.

Relative Density.—Four determinations were made with excellent crystals of different crops belonging to two independent preparations. The numbers obtained were as under.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
3.1592	2.2305
3.0850	2.2344
4.2979	2.2314
4.5574	2.2357

Mean **2.2330**

The value of Kopp, 2.123, quoted by Perrot, is certainly incorrect.

Molecular Volume.— $\frac{M}{d} = \frac{436.9}{2.233} = 195.65$.

Distance Ratios.—The data given in the previous memoir, $\beta = 75^\circ 0'$ and

$$a : b : c = 0.7379 : 1 : 0.5020,$$

when combined with the molecular volume, afford the following distance ratios.

$$\chi : \psi : \omega = 6.0342 : 8.1775 : 4.1051.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Three sections were

ground parallel to the symmetry plane, as the indications afforded by the first two differed somewhat from those of Murmann and Rotter; they exhibited the following extinction angles with respect to the normal to $c\{001\}$.

Section 1	6° 15'
Section 2	6 25
Section 3	6 15
Mean.....	<u>6 18</u>

FIG. 12.

The direction is behind the normal, nearer to the vertical axis of the crystal, as represented in Fig. 12. Murmann and Rotter give $5^{\circ} 26'$.

This direction is again the second median line, the first median line being at right angles to this, also in the symmetry plane.

The *sign of double refraction* is again positive. The crystallographical axial angle ac being $75^{\circ} 0'$, the following statement represents the orientations of the axes of the optical ellipsoid.

The first median line ϵ lies in the obtuse angle of the morphological axes ac , and is inclined $6^{\circ} 18'$ to the axis a .

The second median line α also lies in the obtuse axial angle ac , and is inclined $8^{\circ} 42'$ to the vertical axis c .

The first median line is also inclined $83^{\circ} 42'$ to the normal to $c\{001\}$, and $8^{\circ} 42'$ to the normal to $a\{100\}$.

Optic Axial Angle.—The observations were carried out with six excellent section-plates, three perpendicular to each median line as usual. Owing to the deep green colour of the crystals, a considerable amount of absorption occurs at the red end of the spectrum, so that very intense monochromatic light is necessary for measurements with the thick plates rendered imperative by the feebleness of the double refraction, the receiving slit of the monochromatic illuminator being required to be somewhat more widely opened than usual.

Determination of apparent Angle in Air of Potassium Nickelous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	130° 15'	130° 30'	130° 10'	130° 18'
C	130 20	130 35	130 15	130 23
Na.....	131 2	131 5	130 45	130 57
Tl	131 42	131 40	131 20	131 34
F.....	132 30	132 22	132 14	132 22

Murmann and Rotter state that the apparent angle in air is less for red than for blue, which agrees with the above, and give $131^{\circ} 45'$ as the angle, presumably for sodium light.

Determination of true Optic Axial Angle of Potassium Nickelous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean value of 2Va.
Li.....	{ 1	66° 50'	1a	91° 0'	75° 21'	75° 21'
	{ 2	67 0	2a	91 15	75 22	
	{ 3	66 50	3a	91 4	75 19	
C	{ 1	66 43	1a	90 52	75 20	75 19
	{ 2	66 53	2a	91 7	75 20	
	{ 3	66 45	3a	90 58	75 18	
Na.....	{ 1	66 22	1a	90 27	75 16	75 16
	{ 2	66 32	2a	90 42	75 16	
	{ 3	66 25	3a	90 35	75 16	
Tl.....	{ 1	66 1	1a	90 0	75 13	75 13
	{ 2	66 11	2a	90 15	75 13	
	{ 3	66 6	3a	90 5	75 14	
F	{ 1	65 30	1a	89 24	75 8	75 9
	{ 2	65 40	2a	89 39	75 8	
	{ 3	65 40	3a	89 30	75 12	

The immersion liquid employed was monobromonaphthalene.

The order of dispersion was confirmed by observations in toluene, whose refractive index is exactly the same for sodium light as the β index of the crystal, and only very slightly less than the mean index of the crystal; also in benzene whose index is slightly higher than the mean index of the crystal. In both cases the angle for red was clearly larger than for blue.

Murmann and Rotter represent the true optic axial angle as $75^{\circ} 36'$.

Dispersion of the Median Lines.—The observations in toluene and benzene indicated that the optic axis which emerges nearly perpendicular to the possible face $r\{201\}$ remains almost constant for all colours, whilst the other axis which emerges nearly normal to $r'\{201\}$ is clearly dispersed, being nearer to the stationary axis for blue than

for red. Hence the first median line is dispersed so that it lies nearer to the morphological axis a for red than it does for blue, the amount indicated in toluene being about $10'$ between Li and F. This order of dispersion of the median lines agrees with the observation of Murmann and Rotter, but the dispersion of the axes is the inverse. The observations above given were repeated and confirmed with each of the three sections.

Effect of Rise of Temperature on the Optic Axial Angle.—Section 3 was heated to 70° , and the angle for sodium light found to be $133^\circ 25'$. Hence the apparent angle in air increases by about $2\frac{1}{2}^\circ$ for 50° rise of temperature.

Refractive Indices.—The following table represents the results obtained with six prisms, ground upon four different crystals.

Determination of Refractive Indices of Potassium Nickelous Sulphate.

Index.	Nature of light.	Crystal 1, 1 prism.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Crystal 4, 1 prism.	Mean refractive index.
α Vibrations parallel 2nd median line π .	Li	—	1·4809	1·4807	1·4812	1·4809
	C	—	1·4813	1·4812	1·4815	1·4813
	Na	—	1·4836	1·4834	1·4839	1·4836
	Tl	—	1·4861	1·4857	1·4862	1·4860
	F	—	1·4889	1·4886	1·4893	1·4889
	G	—	1·4933	1·4931	1·4934	1·4933
β Vibrations parallel sym- metry axis b η .	Li	1·4888	1·4889	1·4889	1·4889	1·4889
	C	1·4892	1·4893	1·4895	1·4893	1·4893
	Na	1·4915	1·4914	1·4919	1·4917	1·4916
	Tl	1·4938	1·4941	1·4942	1·4942	1·4941
	F	1·4970	1·4971	1·4974	1·4973	1·4972
	G	1·5014	1·5015	1·5017	1·5013	1·5015
γ Vibrations parallel 1st median line ϵ .	Li	1·5024	1·5025	1·5017	—	1·5022
	C	1·5028	1·5029	1·5021	—	1·5026
	Na	1·5053	1·5052	1·5049	—	1·5051
	Tl	1·5077	1·5078	1·5075	—	1·5077
	F	1·5111	1·5110	1·5107	—	1·5109
	G	1·5155	1·5154	1·5151	—	1·5153

There was a considerable amount of absorption at the red end of the spectrum produced by each of the prisms, but the lime-light afforded ample illumination of the refracted images of the spectrometer slit for Li and C light, even when the openings of the receiving and exit slits of the monochromatic illuminator were as fine as usual.

Three rough values of the indices for the middle portion of the spectrum are given by Murmann and Rotter, $\alpha = 1·483$, $\beta = 1·490$, and $\gamma = 1·502$; they also state that β for red = $1·487$, for yellow = $1·489$, for green = $1·492$, and for violet = $1·495$.

The intermediate index β , in a vacuum, is accurately expressed, for any wave-length λ as far as F, by the formula

$$\beta = 1.4798 + \frac{437\ 765}{\lambda^2} - \frac{496\ 200\ 000\ 000}{\lambda^4} + \dots$$

The α index is closely reproduced by the formula if the constant 1.4798 is diminished by 0.0080, and the γ values if it is increased by 0.0135. The positive nature of the double refraction is evident from the fact that the former is the smaller difference from β .

Effect of Rise of Temperature on Refraction.—The indices were found to be lower by about 0.0012 at 70° than they are at the ordinary temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The refractive indices furnish the following ratios.

$$a : b : c = 1.0054 : 1 : 0.9910.$$

Molecular Optical Constants.—The calculated values of these constants are given below.

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1276	0.1294	0.1324
	{ G. 0.1303	0.1321	0.1352
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 55.75	56.54	57.84
	{ G. 56.93	57.73	59.06
Specific dispersion, $n_G - n_C \dots$	0.0027	0.0027	0.0028
Molecular dispersion, $m_G - m_C \dots$	1.18	1.19	1.22
Molecular refraction, $\frac{n - 1}{d} M \dots$	C. 94.25	95.81	98.42

As refraction and density vary together with change of temperature, these values will be independent of temperature.

Rubidium Nickelous Sulphate, $Rb_2Ni(SO_4)_2 \cdot 6H_2O$.

The crystals of this salt exhibit a perfect *cleavage* parallel to $r'\{201\}$, in common with the other salts of the series. It was proved to be exactly 180° from r' by goniometrical measurement, the fractures yielding sharp images.

Volume.

Relative Density.—The following four determinations were made with beautifully clear crystals of this salt, derived from different crops.

Weight of salt employed.	Sp. gr. at 20°/4°.
5.1109	2.5817
4.4324	2.5833
3.8084	2.5807
3.5480	2.5848

Mean 2.5826

Perrot attributes a density of 2.56 to the salt.

$$\text{Molecular Volume. } \frac{M}{d} = \frac{529.3}{2.583} = 204.92.$$

Distance Ratios.—The axial angle ac being $73^\circ 57'$, and the axial ratio

$$a : b : c = 0.7350 : 1 : 0.5022,$$

as previously given, the distance ratios obtained by combination with the molecular volume are

$$\chi : \psi : \omega = 6.1214 : 8.3284 : 4.1825.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two sections ground parallel to the symmetry plane $b\{010\}$ afforded identical extinction angles in the stauroscope, namely, $3^\circ 25'$, with respect to the normal to $c\{001\}$, the direction being backwards, as represented in Fig. 13. This

FIG. 13.

axis of the optical ellipsoid is, as usual in the series, the second median line, and the other at right angles in the symmetry plane is the first median line.

The *sign of double refraction* is positive, determined as before. The morphological axial angle ac being $73^\circ 57'$, the orientations are as under.

The first median line c lies in the obtuse angle of the morphological axes ac , and is inclined $3^\circ 25'$ to the axis a .

The second median line a lies also in the obtuse angle ac , and is inclined $12^\circ 38'$ to the vertical axis c .

The first median line is also inclined $86^\circ 35'$ to the normal to $c\{001\}$, and $12^\circ 38'$ to the normal to $a\{100\}$. Perrot gives $85^\circ 45'$ for the former value.

Optic Axial Angle.—Six section-plates were ground, three perpendicular to the first, and three to the second median line. The angle in air is very large and difficult to observe properly, but the

following trustworthy measurements were obtained for the three brighter colours.

Determination of apparent Angle in Air of Rubidium Nickelous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
C	155° 55'	156° 15'	156° 0'	156° 3'
Na	156 45	157 25	156 50	157 0
Tl	157 30	158 40	158 5	158 5

Determination of true Optic Axial Angle of Rubidium Nickelous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean value of 2Va.
Li	{ 1	72° 55'	1a	86° 12'	82° 2'	82° 5'
	{ 2	73 12	2a	86 0	82 20	
	{ 3	72 56	3a	86 32	81 52	
C	{ 1	72 52	1a	86 9	82 2	82 4
	{ 2	73 3	2a	85 51	82 18	
	{ 3	72 48	3a	86 22	81 52	
Na	{ 1	72 32	1a	85 47	82 0	82 0
	{ 2	72 37	2a	85 34	82 10	
	{ 3	72 28	3a	85 58	81 50	
Tl	{ 1	72 7	1a	85 22	81 56	81 56
	{ 2	72 5	2a	85 5	82 4	
	{ 3	72 8	3a	85 36	81 48	
F	{ 1	71 38	1a	85 3	81 46	81 48
	{ 2	71 30	2a	84 35	81 56	
	{ 3	71 34	3a	85 6	81 42	

The immersion liquid for 2Ha and 2Ho was in every case monobromonaphthalene. The order of dispersion was confirmed by observation with white light in toluene, whose refractive index is near the mean index of the crystal. One brush was practically devoid of colour, while the other was brightly fringed with red inside (nearest the first median line), and blue outside, indicating a larger angle for red than for blue.

Perrot gives 81° 43' as the true angle for yellow light.

Dispersion of the Median Lines.—The optic axis which remains nearly constant for all wave-lengths is the one which emerges nearly normal to $r\{201\}$, the other, which is furthest removed from the stationary axis for red, is that emerging about 10° from $r\{201\}$. Hence the first median line is dispersed in such order that it lies nearer the morphological axis a for red than it does for blue, the amount of dispersion being about 10'.

Effect of Rise of Temperature on the Optic Axial Angle.—Section 2 was heated to 70°, when the apparent angle in air was found to be 158° 55'. Hence 2E increases $1\frac{1}{2}^\circ$ for 50° rise of temperature.

Refractive Indices.—Six prisms were employed, ground upon three crystals, a complementary pair upon each. The results are as follows.

Determination of Refractive Indices of Rubidium Nickelous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Mean refractive index.	Perrot's values.
α Vibrations parallel 2nd median line α .	Li	1·4870	1·4869	1·4864	1·4868	—
	C	1·4874	1·4872	1·4871	1·4872	1·4874
	Na	1·4894	1·4897	1·4894	1·4895	1·4896
	Tl	1·4922	1·4922	1·4917	1·4920	—
	F	1·4950	1·4951	1·4947	1·4949	1·4951
	G	1·4999	1·4995	1·4993	1·4996	—
β Vibrations parallel sym- metry axis β β .	Li	1·4935	1·4933	1·4932	1·4933	—
	C	1·4939	1·4937	1·4936	1·4937	1·4943
	Na	1·4962	1·4961	1·4961	1·4961	1·4967
	Tl	1·4989	1·4986	1·4986	1·4987	—
	F	1·5020	1·5017	1·5015	1·5017	1·5022
	G	1·5063	1·5062	1·5060	1·5062	—
γ Vibrations parallel 1st median line γ .	Li	1·5023	1·5023	1·5022	1·5023	—
	C	1·5027	1·5027	1·5026	1·5027	1·5033
	Na	1·5054	1·5051	1·5051	1·5052	1·5058
	Tl	1·5080	1·5079	1·5075	1·5078	—
	F	1·5112	1·5111	1·5108	1·5110	1·5118
	G	1·5157	1·5154	1·5158	1·5156	—

The intermediate axis β , corrected to a vacuum, is accurately expressed, for any wave length λ , by the following formula.

$$\beta = 1.4838 + \frac{454\ 708}{\lambda^2} - \frac{486\ 600\ 000\ 000}{\lambda^4} + \dots$$

The α indices are given by the formula very closely if the constant 1·4838 is diminished by 0·0066 and the γ values if it is increased by 0·0091. The former being the lesser difference determines the sign of double refraction to be positive.

Effect of Rise of Temperature on Refraction.—Determinations of refractive index at 70° indicate that the indices are lowered about 0·0015 for 50° rise of temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The indices just given lead to the following values of these ratios

$$a : b : c = 1.0044 : 1 : 0.9940.$$

Molecular Optical Constants.—The indices when combined with the density and molecular weight yield the following numbers for these constants.

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1115 G. 0.1139	0.1127 0.1151	0.1145 0.1169
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$	{ C. 59.00 G. 60.27	59.67 60.94	60.59 61.90
Specific dispersion, $n_G - n_C$	0.0024	0.0024	0.0024
Molecular dispersion, $m_G - m_C$	1.27	1.27	1.31
Molecular refraction $\frac{n - 1}{d} M$	C. 99.92	101.25	103.09

These values are independent of temperature, as density and refraction vary in the same direction on change of temperature.

Cæsium Nickelous Sulphate, $\text{Cs}_2\text{Ni}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

A well-defined *cleavage* parallel $r'\{201\}$ is exhibited by the crystals of this salt in common with the rest of the series. Its direction was proved to be precisely 180° from r' by measurement on the goniometer, the cleaved surfaces reflecting sharp images of the signal.

Volume.

Relative Density.—Four determinations were made with different crops derived from two independent preparations. The results are as under.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
3.5401	2.8645
3.1240	2.8701
4.1118	2.8617
4.9850	2.8649

Mean 2.8653

$$\text{Molecular Volume.} \frac{M}{d} = \frac{624.3}{2.865} = 217.91.$$

Distance Ratios.—The axial ratio was previously shown to be

$$a : b : c = 0.7270 : 1 : 0.4984,$$

and the axial angle $\beta 72^\circ 58'$. Combining these with the molecular volume, we obtain the following distance ratios.

$$\chi : \psi : \omega = 6.2289 : 8.5680 : 4.2703.$$

Optics.

The crystals of cæsium nickelous sulphate exhibit optical properties of a particularly interesting nature. The sign of double refraction is negative, the reverse of what usually obtains in the series, and this is accompanied by a reversal of the median lines. Moreover, this

exceptional optical character will be subsequently shown, when a comparison of the potassium, rubidium, and caesium salts containing nickel as the second metal is made, to have an important bearing on the object of this investigation.

Orientation of Axes of Optical Ellipsoid.—Two sections ground parallel to the symmetry plane $b\{010\}$ exhibited the following extinction angles in the stauroscope with respect to the normal to $c\{001\}$.

Section 1	7°	10
Section 2	7	0
Mean	7	5

FIG. 14.

The direction is forwards, as indicated in Fig. 14.

This direction is, however, the first median line, the second median line being the direction at right angles lying also in the symmetry plane, the usual order of first and second median lines for the series being thus reversed.

That the *sign of double refraction* is negative is indicated both by the phenomena which are produced when a quarter undulation mica plate is introduced while observing the interference figure afforded by a section-plate perpendicular to the first median line, and by the relations of the maximum and minimum refractive indices to the intermediate index, as will hereafter be shown.

The morphological axial angle being $72^\circ 58'$, the orientations may be stated as follows.

The first median line, in this case a , lies in the obtuse angle of the morphological axes ac , and is inclined $24^\circ 7'$ to the vertical axes c .

The second median line c lies in the acute axial angle ac , and is inclined $7^\circ 5'$ to the axis a .

The first median line is also inclined $7^\circ 5'$ to the normal to $c\{001\}$ and $65^\circ 53'$ to the normal to $a\{100\}$.

Optic Axial Angle.—Three pairs of section-plates were ground perpendicular to the two directions of extinction in the symmetry plane, and it was on measuring the apparent optic axial angle in monobromonaphthalene exhibited by the first pair of sections, that it was found that the angle afforded by the plate perpendicular to what is generally the first median line in the series, was larger than the angle indicated by the section ground in the direction at right angles to this, and that therefore the median lines are reversed. Both angles are very large, so much so that none of the sections exhibited the optic axial brushes in air, only portions of rings. The following are the measurements with the various sections in monobromonaphthalene.

Determination of true Optic Axial Angle of Cesium Nickelous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2H α .	No. of section perp. 2nd median line.	Observed values of 2H α .	Calculated values of 2V α .	Mean value of 2V α .
Li.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 78^{\circ} 20' \\ 78 \ 16 \\ 78 \ 24 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 82^{\circ} 51' \\ 83 \ 0 \\ 83 \ 5 \end{array} \right.$	$\left\{ \begin{array}{l} 87^{\circ} 20' \\ 87 \ 12 \\ 87 \ 14 \end{array} \right.$	$\left. \right\} 87^{\circ} 15'$
C.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 78 \ 15 \\ 78 \ 12 \\ 78 \ 18 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 82 \ 45 \\ 82 \ 54 \\ 82 \ 57 \end{array} \right.$	$\left\{ \begin{array}{l} 87 \ 21 \\ 87 \ 14 \\ 87 \ 16 \end{array} \right.$	$\left. \right\} 87 \ 17$
Na	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 78 \ 2 \\ 77 \ 59 \\ 78 \ 1 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 82 \ 28 \\ 82 \ 24 \\ 82 \ 30 \end{array} \right.$	$\left\{ \begin{array}{l} 87 \ 22 \\ 87 \ 22 \\ 87 \ 20 \end{array} \right.$	$\left. \right\} 87 \ 21$
Tl.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 77 \ 42 \\ 77 \ 44 \\ 77 \ 42 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 81 \ 54 \\ 81 \ 53 \\ 82 \ 1 \end{array} \right.$	$\left\{ \begin{array}{l} 87 \ 29 \\ 87 \ 31 \\ 87 \ 26 \end{array} \right.$	$\left. \right\} 87 \ 29$
F.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 77 \ 23 \\ 77 \ 15 \\ 77 \ 18 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 81 \ 10 \\ 81 \ 10 \\ 81 \ 13 \end{array} \right.$	$\left\{ \begin{array}{l} 87 \ 43 \\ 87 \ 38 \\ 87 \ 38 \end{array} \right.$	$\left. \right\} 87 \ 40$

The order of dispersion was confirmed by observation of the interference figures afforded by Sections 1, 2, and 3 in cedar oil, whose refractive index is very nearly the same as the mean index of the crystal. Both the order of the colours in white light, and actual measurements for C and Tl light, indicate a larger angle for blue than for red light. One of the axial brushes was practically immovable for all changes of wave-length, whilst the other axis was considerably dispersed, being farthest from the immovable axis for blue light.

Dispersion of the Median Lines.—The observations in cedar oil were supplemented by similar ones in benzene, whose refractive index is only slightly different from the mean index of the crystal but on the other side of that mean to cedar oil. Hence the mean of the two series of observations will probably be very near the truth. In cedar oil, the immovable axis was the one which emerges between $\alpha\{100\}$

and $r\{201\}$, the axis which showed considerable dispersion being approximately normal to $s'\{\bar{1}01\}$ (*vide* spherical projection, previous memoir, 463). In benzene, both axes showed dispersion, but only a very small amount, apparently the axis normal to $s'\{\bar{1}01\}$ least. In both cases, the first median line was nearer to the vertical axis c for blue than for red. The true dispersion of the axes themselves would appear to be that the axis emerging between $a\{100\}$ and $r\{201\}$ is very slightly dispersed and the other normal to $s'\{\bar{1}01\}$ more so, the order for the two ends of the spectrum being as follows.

red blue	1st median line	red blue
└───┘		└───┘
axis between	for for	axis normal
$a\{100\}$ and $r\{201\}$.	red. blue.	to $s'\{\bar{1}01\}$.

From the readings in monochromatic light, the first median line would appear to be nearer the vertical morphological axis c by about $15'$ for F than it is for Li light.

The effect of rise of temperature could not be determined as the optic axes did not emerge in air owing to their large internal angle.

Refractive Indices.—These were determined with six prisms, ground upon three different crystals, a pair upon each. The results are given in the accompanying table.

Determination of Refractive Indices of Cesium Nickelous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Mean refrac- tive index.
α Vibrations par- allel 1st median line α . ($7^\circ 5'$ from normal to $c\{001\}$).	Li	1.5067	1.5060	1.5056	1.5061
	C	1.5070	1.5064	1.5060	1.5065
	Na	1.5093	1.5086	1.5082	1.5087
	Tl	1.5118	1.5110	1.5109	1.5112
	F	1.5150	1.5145	1.5143	1.5146
	G	1.5198	1.5190	1.5189	1.5192
β Vibrations parallel symmetry axis b (b).	Li	1.5103	1.5097	1.5101	1.5100
	C	1.5106	1.5101	1.5105	1.5104
	Na	1.5130	1.5125	1.5131	1.5129
	Tl	1.5156	1.5151	1.5156	1.5154
	F	1.5189	1.5185	1.5187	1.5187
	G	1.5229	1.5238	1.5238	1.5235
γ Vibrations parallel 2nd median line γ .	Li	1.5134	1.5130	1.5135	1.5133
	C	1.5138	1.5134	1.5139	1.5137
	Na	1.5164	1.5161	1.5162	1.5162
	Tl	1.5189	1.5189	1.5189	1.5189
	F	1.5221	1.5221	1.5221	1.5221
	G	1.5263	1.5261	1.5273	1.5266

It will be observed that the directions of vibration for the three

indices are as usual in the series, the direction for α , however, usually the second median line, is, in this salt, the first median line, and for γ the second median line instead of the first median line as usual.

The intermediate index β , corrected to a vacuum, is accurately expressed, for any wave-length λ to near F, by the formula

$$\beta = 1.4980 + \frac{634\ 374}{\lambda^2} - \frac{3\ 582\ 100\ 000\ 000}{\lambda^4} + \dots$$

The α indices are closely reproduced if the constant 1.4980 is diminished by 0.0041, and the γ values if it is increased by 0.0033. The former difference being greater than the latter, instead of less as usual throughout the series, determines the sign of double refraction to be negative.

Effect of Rise of Temperature on Refraction.—Determinations at 70° indicate that the indices are lowered about 0.0016 by 50° rise of temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The indices of refraction furnish the following ratios along the three rectangular directions of vibration.

$$a : b : c = 1.0028 : 1 : 0.9978.$$

Molecular Optical Constants.—The calculated values of these constants are as under.

Axis of optical ellipsoid.....	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1039	0.1045	0.1051
	{ G. 0.1061	0.1068	0.1073
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 64.84	65.26	65.61
	{ G. 66.20	66.66	66.99
Specific dispersion, $n_G - n_C \dots$	0.0022	0.0023	0.0022
Molecular dispersion, $m_G - m_C \dots$	1.36	1.40	1.38
Molecular refraction, $\frac{n - 1}{d} M \dots$	C. 110.46	111.31	112.03

These values will, as in the cases of the previous salts, be for the same reason independent of temperature.

SALTS CONTAINING COBALT.

Potassium Cobaltous Sulphate, $K_2Co(SO_4)_2 \cdot 6H_2O$.

Cleavage occurs readily parallel $r'\{201\}$; the position of the cleaved surfaces was goniometrically verified to be 180° from the r' faces, the images of the signal being sharply defined.

Volume.

Relative Density.—Four determinations with good crystals yielded the following results.

Weight of salt employed.	Sp. gr. at 20°/4°.
4.2793	2.2096
4.2476	2.2123
4.2982	2.2127
4.4105	2.2142

Mean **2.2122**

Schiff obtained 2.154.

$$\text{Molecular Volume.} \frac{M}{d} = \frac{437.6}{2.212} = 197.83.$$

Distance Ratios.—The axial angle β was previously shown to be $75^\circ 5'$, and the axial ratio

$$a : b : c = 0.7404 : 1 : 0.5037.$$

Combination of these with the molecular volume affords the following distance ratios.

$$\psi : \chi : w = 6.0625 : 8.1881 : 4.1243.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two sections were ground parallel to the symmetry plane $b\{010\}$, and were found to exhibit extinction angles in each case of $4^\circ 50'$ with the normal to $c\{001\}$, the direction being backwards, as represented in Fig. 15. Murmann and Rotter found it $4^\circ 49'$.

FIG. 15.

This direction is as usual in the series the second median line, the first median line being at right angles to it in the symmetry plane.

The *sign of double refraction* is positive, determined by the relations of the three refractive indices and confirmed by the quarter undulation mica. As the morphological axial angle ac is $75^\circ 5'$, the following are the orientations.

The first median line ϵ lies in the obtuse angle of the morphological axes ac , and is inclined $4^\circ 50'$ to the axis a .

The second median line a lies also in the obtuse axial angle ac , and is inclined $10^{\circ} 5'$ to the vertical axis c .

The first median line is also inclined $85^{\circ} 10'$ to the normal to $c\{001\}$ and $10^{\circ} 5'$ to the normal to $a\{100\}$.

Optic Axial Angle.—Six section-plates were ground, three perpendicular to the first, and three to the second median line. Owing to the large amount of absorption at the blue end of the spectrum by the deep brownish-red crystals of this salt, measurements of the apparent optic axial angle in air and in monobromonaphthalene for F light are more difficult to obtain than usual, and require a powerful source of light; it was consequently necessary to open the receiving slit of the monochromatic illuminator more than usual for the observations with F light.

Determination of the apparent Angle in Air of Potassium Cobaltous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	114° 50'	113° 12'	113° 46'	113° 56'
C	114 54	113 16	113 51	114 0
Na	115 26	113 40	114 8	114 25
Tl	116 7	114 12	114 28	114 56
F	117 0	115 0	115 0	115 40

Murmann and Rotter give $110^{\circ} 10'$ as the apparent angle in air; they further state that both axes are nearer to the median line for red than for blue, hence the angle was larger for blue, which agrees with the above measurements.

Determination of true Optic Axial Angle of Potassium Cobaltous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of $2H\alpha$.	No. of section perp. 2nd median line.	Observed values of $2H\alpha$.	Calculated values of $2V\alpha$.	Mean $2V\alpha$.
Li	{ 1	61° 32'	1a	96° 1'	69° 5'	68° 38'
	{ 2	60 48	2a	96 16	68 24	
	{ 3	60 52	3a	96 21	68 25	
C	{ 1	61 30	1a	95 48	69 8	68 39
	{ 2	60 46	2a	96 11	68 24	
	{ 3	60 49	3a	96 15	68 25	
Na	{ 1	61 21	1a	95 23	69 12	68 41
	{ 2	60 34	2a	95 45	68 26	
	{ 3	60 37	3a	95 51	68 26	
Tl	{ 1	61 14	1a	94 55	69 18	68 44
	{ 2	60 21	2a	95 15	68 28	
	{ 3	60 25	3a	95 23	68 28	
F	{ 1	61 0	1a	94 20	69 22	68 48
	{ 2	60 3	2a	94 35	68 31	
	{ 3	60 10	3a	94 50	68 30	

The immersion liquid employed was monobromonaphthalene. Different crops of this salt appear to exhibit somewhat greater differences of optic axial angle than usual. The crystals investigated were from the same preparation as that used for the morphological work, and shown by analysis to be pure.

Murmann and Rotter give the true angle as $66^{\circ} 56'$, but only one plate was employed, and the true angle calculated by use of the mean refractive index of the salt, and the index of the immersion oil employed. They further state that the true optic axial angle is less for red than for blue.

The order of dispersion was confirmed with Sections 1 and 3 by measurements while immersed in toluene, whose refractive index is almost exactly the mean of the crystal indices. The amount of dispersion indicated was very small, which agrees with the calculated amount shown in the table of results, but the angle for blue was decidedly larger than that for red.

Dispersion of the Median Lines.—The observations in toluene indicated clearly that the first median line is dispersed so that for red it lies nearer to the morphological axis a than for blue. This agrees with the observation of Murmann and Rotter. The amount indicated was $10-15'$. Both axes exhibited slight dispersion in the same direction, the one near the normal to $r'\{201\}$ least.

Effect of Rise of Temperature on the Optic Axial Angle.—Measurement of the apparent angle in air with Section 1 at 70° indicated that $2E$ increases about $2^{\circ} 15'$ for 50° rise of temperature.

Refractive Indices.—The six prisms employed were ground upon five different crystals, and the results are given in the accompanying table. The readings for F light were quite trustworthy, the absorption in the crystal not being very material.

As was observed with respect to the optic axial angle, different crops exhibit slight differences of refractive index.

Murmann and Rotter ascribe the values to β for red 1.482 , for yellow 1.487 , and for green 1.490 .

The intermediate axis β , corrected to a vacuum, is expressed, for any wave-length as far as F, by the formula

$$\beta = 1.4737 + \frac{514\ 824}{\lambda^2} - \frac{1\ 962\ 700\ 000\ 000}{\lambda^4} + \dots$$

The α indices are closely reproduced if the constant 1.4737 is diminished by 0.0058 , and the γ values if it is increased by 0.0138 . The former difference being the smaller, determines that the sign of double refraction is, as previously stated, positive.

Determination of Refractive Indices of Potassium Cobaltous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 1 prism.	Crystal 3, 1 prism.	Crystal 4, 1 prism.	Crystal 5, 1 prism.	Mean refractive index.
α Vibs. par. 2nd median line	Li	1·4771	1·4789	1·4781	—	—	1·4780
	C	1·4775	1·4793	1·4785	—	—	1·4784
	Na	1·4795	1·4817	1·4808	—	—	1·4807
	Tl	1·4819	1·4839	1·4831	—	—	1·4830
	F	1·4850	1·4871	1·4861	—	—	1·4861
	G	1·4892	1·4916	—	—	—	1·4904
β Vibs. par. symmetry axis b ($\hbar$)	Li	1·4832	1·4849	1·4842	1·4836	1·4829	1·4838
	C	1·4836	1·4853	1·4846	1·4840	1·4833	1·4842
	Na	1·4860	1·4877	1·4869	1·4866	1·4853	1·4865
	Tl	1·4884	1·4900	1·4895	1·4889	1·4875	1·4889
	F	1·4915	1·4932	1·4924	1·4919	1·4905	1·4919
	G	1·4958	1·4972	—	1·4964	1·4949	1·4961
γ Vibs. par. 1st median line ϵ .	Li	1·4975	—	—	1·4973	1·4971	1·4973
	C	1·4980	—	—	1·4977	1·4975	1·4977
	Na	1·5006	—	—	1·5004	1·5001	1·5004
	Tl	1·5032	—	—	1·5028	1·5023	1·5028
	F	1·5065	—	—	1·5058	1·5055	1·5059
	G	1·5106	—	—	1·5111	1·5097	1·5105

Effect of Rise of Temperature on Refraction.—Determinations of refractive index at 70° indicated a lowering of the indices by about 0·0020 for 50° rise of temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The values of these constants, derived from the indices, are as under.

$$a : b : c = 1·0039 : 1 : 0·9907.$$

Molecular Optical Constants.—The following values were deduced from the indices, combined with the density and molecular weight.

Axis of optical ellipsoid	α .	$\hbar$.	ϵ .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n..$	{ C. 0·1282	0·1295	0·1325
	{ G. 0·1309	0·1322	0·1354
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m.$	{ C. 56·08	56·66	58·00
	{ G. 57·28	57·84	59·26
Specific dispersion, $n_g - n_c$	0·0027	0·0027	0·0029
Molecular dispersion, $m_g - m_c$	1·20	1·18	1·26
Molecular refraction, $\frac{n - 1}{d} M$	C. 94·72	95·87	98·54

These values will be substantially constant for all temperatures at which the crystals of the salt are capable of existing, as refraction and density vary in the same direction for change of temperature.

Rubidium Cobaltous Sulphate, $\text{Rb}_2\text{Co}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

The crystals exhibited an excellent *cleavage* parallel $r'\{201\}$. The direction was proved to be exactly 180° from r' by goniometrical measurement, the fractures affording well-defined images of the signal.

Volume.

Relative Density.—Four determinations were made with different crops of clear crystals derived from two independent preparations; the two last determinations were carried out with crystals which were respectively dark brownish-red and ruby-red, in order to ascertain if any difference in density were apparent. The results will be seen, from the table given below, to be nearly identical in these two cases.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
3.7399	2.5529
4.8409	2.5571
5.9019	2.5573
4.9051	2.5591

Mean $\overline{2.5566}$

Perrot ascribes the density 2.56 to this salt, his individual values varying from 2.540 to 2.577.

Molecular Volume.— $\frac{M}{d} = \frac{530}{2.557} = 207.28$.

Distance Ratios.—The axial angle was previously shown to be $73^\circ 59'$, and the ratio of the axes

$$a : b : c = 0.7391 : 1 : 0.5011.$$

Upon combination of these constants with the molecular volume, the following distance ratios are obtained.

$$\chi : \psi : \omega = 6.1718 : 8.3505 : 4.1844.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two section-plates ground parallel to the symmetry plane $b\{010\}$ exhibited the following extinction angles with respect to the normal to $c\{001\}$.

Section 1.	$0^\circ 10'$ forwards.
Section 2.	0 10 backwards.

Mean $\overline{0 \quad 0}$

The direction of extinction is thus found to be exactly normal to the basal plane, as represented in Fig. 16. This direction is the

second median line, the first median line being coincident with the crystallographical axis a .

FIG. 16.

The *sign of double refraction* is positive, proved as in the cases of the foregoing salts. The morphological axial angle being $73^{\circ} 59'$, the orientation is as follows.

The first median line t is coincident with the morphological axis a .

The second median line a lies in the obtuse angle of the morphological axes ac ; it is precisely normal to $c\{001\}$, and is inclined $16^{\circ} 1'$ to the vertical axis c .

The first median line is consequently inclined exactly 90° to the normal to $c\{001\}$, and is inclined $16^{\circ} 1'$ to the normal to $a\{100\}$.

Perrot states that the first median line is inclined $85^{\circ} 15'$ to the normal to $\{001\}$, which differs somewhat from the orientation of the principal optical directions afforded by the author's crystals.

Optic Axial Angle.—Three section-plates were ground perpendicular to the first median line. The fact that the second median line is exactly normal to the basal plane rendered it unnecessary to grind plates perpendicular to this median line, and three crystals exhibiting plane and large $c\{001\}$ faces were consequently employed for the observations of 2Ho. Powerful illumination is required with F light, owing to absorption by the red crystals.

Determination of apparent Angle in Air of Rubidium Cobaltous Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	$129^{\circ} 28'$	$129^{\circ} 24'$	$130^{\circ} 25'$	$129^{\circ} 46'$
C.....	129 34	129 28	130 30	129 51
Na	129 54	129 48	130 51	130 10
Tl.....	130 9	130 6	131 12	130 29
F.....	130 35	130 33	131 38	130 55

Determination of true Optic Axial Angle of Rubidium Cobaltous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of plate perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean 2Va.
Li.....	{ 1	66° 18'	1a	90° 12'	75° 20'	75° 15'
	{ 2	66 38	2a	91 4	75 10	
	{ 3	66 56	3a	91 22	75 15	
C	{ 1	66 15	1a	90 9	75 19	75 14
	{ 2	66 33	2a	90 59	75 8	
	{ 3	66 53	3a	91 18	75 14	
Na . . .	{ 1	66 0	1a	89 52	75 16	75 11
	{ 2	66 18	2a	90 46	75 4	
	{ 3	66 39	3a	91 0	75 12	
Tl	{ 1	65 40	1a	89 30	75 12	75 8
	{ 2	66 0	2a	90 26	75 0	
	{ 3	66 24	3a	90 40	75 11	
F	{ 1	65 12	1a	89 0	75 6	75 3
	{ 2	65 36	2a	90 0	74 54	
	{ 3	66 2	3a	90 12	75 8	

The immersion liquid was monobromonaphthalene in each case for the observation of 2Ha and 2Ho. The order of dispersion was confirmed by observations in toluene, whose index of refraction is the same as the β index of the crystals; the angle for red was clearly larger than that for blue.

Perrot gives the angle as 75° 5' for sodium light.

Dispersion of the Median Lines.—The observations in toluene clearly indicated that the optic axis which emerges between $a\{100\}$ and $r\{201\}$ is invariable for all colours, whilst the other axis is dispersed so that for red it is furthest removed from the immovable axis. The first median line is consequently dispersed so that for blue it is 8—10' nearer to the immovable axis than for red.

Effect of Rise of Temperature on the Optic Axial Angle.—Measurements with Section I at 70° indicate that the apparent optic axial angle in air increases about 2° for 50° rise of temperature.

Refractive Indices.—Six prisms were ground upon four different crystals; the results obtained with them are given in the accompanying table. The absorption for F light was insufficient to cause material trouble in obtaining good readings for this wave-length.

As in the case of the potassium cobalt salt, different crops are found to exhibit slight differences of refractive index. Perrot remarked this, and convinced himself that they were not due to impurity, a conclusion which is fully borne out by the present work with preparations of undoubted purity.

The intermediate index β , corrected for a vacuum, is accurately expressed, for any wave-length λ as far as F, by the formula

$$\beta = 1.4789 + \frac{507\ 763}{\lambda^2} - \frac{1\ 841\ 600\ 000\ 000}{\lambda^4} + \dots$$

The α indices are very closely reproduced if the constant 1.4789 is diminished by 0.0057, and the γ indices if it is increased by 0.0098. The positive nature of the double refraction is evident from the fact that the former is the smaller difference from β .

Determination of Refractive Indices of Rubidium Cobaltous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 1 prism.	Crystal 4, 1 prism.	Mean refractive index.	Perrot's values.
α Vibs. par. 2nd median line	Li	1.4825	1.4840	—	1.4834	1.4833	—
	C	1.4829	1.4844	—	1.4838	1.4837	1.4834
	Na	1.4852	1.4864	—	1.4861	1.4859	1.4860
	Tl	1.4875	1.4889	—	1.4882	1.4882	—
	F	1.4904	1.4916	—	1.4910	1.4910	1.4912
	G	1.4948	1.4960	—	—	1.4954	—
β Vibs. par. symmetry axis b ($\hbar$)	Li	1.4878	1.4891	1.4893	1.4893	1.4889	—
	C	1.4882	1.4895	1.4896	1.4897	1.4893	1.4890
	Na	1.4904	1.4919	1.4919	1.4920	1.4916	1.4916
	Tl	1.4930	1.4943	1.4943	1.4942	1.4940	—
	F	1.4960	1.4972	1.4969	1.4970	1.4968	1.4970
	G	1.5001	1.5015	1.5018	—	1.5011	—
γ Vibs. par. 1st median line r	Li	1.4976	1.4988	1.4990	—	1.4985	—
	C	1.4983	1.4991	1.4995	—	1.4989	1.4985
	Na	1.5005	1.5018	1.5018	—	1.5014	1.5012
	Tl	1.5033	1.5040	1.5041	—	1.5038	—
	F	1.5062	1.5073	1.5070	—	1.5068	1.5069
	G	1.5104	1.5119	1.5119	—	1.5114	—

Effect of Rise of Temperature on Refraction.—Determinations at 70° indicate that the indices are lowered about 0.0016 by 50° rise of temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—These are as follows.

$$a : b : c = 1.0038 : 1 : 0.9935.$$

Molecular Optical Constants.—The calculated values of these constants, obtained by combining the refractive indices with the molecular weight and the density, are as under.

Axis of optical ellipsoid	α .	$\hbar$.	r .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1119	0.1130	0.1149
	{ G. 0.1142	0.1153	0.1173
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$	{ C. 59.31	59.90	60.89
	{ G. 60.53	61.12	62.18
Specific dispersion, $n_G - n_C \dots$	0.0023	0.0023	0.0024
Molecular dispersion, $m_G - m_C \dots$	1.22	1.22	1.29
Molecular refraction, $\frac{n - 1}{d} M \dots$	C. 100.34	101.50	103.49

These values will again be independent of temperature, as change of the latter causes both refraction and density to vary in the same direction.

Cæsium Cobaltous Sulphate, $\text{Cs}_2\text{Co}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

There is a good *cleavage* parallel $r'\{201\}$. The cleaved surfaces were proved by direct measurement to be exactly 180° from r' ; this was rendered possible by the perfection of the reflection of the goniometer signal from the plane fracture.

Volume.

Relative Density.—Four determinations were made with different crops of perfectly transparent crystals derived from two independent preparations. The results are given beneath.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
4.6173	2.8391
4.1226	2.8372
3.7098	2.8365
4.4523	2.8350
Mean.....	2.8370

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{625}{2.837} = 220.30.$$

Distance Ratios.—It was previously shown that the axial angle β is $72^\circ 52'$, and the axial ratio

$$a : b : c = 0.7270 : 1 : 0.4968.$$

When combined with the molecular volume the following distance ratios are afforded.

$$\chi : \psi : \omega = 6.2595 : 8.6101 : 4.2775.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two sections were ground parallel to the symmetry plane $b\{010\}$. Each exhibited an extinc-

FIG. 17.

tion angle with respect to the normal to the basal plane $c\{001\}$ of $9^\circ 0'$ forwards, as represented in Fig. 17.

This direction is the second median line, as is normally the case throughout the series, the first median line being the other direction of extinction, at 90° to this, in the symmetry plane.

The *sign of double refraction* is positive, proved as before by the quarter undulation mica and the relations of the refractive indices. The morphological axial angle ac being $72^\circ 52'$, the orientation is as follows.

The first median line ϵ lies in the acute angle of the morphological axes ac , and is inclined $9^\circ 0'$ to the axis a .

The second median line a lies in the obtuse axial angle ac , and is inclined $26^\circ 8'$ to the vertical axis c .

The first median line is also inclined $99^\circ 0'$ to the normal to $c\{001\}$, and $26^\circ 8'$ to the normal to $a\{100\}$.

Optic Axial Angle.—Six section-plates were ground, three perpendicular to the first and three to the second median line. The apparent angle in air is very large, and consequently difficult to observe, particularly in the case of such feebly doubly refractive substances as the cæsium salts of this series, whose section-plates require to be relatively very thick to yield well-defined interference figures. The three sections perpendicular to the first median line were very good ones, but 2E was only afforded approximately. For sodium light the values obtained were $157^\circ 50'$, $154^\circ 10'$, and $158^\circ 0'$. The mean angle $156^\circ 40'$ for 2E is probably as near the truth as can be obtained for so large an angle.

Determination of true Optic Axial Angle of Cæsium Cobaltous Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean 2Va.
Li.....	$\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 73^\circ 45' \\ 72 \quad 35 \\ 74 \quad 8 \end{Bmatrix}$	$\begin{Bmatrix} 1a \\ 2a \\ 3a \end{Bmatrix}$	$\begin{Bmatrix} 87^\circ 52' \\ 86 \quad 55 \\ 87 \quad 55 \end{Bmatrix}$	$\begin{Bmatrix} 81^\circ 43' \\ 81 \quad 26 \\ 81 \quad 56 \end{Bmatrix}$	$81^\circ 42'$
C.....	$\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 73 \quad 40 \\ 72 \quad 30 \\ 74 \quad 3 \end{Bmatrix}$	$\begin{Bmatrix} 1a \\ 2a \\ 3a \end{Bmatrix}$	$\begin{Bmatrix} 87 \quad 47 \\ 86 \quad 52 \\ 87 \quad 52 \end{Bmatrix}$	$\begin{Bmatrix} 81 \quad 42 \\ 81 \quad 24 \\ 81 \quad 54 \end{Bmatrix}$	$81 \quad 40$
Na.....	$\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 73 \quad 25 \\ 72 \quad 5 \\ 73 \quad 50 \end{Bmatrix}$	$\begin{Bmatrix} 1a \\ 2a \\ 3a \end{Bmatrix}$	$\begin{Bmatrix} 87 \quad 35 \\ 86 \quad 40 \\ 87 \quad 42 \end{Bmatrix}$	$\begin{Bmatrix} 81 \quad 38 \\ 81 \quad 13 \\ 81 \quad 51 \end{Bmatrix}$	$81 \quad 34$
Tl.....	$\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 73 \quad 10 \\ 71 \quad 40 \\ 73 \quad 35 \end{Bmatrix}$	$\begin{Bmatrix} 1a \\ 2a \\ 3a \end{Bmatrix}$	$\begin{Bmatrix} 87 \quad 25 \\ 86 \quad 23 \\ 87 \quad 30 \end{Bmatrix}$	$\begin{Bmatrix} 81 \quad 34 \\ 81 \quad 5 \\ 81 \quad 48 \end{Bmatrix}$	$81 \quad 29$
F.....	$\begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$	$\begin{Bmatrix} 72 \quad 50 \\ 71 \quad 10 \\ 73 \quad 16 \end{Bmatrix}$	$\begin{Bmatrix} 1a \\ 2a \\ 3a \end{Bmatrix}$	$\begin{Bmatrix} 87 \quad 10 \\ 86 \quad 3 \\ 87 \quad 14 \end{Bmatrix}$	$\begin{Bmatrix} 81 \quad 28 \\ 80 \quad 55 \\ 81 \quad 43 \end{Bmatrix}$	$81 \quad 22$

The true angle was determined as usual by immersion in monobromonaphthalene and observation of $2H\alpha$ and $2H\delta$. Very intense light is essential in order to measure the angles for F light.

The order of dispersion was proved by measurements with Sections 1, 2, and 3 while immersed in cedar oil, whose index of refraction is near the mean index of the crystal; the angle in each case was about $20'$ larger for red than for greenish-blue.

Dispersion of the Median Lines.—The observations in cedar oil further indicated that the optic axis which emerges between $a\{100\}$ and $r\{201\}$ is immovable for all colours, while the other axis is dispersed so that for red it is furthest from the fixed axis. The first median line is therefore dispersed so that for F light it lies about $10'$ nearer to the morphological axis a than it does for Li light.

No observations of the optic axial angle were made at higher temperatures, owing to the difficulty of obtaining exact measurements of $2E$ on account of the exceptionally large angle. The error of the measurements was about as great as the usual alteration of the angle for 50° rise of temperature.

Refractive Indices.—Six prisms were ground upon four different crystals. The results are given in the accompanying table.

Determination of Refractive Indices of Cesium Cobaltous Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 2 prisms.	Crystal 3, 1 prism.	Crystal 4, 1 prism.	Mean refractive index.
α Vibrations parallel 2nd median line a .	Li	1.5022	1.5028	—	1.5033	1.5028
	C	1.5026	1.5032	—	1.5038	1.5032
	Na	1.5055	1.5054	—	1.5062	1.5057
	Tl	1.5077	1.5078	—	1.5083	1.5079
	F	1.5109	1.5111	—	1.5117	1.5112
	G	1.5159	1.5160	—	—	1.5159
β Vibrations parallel symmetry axis b .	Li	1.5054	1.5057	1.5059	—	1.5057
	C	1.5058	1.5061	1.5033	—	1.5061
	Na	1.5082	1.5086	1.5087	—	1.5085
	Tl	1.5108	1.5111	1.5111	—	1.5110
	F	1.5140	1.5142	1.5145	—	1.5142
	G	1.5188	1.5189	1.5188	—	1.5188
γ Vibrations parallel 1st median line c .	Li	1.5096	1.5102	1.5103	1.5109	1.5102
	C	1.5099	1.5106	1.5107	1.5113	1.5106
	Na	1.5127	1.5131	1.5131	1.5139	1.5132
	Tl	1.5152	1.5158	1.5154	1.5159	1.5156
	F	1.5185	1.5191	1.5184	1.5189	1.5187
	G	1.5237	1.5236	1.5237	—	1.5237

The amount of absorption in the greenish-blue is not sufficient to

so seriously diminish the intensity of the images of the spectrometer slit for F light as to render the readings untrustworthy, the lime-light furnishing ample light even when the slits of the monochromatic illuminator are very fine.

The intermediate index β reduced to a vacuum is accurately expressed, for any wave-length λ as far as near F, by the following formula

$$\beta = 1.4952 + \frac{532\ 337}{\lambda^2} - \frac{1\ 971\ 500\ 000\ 000}{\lambda^4} + \dots$$

The α indices are closely reproduced by the formula if the constant 1.4952 is diminished by 0.0029, and the γ indices if it is increased by 0.0046. The former number, being the smaller, determines the positive nature of the double refraction.

Effect of Rise of Temperature on Refraction.—Determinations at 70° indicated that the refractive indices are lower at that temperature by about 0.0018 than they are at the ordinary temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—These are as follows for the crystals of this salt.

$$a : b : c = 1.0019 : 1 : 0.9969.$$

Molecular Optical Constants.—The refractive indices lead to the following constants.

Axis of optical ellipsoid.....	α .	β .	γ .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1043 G. 0.1065	0.1048 0.1070	0.1056 0.1079
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$			
Specific dispersion, $n_G - n_C \dots$	C. 0.0022	0.0022	0.0023
Molecular dispersion, $m_G - m_C \dots$	1.38	1.38	1.41
Molecular refraction, $\frac{n - 1}{d} M \dots$	110.95	111.58	112.57

As in the cases of the other salts of the series these constants will be independent of temperature, refraction and density varying together for change of temperature.

SALTS CONTAINING COPPER.

Potassium Copper Sulphate, $K_2Cu(SO_4)_2 \cdot 6H_2O$.

The crystals of this salt exhibit *cleavage* parallel $r'\{\bar{2}01\}$ in common with the other salts of the series. Good images of the goniometer signal were obtained from the fractures at exactly 180° from the r' faces.

It was remarked in the former memoir concerning the morphology of the series (p. 397) that the crystals of this salt were usually more

or less opaque, and that the analytical results for SO_3 were slightly low. It was pointed out that this was certainly not due to the presence of any other salt crystallising isomorphically with it, and that, therefore, it would not affect the angles of the crystals. It was suggested that it was in all probability due to a minute quantity of foreign matter mechanically included. Further work with the salt confirms this supposition. When a thin crystal or a section-plate is examined with a high power of the microscope the opacity is seen to be due to myriads of small cavities containing mother liquor. Powdering the salt and subsequent drying for an hour at the ordinary temperature on porous porcelain, of course, removes the larger ones; but the finest powder still consists of particles retaining minute cavities. Hence the slightly low analytical results. For the goniometrical investigation this property is naturally of no consequence, and, even if it were, the crystals selected for measurement were the most transparent that could be obtained. The same applies to the optical investigation, which was necessarily carried out with crystals of the highest procurable transparency. The determinations of density are more liable to be affected by it, the effect being to lower the results. The greatest care, however, was taken to render its effect as minute as possible by bestowing special trouble on the powdering and air drying, passing the dust through a sieve of the finest platinum gauze in order to weed out all coarser particles. Evidence will be subsequently given in comparing the densities of the salts of the series, that the error has in all probability been reduced to within the limits of experimental error in carrying out density determinations.

Volume.

Relative Density.—Four determinations were carried out with the precautions above specified, the crystals employed being as transparent as could be obtained, of small size, and from different preparations. The results are stated below.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
4.4867	2.2228
4.1229	2.2244
4.5166	2.2227
4.1518	2.2257

Mean 2.2239

This mean value is considerably higher than the numbers given by Joule and Playfair, 2.164, and by Schiff, 2.137, and is, undoubtedly, very near the truth. The air-drying was not sufficiently prolonged to admit any source of error due to efflorescence, it being only con-

tinned until the particles exhibited no tendency to adhere to a glass rod.

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{441}{2.224} = 198.29.$$

Distance Ratios.—The axial angle β was previously shown to be $75^\circ 32'$, and the axial ratio

$$a : b : c = 0.7490 : 1 : 0.5088.$$

The distance ratios obtained by combining these with the molecular volume are

$$\chi : \psi : \omega = 6.0893 : 8.1300 : 4.1366.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two sections were ground parallel to the symmetry plane $b\{010\}$ upon two very small, but well developed and transparent, crystals. All the sections and prisms for the optical investigation of this salt were ground upon very small crystals on account of their greater transparency; there is no difficulty in carrying this out with the new grinding instrument, delicate manipulation being all that is required.

The peculiarity of the three salts containing copper, owing to the marked individuality of the latter metal as compared with that exerted by each of the dyad metals in the salts previously described, a phenomenon which was pointed out with respect to the morphology, at once begins to manifest itself in the orientation of the axes of optical elasticity. The two sections exhibited extinctions on the forward side of the normal to the basal plane, as shown in Fig. 18, whereas the potassium salts of the series usually show it behind the normal. The angles were

Section 1	4° 0'
Section 2	4 10
Mean	<hr/> 4 5

FIG. 18.

Murmann and Rotter found an extinction angle of $4^{\circ} 23'$ on the same side.

This direction is the second median line, the first median line being as usual the direction at right angles in the symmetry plane.

The sign of *double refraction* is positive. The morphological axial angle being $75^{\circ} 32'$, the following is the orientation.

The first median line, ϵ , lies in the acute angle of the morphological axes ac , and is inclined $4^{\circ} 5'$ to the axis a .

The second median line, a , lies in the obtuse angle ac , and is inclined $18^{\circ} 33'$ to the vertical axis c .

The first median line is also inclined $94^{\circ} 5'$ to the normal to $c\{001\}$, and $18^{\circ} 33'$ to the normal to $a\{100\}$.

Optic Axial Angle.—Six sections were ground, three perpendicular to each median line, out of small, but transparent, crystals. The results obtained with them are given in the accompanying tables.

Determination of apparent Angle in Air of Potassium Copper Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	$71^{\circ} 40'$	$71^{\circ} 30'$	$70^{\circ} 46'$	$71^{\circ} 19'$
C	71 47	71 37	70 50	71 25
Na.....	72 42	72 18	71 20	72 7
Tl.....	73 43	73 5	72 20	73 3
F	75 0	74 0	73 40	74 13

The apparent angle in air was given by Murmann and Rotter as $76^{\circ} 12'$.

Determination of true Optic Axial Angle of Potassium Copper Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of $2Ha$.	No. of section perp. 2nd median line.	Observed values of $2Ho$.	Calculated values of $2Va$.	Mean $2Va$.
Li.....	{ 1	$41^{\circ} 20'$	$1a$	$111^{\circ} 55'$	$46^{\circ} 8'$	$46^{\circ} 1'$
	{ 2	41 13	$2a$	111 45	46 4	
	{ 3	41 0	$3a$	111 56	45 50	
C	{ 1	41 22	$1a$	111 50	46 12	$46 6$
	{ 2	41 17	$2a$	111 37	46 10	
	{ 3	41 5	$3a$	111 50	45 56	
Na.....	{ 1	41 32	$1a$	110 35	46 40	$46 32$
	{ 2	41 33	$2a$	110 45	46 38	
	{ 3	41 15	$3a$	110 56	46 18	
Tl.....	{ 1	41 45	$1a$	109 35	47 8	$47 0$
	{ 2	41 46	$2a$	109 40	47 7	
	{ 3	41 25	$3a$	109 50	46 44	
F	{ 1	42 0	$1a$	108 10	47 44	$47 33$
	{ 2	42 0	$2a$	108 35	47 38	
	{ 3	41 40	$3a$	108 35	47 18	

Monobromonaphthalene was employed as immersion liquid in the measurements of $2Ha$ and $2Ho$.

An indirect determination of Murmann and Rotter by use of one plate and the mean index of refraction, afforded the angle $48^{\circ} 53'$. The dispersion is considerable, as will be seen from the table. Its order and approximate amount were confirmed by observations in toluene, whose refractive index is very near the mean index of the crystal.

Dispersion of the Median Lines.—The observations in toluene, with the two larger sections perpendicular to the first median line, indicated that the optic axis nearly normal to $r'\{201\}$ is almost fixed for all colours, whilst the other axis, not far from normal to $a\{100\}$, is dispersed, so that for blue it is furthest from the stationary axis. Hence the first median line is dispersed so that for blue it lies nearer to the crystallographic axis a than for red, the amount of difference between Li and F being a little over half a degree.

Effect of Rise of Temperature on Optic Axial Angle.—Section 1 was heated to 70° , and at this temperature the apparent angle in air 2E for C light was $80^{\circ} 30'$, for Na light $81^{\circ} 30'$, and for Tl light $82^{\circ} 30'$. Hence the apparent angle in air increases by no less than 9° for 50° rise of temperature, an amount of change larger than has been observed in any other salt of the series, excepting the abnormal cæsium magnesium salt.

Refractive Indices.—The following are the results of the determinations of refractive index, with six prisms, whose grinding upon the

Determination of Refractive Indices of Potassium Copper Sulphate.

Index.	Nature of light.	Crystal 1, 1 prism.	Crystal 2, 1 prism.	Crystal 3, 2 prisms.	Crystal 4, 2 prisms.	Mean refractive index.
α Vibrations parallel 2nd median line α .	Li	—	1.4808	1.4809	1.4803	1.4807
	C	—	1.4812	1.4814	1.4807	1.4811
	Na	—	1.4836	1.4837	1.4835	1.4836
	Tl	—	1.4863	1.4862	1.4858	1.4861
	F	—	1.4895	1.4893	1.4890	1.4893
	G	—	1.4945	1.4942	—	1.4944
β Vibrations parallel symmetry axis β β .	Li	1.4834	1.4836	1.4837	1.4830	1.4834
	C	1.4838	1.4840	1.4841	1.4834	1.4838
	Na	1.4864	1.4865	1.4866	1.4859	1.4864
	Tl	1.4890	1.4891	1.4890	1.4886	1.4889
	F	1.4923	1.4924	1.4922	1.4920	1.4922
	G	—	1.4977	1.4972	—	1.4975
γ Vibrations parallel 1st median line γ .	Li	1.4993	—	1.4992	1.4985	1.4990
	C	1.4997	—	1.4996	1.4989	1.4994
	Na	1.5021	—	1.5022	1.5017	1.5020
	Tl	1.5048	—	1.5049	1.5044	1.5047
	F	1.5081	—	1.5083	1.5080	1.5081
	G	—	—	1.5134	—	1.5134

small crystals, which alone are sufficiently transparent, required unusually delicate manipulation.

The excellent agreement of the values obtained from the four different crystals affords noteworthy testimony of what can be done with the new grinding instrument. Two of the crystals employed, chosen in preference to larger ones on account of their perfect transparency and perfection of development, were scarcely larger than the head of an ordinary pin, and on one of these (crystal 3) two prisms were ground having faces which were verified to be accurately orientated in the directions theoretically desirable in order to afford all three indices. The images of the spectrometer, "Websky" slit, refracted through these miniature prisms, moreover, were quite brilliant, even up to G, using the ordinary eye-piece affording images apparently an inch long, and illuminating in the ordinary manner with the slits of the monochromatic illuminator quite fine, the lime-light being the source of light.

The rough β indices given by Murmann and Rotter differ considerably from the values now presented. They are for end red, 1.488; bright red, 1.489; yellow, 1.491; green, 1.493; greenish-blue, 1.495; and blue, 1.498.

The β indices, corrected to a vacuum, are accurately afforded, for any wave-length λ to near F, by the following formula.

$$\beta = 1.4700 + \frac{729\ 070}{\lambda^2} - \frac{5\ 063\ 700\ 000\ 000}{\lambda^4} + \dots$$

The α indices are closely reproduced (to within 0.0001) if the constant 1.4700 is diminished by 0.0028, and the γ indices if it is increased by 0.0157. The former difference, being smallest, determines that the double refraction is positive, and the relatively very small amount of that difference causes the optic axial angle also to be small.

Effect of Rise of Temperature on the Refraction.—Some difficulty was experienced in obtaining trustworthy determinations of refractive index at 70° in the case of this salt, on account of decomposition and the resulting opacity. The results indicated conclusively, however, that the indices are lowered to about the same extent as was found for the other salts of the series, the particular mean amount indicated being 0.0020, with a possible error, from the cause mentioned, of 0.0005.

Ratio of Axes of Optical Velocity Ellipsoid.—The indices given afford the following ratios.

$$a : b : c = 1.0019 : 1 : 0.9896.$$

Molecular Optical Constants.—These are as under.

Axis of optical ellipsoid	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	C. 0.1281	0.1287	0.1322
	G. 0.1311	0.1318	0.1353
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$	C. 56.48	56.75	58.30
	G. 57.81	58.11	59.68
Specific dispersion, $n_g - n_c$	0.0030	0.0031	0.0031
Molecular dispersion, $m_g - m_c$	1.33	1.36	1.38
Molecular refraction, $\frac{n - 1}{d} M$	C. 95.48	96.01	99.11

These values will be independent of temperature, as alteration of the latter evokes changes of refraction and density in the same direction.

Rubidium Copper Sulphate, $\text{Rb}_2\text{Cu}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

The common *cleavage* of the series, parallel $r'\{\bar{2}01\}$, was verified on this salt, which exhibits it as well developed as usual. The cleaved surfaces reflected sharp images of the signal, at exactly 180° to r' .

Volume.

Relative Density.—Four determinations were made with crystals as free as possible from traces of opacity, belonging to different crops derived from two distinct preparations. The same precautions were taken as in the case of the potassium salt. The results were as stated below.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
3.4659	2.5652
3.5799	2.5682
4.9711	2.5735
4.2749	2.5726

Mean **2.5699**

Perrot found the density 2.58 at 15° .

$$\text{Molecular Volume.} \quad \frac{M}{d} = \frac{533.4}{2.570} = 207.55.$$

Distance Ratios.—The axial angle was previously shown to be $74^\circ 42'$, and the axial ratio:

$$a : b : c = 0.7490 : 1 : 0.5029.$$

Combination with the molecular volume affords the following distance ratios.

$$\chi : \psi : \omega = 6.2147 : 8.2974 : 4.1727.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two clear sections parallel to the symmetry plane $b\{010\}$ were obtained by grinding,

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Determination of true Optic Axial Angle of Rubidium Copper Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean 2Va.
Li.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 39^{\circ} 49' \\ 39 \ 44 \\ 39 \ 51 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 113^{\circ} \ 5' \\ 112 \ 50 \\ 112 \ 40 \end{array} \right.$	$\left\{ \begin{array}{l} 44^{\circ} \ 24' \\ 44 \ 23 \\ 44 \ 32 \end{array} \right.$	$\left. \right\} 44^{\circ} \ 26'$
C.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 39 \ 50 \\ 39 \ 45 \\ 39 \ 52 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 112 \ 55 \\ 112 \ 43 \\ 112 \ 35 \end{array} \right.$	$\left\{ \begin{array}{l} 44 \ 28 \\ 44 \ 26 \\ 44 \ 34 \end{array} \right.$	$\left. \right\} 44 \ 29$
Na.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 39 \ 53 \\ 39 \ 49 \\ 39 \ 55 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 112 \ 15 \\ 112 \ 0 \\ 111 \ 55 \end{array} \right.$	$\left\{ \begin{array}{l} 44 \ 40 \\ 44 \ 40 \\ 44 \ 46 \end{array} \right.$	$\left. \right\} 44 \ 42$
Tl.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 39 \ 57 \\ 39 \ 54 \\ 39 \ 59 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 111 \ 30 \\ 111 \ 17 \\ 111 \ 10 \end{array} \right.$	$\left\{ \begin{array}{l} 44 \ 54 \\ 44 \ 54 \\ 45 \ 2 \end{array} \right.$	$\left. \right\} 44 \ 57$
F.....	$\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$	$\left\{ \begin{array}{l} 40 \ 2 \\ 40 \ 0 \\ 40 \ 5 \end{array} \right.$	$\left\{ \begin{array}{l} 1a \\ 2a \\ 3a \end{array} \right.$	$\left\{ \begin{array}{l} 110 \ 40 \\ 110 \ 20 \\ 110 \ 20 \end{array} \right.$	$\left\{ \begin{array}{l} 45 \ 12 \\ 45 \ 14 \\ 45 \ 20 \end{array} \right.$	$\left. \right\} 45 \ 15$

Monobromonaphthalene was the immersion liquid for 2Ha and 2Ho. The order of dispersion was confirmed by observations in toluene, which clearly indicated that the angle for blue is greater than that for red.

Perrot found it $44^{\circ} 30'$ for sodium light, using a plate approximately perpendicular to the first median line, and knowing the β index of refraction.

Dispersion of the Median Lines.—The observations in toluene further indicated that the optic axis which emerges between $r\{20\bar{1}\}$ and $s\{10\bar{1}\}$ is almost constant for all colours, whilst the other axis nearly normal to $a\{100\}$ is dispersed so that for red it is nearer to the fixed axis than it is for blue. The first median line, therefore, is dispersed so that for blue it lies nearer to the morphological axis a than it does for red; the amount of dispersion indicated between Li and F was about $15'$.

Effect of Rise of Temperature on Optic Axial Angle.—Section 2 was heated to 70° , and the apparent angle in air measured at that temperature. It was found to be $72^{\circ} 45'$ for sodium light. Hence 2E increases about $3^{\circ} 45'$ for 50° rise of temperature.

Refractive Indices.—The indices were determined with six prisms ground upon four different crystals. The results are as given in the accompanying table (p. 440).

The β indices, corrected to a vacuum, are accurately expressed, for any wave-length λ to beyond F, by the following formula.

$$\beta = 1.4791 + \frac{392 \ 159}{\lambda^2} + \frac{726 \ 600 \ 000 \ 000}{\lambda^4} + \dots$$

Determination of Refractive Indices of Rubidium Copper Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2, 1 prism.	Crystal 3, 2 prisms.	Crystal 4, 1 prism.	Mean refractive index.	Perrot's values.
α Vibs. par. 2nd median line	Li	1·4858	1·4853	1·4863	1·4860	1·4858	—
	C	1·4862	1·4857	1·4867	1·4863	1·4862	1·4859
	Na	1·4886	1·4881	1·4892	1·4887	1·4886	1·4885
	Tl	1·4911	1·4906	1·4917	1·4915	1·4912	—
	F	1·4943	1·4938	1·4948	1·4944	1·4943	1·4944
	G	—	1·4988	1·4994	1·4990	1·4991	—
β Vibs. par. symmetry axis <i>b</i> (<i>h</i>)	Li	1·4878	1·4873	1·4883	—	1·4878	—
	C	1·4882	1·4877	1·4887	—	1·4882	1·4880
	Na	1·4906	1·4901	1·4912	—	1·4906	1·4906
	Tl	1·4932	1·4928	1·4938	—	1·4933	—
	F	1·4966	1·4962	1·4971	—	1·4966	1·4968
	G	1·5012	1·5010	1·5018	—	1·5013	—
γ Vibs. par. 1st median line <i>c</i>	Li	1·5004	—	1·5006	1·5010	1·5007	—
	C	1·5008	—	1·5010	1·5014	1·5011	1·5008
	Na	1·5030	—	1·5037	1·5041	1·5036	1·5032
	Tl	1·5061	—	1·5064	1·5067	1·5064	—
	F	1·5095	—	1·5098	1·5102	1·5098	1·5095
	G	1·5142	—	1·5150	1·5151	1·5148	—

The α indices are very closely reproduced if the constant 1·4791 is diminished by 0·0021, and the γ indices if it is increased by 0·0130. The former difference being the smaller determines the positive nature of the double refraction; it is even relatively smaller than in the potassium salt, and hence the optic axial angle is less.

Effect of Rise of Temperature on the Refraction.—Determinations at 70° indicate that the indices are lowered about 0·0015 by 50° rise of temperature.

Ratio of Axes of Optical Velocity Ellipsoid.—The calculated values of these ratios are as under.

$$\alpha : \beta : \gamma = 1·0013 : 1 : 0·9914.$$

Molecular Optical Constants.—The refractive indices combined with the molecular weight and the density afford the following optical constants.

Axis of optical ellipsoid.....	α .	β .	γ .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0·1118	0·1122	0·1147
	{ G. 0·1143	0·1148	0·1174
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 59·65	59·86	61·20
	{ G. 60·99	61·22	62·61
Specific dispersion, $n_G - n_C \dots$	0·0025	0·0026	0·0027
Molecular dispersion, $m_G - m_C \dots$	1·34	1·36	1·41
Molecular refraction, $\frac{n - 1}{d} M \dots$	C. 100·99	101·41	104·09

These constants are again practically independent of temperature, as the refraction has been shown to diminish with rise of temperature, that is, to change in the same direction as does the density when the temperature is altered.

Cæsium Copper Sulphate, $\text{Cs}_2\text{Cu}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

There is an excellent *cleavage* parallel $r'\{201\}$, as usual in the series. Perfect reflections, at 180° from the r' faces, were obtained from the fractured surfaces.

The crystals of this salt are readily obtained quite free from turbidity, as stated in the former communication. Both the volume work and the optical investigation were carried out with very perfect specimens.

Volume.

Relative Density.—Four determinations were made with different crops belonging to two distinct preparations. The results were as given below.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
4.3268	2.8500
5.0172	2.8561
4.5351	2.8535
3.4604	2.8565
Mean	<u>2.8540</u>

$$\text{Molecular Volume.} - \frac{M}{d} = \frac{628.4}{2.854} = 220.19.$$

Distance Ratios.—The axial angle β was previously found to be $73^\circ 50'$, and the axial ratio

$$a : b : c = 0.7429 : 1 : 0.4946.$$

The distance ratios obtained by combining these with the molecular volume are

$$\chi : \psi : \omega = 6.3481 : 8.5450 : 4.2263.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Two sections ground parallel to the symmetry plane $b\{010\}$, exhibited the following extinction angles with respect to the normal to the basal plane $c\{001\}$.

Section 1	$26^\circ 45'$
Section 2	$26 \quad 50$
	<u>26 47</u>

The direction is forwards, as indicated in Fig. 20.

This is again the second median line, the first median line being at right angles to it in the symmetry plane.

The *sign of double refraction* is positive, demonstrated and proved as in the cases previously considered. The morphological axial angle being $73^{\circ} 50'$, the orientations are as follows.

The first median line ϵ lies in the acute angle of the morphological axes ac , and is inclined $26^{\circ} 47'$ to the morphological axis a .

The second median line α lies in the obtuse angle ac , and is inclined $42^{\circ} 57'$ to the vertical axis c .

The first median line is also inclined $116^{\circ} 47'$ to the normal to $c\{001\}$, and $42^{\circ} 57'$ to the normal to $a\{100\}$, and is only $27'$ removed from being exactly normal to $r'\{20\bar{1}\}$.

Optic Axial Angle.—Six excellent sections were ground, three perpendicular to the first, and three to the second, median line, and afforded the results given in the accompanying tables.

Determination of apparent Angle in Air of Cæsium Copper Sulphate.

Nature of light.	Section 1.	Section 2.	Section 3.	Mean 2E.
Li	$67^{\circ} 30'$	$67^{\circ} 25'$	$67^{\circ} 22'$	$67^{\circ} 26'$
C	$67 35$	$67 32$	$67 28$	$67 32$
Na	$68 5$	$68 11$	$68 0$	$68 5$
Tl	$68 40$	$68 50$	$68 29$	$68 40$
F	$69 27$	$69 40$	$69 5$	$69 24$

Monobromonaphthalene was used as immersion liquid in the measurement of $2Ha$ and $2Ho$. The order of dispersion was confirmed by observations in cedar oil, whose refractive index is very nearly the same as the mean index of the crystal; the angle was clearly larger for blue than for red.

Dispersion of the Median Lines.—The observations in cedar oil indicated that the optic axis emerging nearly parallel to the morphological axis a is practically immovable for all colours, whilst the other axis nearly normal to $s'\{10\bar{1}\}$ is dispersed, so that for blue it is further removed from the fixed axis than for red. Hence the first median

Determination of true Optic Axial Angle of Cæsium Copper Sulphate.

Nature of light.	No. of section perp. 1st median line.	Observed values of 2H α .	No. of section perp. 2nd median line.	Observed values of 2H β .	Calculated values of 2V α .	Mean 2V α .
Li	{ 1 2 3	{ 39° 3' 38 55 39 6	{ 1a 2a 3a	{ 115° 31' 115 30 115 35	{ 43° 7' 43 0 43 10	{ 43° 6'
C	{ 1 2 3	{ 39 4 38 58 39 7	{ 1a 2a 3a	{ 115 22 115 22 115 27	{ 43 10 43 4 43 12	{ 43 9
Na	{ 1 2 3	{ 39 8 39 7 39 11	{ 1a 2a 3a	{ 114 38 114 40 114 45	{ 43 24 43 22 43 25	{ 43 24
Tl	{ 1 2 3	{ 39 13 39 16 39 15	{ 1a 2a 3a	{ 113 47 113 56 113 52	{ 43 40 43 41 43 40	{ 43 40
F	{ 1 2 3	{ 39 21 39 30 39 20	{ 1a 2a 3a	{ 112 35 113 0 112 52	{ 44 4 44 7 43 59	{ 44 3

line is dispersed in such a manner that for red it is nearer to the morphological axis a than it is for blue; the amount indicated was about 20—25'. It is interesting to note that the only other salt of the series which exhibits this order of dispersion of the median lines is the cæsium magnesium salt, and that in both these cases this reversal of the usual order of dispersion for the series is accompanied by the exhibition of an exceptionally small optic axial angle, and near approach to coincidence of the α and β refractive indices.

Effect of Rise of Temperature on the Optic Axial Angle.—Section 2 was heated to 70°, at which temperature the apparent angle in air measured 66° 40' for sodium light. Hence 2E decreases about 1½° for 50° rise of temperature.

Refractive Indices.—Seven excellent prisms were ground upon four crystals, and furnished the results given in the accompanying table.

The β indices, corrected to a vacuum, are accurately represented, for any wave-length λ almost as far as G, by the formula

$$\beta = 1.4940 + \frac{417\ 147}{\lambda^2} + \frac{580\ 700\ 000\ 000}{\lambda^4} + \dots$$

The α indices are very closely reproduced if the constant 1.4940 is diminished by 0.0015, and the γ indices if it is increased by 0.0091. The relations of these two differences from β determine the sign of double refraction to be again positive.

Effect of Rise of Temperature on Refraction.—Determinations at 70° yielded refractive indices about 0.0016 lower than the values at the ordinary temperature.

Determination of Refractive Indices of Cæsium Copper Sulphate.

Index.	Nature of light.	Crystal 1, 1 prism.	Crystal 2, 2 prisms.	Crystal 3, 2 prisms.	Crystal 4, 2 prisms.	Mean refractive index.
α Vibrations parallel 2nd median line α	Li	—	1·5018	1·5017	1·5017	1·5017
	C	—	1·5022	1·5021	1·5020	1·5021
	Na	—	1·5049	1·5048	1·5047	1·5048
	Tl	—	1·5075	1·5074	1·5074	1·5074
	F	—	1·5109	1·5108	1·5107	1·5108
	G	—	1·5160	1·5155	1·5163	1·5159
β Vibrations parallel sym- metry axis β β	Li	1·5030	1·5035	1·5033	1·5030	1·5032
	C	1·5034	1·5039	1·5037	1·5034	1·5036
	Na	1·5059	1·5064	1·5062	1·5060	1·5061
	Tl	1·5085	1·5091	1·5090	1·5089	1·5089
	F	1·5120	1·5126	1·5123	1·5122	1·5123
	G	1·5171	1·5175	1·5171	1·5179	1·5174
γ Vibrations parallel 1st median line γ	Li	1·5120	1·5125	1·5121	1·5122	1·5122
	C	1·5124	1·5128	1·5125	1·5126	1·5126
	Na	1·5151	1·5156	1·5153	1·5153	1·5153
	Tl	1·5179	1·5185	1·5178	1·5179	1·5180
	F	1·5215	1·5220	1·5213	1·5214	1·5216
	G	1·5268	1·5268	1·5266	1·5263	1·5266

Ratio of Axes of Optical Velocity Ellipsoid.—These ratios are as follows.

$$a : b : c = 1.0009 : 1 : 0.9939.$$

Molecular Optical Constants.—The values of these constants are tabulated below.

Axis of optical ellipsoid	α .	β .	γ .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.1035	0.1037	0.1053
	{ G. 0.1059	0.1061	0.1077
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 65.04	65.20	66.18
	{ G. 66.54	66.70	67.69
Specific dispersion, $n_G - n_C \dots$	0.0024	0.0024	0.0024
Molecular dispersion, $m_G - m_C \dots$	1.50	1.50	1.51
Molecular refraction, $\frac{n - 1}{d} M \dots$	C. 110.64	110.97	112.96

These values are substantially constant for all temperatures at which the salt is capable of existing, as refraction and density are similarly affected by change of temperature.

SALTS CONTAINING CADMIUM.

As stated in the former communication, efforts to prepare a potassium cadmium sulphate containing $6H_2O$ have proved unavailing.

Rubidium Cadmium Sulphate, $\text{Rb}_2\text{Cd}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

The crystals of this salt exhibit a perfect *cleavage* parallel $r'\{\bar{2}01\}$, in common with the other salts of the series. Good images of the goniometer signal are obtained from the freshly cleaved surfaces, at exactly 180° from r' .

Considerable difficulty attended the investigation of this salt, on account of the unfortunate property referred to in the former memoir (p. 407), of efflorescing and becoming perfectly opaque with extreme rapidity after removal from the mother liquor.

Volume.

Relative Density.—In order to determine the density, the device was adopted of weighing the salt under carbon tetrachloride, the liquid used in the determinations. It had previously been found that benzene and carbon tetrachloride were both effective in preventing freshly grown crystals from losing their transparency for some considerable time, several days, as a rule. Four crops of beautifully perfect crystals were employed. In each case the crystals were rapidly dried on blotting paper as they were removed from the mother liquor, and then immediately placed under benzene. They were then well washed with several quantities of pure benzene, in order to remove the last traces of moisture. Two or three at a time were then removed, drained from benzene on blotting paper, given two minutes or so to lose the last traces of that liquid by evaporation, afterwards powdered as rapidly as possible in the agate mortar, and eventually transferred to the pyknometer, which already contained a known weight of carbon tetrachloride, sufficient to amply cover the powder; when sufficient had thus been introduced, the stopper and cap were replaced, the whole fixed in the wire spring stand, and weighed after the usual interval to permit of the surface equilibrium of the pyknometer being attained. A slight correction was made, in accordance with the indications of a duplicate experiment with the liquid alone, for the small amount of loss of carbon tetrachloride by evaporation during the transference of the salt to the pyknometer. The remaining small errors, due to a possible trace of moisture or benzene, on the one hand, and to slight efflorescence during the powdering, on the other, affect the density in opposite directions and thus tend to neutralise each other. The results obtained are as follows.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
4.2949	2.6726
2.5409	2.6712
3.8547	2.6677
3.0104	2.6744
Mean	2.6715

Perrot found the density about 2.70.

$$\text{Molecular Volume.}—\frac{M}{d} = \frac{582.4}{2.672} = 217.96.$$

Distance Ratios.—The axial angle β was previously shown to be $74^\circ 7'$, and the axial ratio

$$a : b : c = 0.7346 : 1 : 0.4931.$$

Combination of these with the molecular volume affords the following distance ratios.

$$\chi : \psi : \omega = 6.2828 : 8.5526 : 4.2174.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—The rapidity of efflorescence is such that section-plates or prisms ground as quickly as possible are absolutely opaque by the time they are completed. It was found possible, however, to obtain prisms sufficiently transparent for the purposes of the determinations of refractive index, by a device which will be described in connection with those determinations.

Five section-plates, parallel to the symmetry plane, were prepared with the greatest rapidity consistent with due care as to their orientation, using a cementing wax which remains soft just sufficiently long for the purpose after becoming moderately cool, in order to avoid hastening the efflorescence by rise of temperature; but all were useless when finished, having the appearance of white chalk. Attempts to obtain sections by cementing the first surface upon one of the glass discs intended for use with the special crystal holder employed for grinding a second parallel surface, by a cold viscoid solution of hard balsam in benzene, covering the whole crystal with a protecting layer of the same, and then grinding the second surface after standing all night to harden, were also fruitless as opacity always supervened during the interval.

The orientation of the optical planes was, however, determined by immersion of transparent crystals, freshly removed from the mother liquor, in toluene, whose refractive index is nearly the same as the mean index of the crystal, contained in the cell of a stage goniometer, forming one of the accessories of a Swift petrological microscope. The crystal was arranged so that the symmetry plane lay parallel to the stage, and the edge $(110):(010)$ was focussed and brought parallel to one of the spider lines, in order to utilise it as reference direction. The extinction angle made with this edge was then determined in the usual manner, and repeated with the crystal rotated 180° , in order to eliminate any error of parallelism between spider line and nicols. It was found in each case that extinction occurred at about 16° from this edge, and apparently exactly normal to the basal plane, that is, to the edge $(001):(011)$. Now the com-

plement of the axial angle ($74^{\circ} 7'$) is $15^{\circ} 53'$, and this angle is the inclination between the reference edge and the normal to the basal plane. The difference between 16° and $15^{\circ} 53'$ is within the error of the determinations, so that the extinction is normal to the basal plane within the usual error of stauroscopical determinations.

Perrot arrived at the same conclusion, but does not state the method of determining it.

This direction is the second median line, and the first median line is the direction at right angles in the symmetry plane, as shown in Fig. 21.

FIG. 21.

The *sign of double refraction* is positive, proved by the relations of the refractive indices, and demonstrated by the quarter undulation mica. Hence the orientation is as follows.

The first median line c is coincident with the morphological axis ac .

The second median line a lies in the obtuse angle of the morphological axes ac , it is precisely normal to $c\{001\}$, and is inclined $15^{\circ} 53'$ to the vertical axis c .

The first median line is consequently inclined exactly 90° to the normal to $c\{001\}$, and $15^{\circ} 53'$ to the normal to $a\{100\}$.

The above method of determining extinctions, by immersion in a liquid of the same refractive index as the crystal, has lately been described by Klein (*Sitz. Ber., Berlin Akademie*, 1895, 5, 91). The author has long been in the habit of having recourse to measurements in such liquids in difficult cases, and the present instance affords a good example of the value of the method. The refracting edges of the crystal in such a medium exert no disturbing influence on the observations, the crystal behaving like a parallel-sided plate.

Optic Axial Angle.—The method has been further utilised, as in the case of rubidium sulphate described in the 1894 memoir (p. 692), and the case of cæsium magnesium sulphate described in the present communication, in order to determine the true optic axial angle of rubidium cadmium sulphate. Two liquids were employed, very

slightly lower and higher respectively than the mean index of the crystals, namely, turpentine and toluene, as in the case of the cæsium magnesium salt, whose indices are nearly the same as those of the salt now under consideration. The mean of the angles in the two liquids is considered as the true angle. As the second median line is perpendicular to the well-developed faces of the basal plane, the obtuse angle of the axes was actually observed. Fresh, perfectly transparent crystals, which had been well washed with benzene were taken and measured, first in toluene, which, being volatile, rapidly dries off the crystal after removal of the cell, and subsequently in turpentine. The results are presented in the accompanying table.

Determination of true Optic Axial Angle of Rubidium Cadmium Sulphate.

Nature of light.	No. of crystal.	Angle in toluene.	Angle in turpentine.	Mean values.	Mean obtuse angle.	Mean acute angle, $2Va$.
Li.....	{ 1	106° 30'	108° 48'	107° 39'	107° 40'	72° 20'
	{ 2	106 36	108 55	107 46		
	{ 3	106 33	108 35	107 34		
C.....	{ 1	106 28	108 48	107 38	107 39	72 21
	{ 2	106 35	108 55	107 45		
	{ 3	106 32	108 35	107 33		
Na.....	{ 1	106 16	108 46	107 31	107 34	72 26
	{ 2	106 27	108 54	107 40		
	{ 3	106 27	108 34	107 30		
Tl.....	{ 1	106 8	108 44	107 26	107 29	72 31
	{ 2	106 21	108 51	107 36		
	{ 3	106 21	108 32	107 26		
F.....	{ 1	105 56	108 41	107 18	107 23	72 37
	{ 2	106 14	108 49	107 31		
	{ 3	106 14	108 29	107 21		

Perrot determined the angle upon two natural plates parallel $\{20\bar{1}\}$, by the indirect method, and obtained the values $71^\circ 59'$ and $72^\circ 15'$ for sodium light.

Dispersion of the Median Lines.—The observations in both liquids indicated that the second median line is dispersed so that for blue it is nearer to the vertical axis of the crystal than for red, and that the first median line is correspondingly dispersed so that for blue it is nearer to the normal to a $\{100\}$ than it is for red. The amount of dispersion is about $10'$ between Li and F. The indications regarding the dispersion of the axes themselves are different in the two liquids, but the mean of the two sets of readings shows that both axes exhibit dispersion, but the one emerging between $r'\{20\bar{1}\}$ and $s'\{10\bar{1}\}$ least.

Measurements at higher temperatures were manifestly impossible with this salt.

Refractive Indices.—It happens fortunately that, in this salt, one of the principal planes, bc , of the optical ellipsoid is parallel to the basal plane, for this enables a pair of faces of the clinodome $q\{011\}$, which are symmetrical to the basal plane, and whose edge is parallel to the first median line, the optical elasticity axis c , to be utilised for the determination of the refractive indices β and γ . Three such crystals were employed, fresh, transparent individuals with largely developed q faces being chosen. In order to prevent efflorescence during the measurements, and also to eliminate the effects of any minute imperfections of the faces, the two faces utilised as a prism were in each case fitted with thin micro-cover glasses cemented by a solution of hard balsam in benzene. Four other prisms were obtained by grinding, in such directions as to afford the indices α and β . This was successfully achieved by choosing crystals of such shape that the wax of the crystal holder of the grinding instrument, while only slightly warm just before setting hard, could be attached to a part of the crystal sufficiently removed from the part where the prism faces were to be ground as not to exert any great influence in accelerating efflorescence; also, as soon as the necessary adjustment of the crystal had been achieved, by goniometrical observation of the reference faces, the part of the crystal projecting out of the wax was coated with the balsam solution, which retarded materially the efflorescence. Working quickly with these precautions, not waiting to polish the ground surfaces, cleaning the crystal from wax entirely by immersion in benzene, thickly coating the first ground prism face with balsam solution immediately on removing the crystal from the benzene, and finally, when the second face had been ground in like manner, without allowing the warm wax to approach dangerously near to the first face while attaching to the holder, the two surfaces were coated with balsam and fitted with cover glasses, and a prism thus obtained which, in each case, proved sufficiently transparent to afford clear refracted images of the spectrometer slit. One of these prisms was ground upon one of the three crystals previously used to determine β and γ .

The results afforded by the seven prisms are given in the accompanying table (p. 450).

The β index, corrected for a vacuum, is accurately expressed for any wave-length λ nearly as far as F by the following formula.

$$\beta = 1.4706 + \frac{602\ 672}{\lambda^2} - \frac{3\ 327\ 400\ 000\ 000}{\lambda^4} + \dots$$

The α index is very closely expressed by the formula if the constant 1.4706 is diminished by 0.0048, and the γ index if it is increased by 0.0100. The positive nature of the double refraction is

Determination of Refractive Indices of Rubidium Cadmium Sulphate.

Index.	Nature of light.	Crystal 1, 2 prisms.	Crystal 2.	Crystal 3.	Crystal 4.	Crystal 5.	Crystal 6.	Mean refrac- tive index.	Perot's values.
α Vibrations parallel 2nd median line <i>a</i> .	Li	1.4774	—	1.4771	1.4773	1.4774	—	1.4773	—
	C	1.4778	—	1.4775	1.4777	1.4778	—	1.4777	1.4773
	Na	1.4798	—	1.4797	1.4797	1.4800	—	1.4798	1.4800
	Tl	1.4823	—	1.4821	1.4824	1.4825	—	1.4823	—
	F	1.4856	—	1.4854	1.4857	1.4859	—	1.4856	1.4856
	G	1.4906	—	—	—	—	—	1.4906	—
β Vibrations parallel symmetry axis <i>b</i> <i>b</i> .	Li	1.4824	1.4816	1.4821	1.4826	1.4816	1.4820	1.4820	—
	C	1.4828	1.4820	1.4824	1.4829	1.4820	1.4824	1.4824	—
	Na	1.4852	1.4842	1.4849	1.4852	1.4844	1.4848	1.4848	1.4851
	Tl	1.4876	1.4868	1.4872	1.4876	1.4867	1.4872	1.4872	—
	F	1.4909	1.4900	1.4903	1.4909	1.4904	1.4905	1.4905	1.4908
	G	1.4955	—	—	—	—	—	1.4955	—
γ Vibrations parallel 1st median line <i>c</i> .	Li	1.4923	1.4915	—	—	—	1.4919	1.4919	—
	C	1.4927	1.4919	—	—	—	1.4923	1.4923	—
	Na	1.4952	1.4943	—	—	—	1.4948	1.4948	1.4952
	Tl	1.4976	1.4969	—	—	—	1.4972	1.4972	—
	F	1.5012	1.5001	—	—	—	1.5008	1.5007	—
	G	1.5061	—	—	—	—	—	1.5061	—

proved by the fact that the former is the smaller of the two differences from β .

Determinations at higher temperatures were out of the question.

Ratio of Axes of Optical Velocity Ellipsoid.—The indices afford the following ratios.

$$a : b : c = 1.0034 : 1 : 0.9933.$$

Molecular Optical Constants.—The calculated values of these constants are appended.

Axis of optical ellipsoid.....	a.	b.	c.
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n$..	{ C. 0.1060 G. 0.1084	0.1068 0.1093	0.1087 0.1113
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m$ {	C. 61.71 G. 63.13	62.23 63.66	63.31 64.81
Specific dispersion, $n_G - n_C$	0.0024	0.0025	0.0026
Molecular dispersion, $m_G - m_C$	1.42	1.43	1.50
Molecular refraction, $\frac{n - 1}{d} M$	C. 104.21	105.23	107.39.

Cæsium Cadmium Sulphate, $\text{Cs}_2\text{Cd}(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$.

There is a good *cleavage* parallel $r'\{201\}$; the cleaved surfaces reflect perfect images of the goniometer signal at exactly 180° from r' .

Volume.

Relative Density.—The crystals of cæsium cadmium sulphate are not nearly so unstable as those of the rubidium salt, but efflorescence does ensue, to some extent, in the course of a day or two when freely exposed to the air. In order to avoid error due to this, the powdered substance was weighed under carbon tetrachloride, as in the case of the rubidium salt. Four crops of freshly grown, perfectly transparent crystals, derived from two independent preparations, were employed for the density determinations, and yielded the following results.

Weight of salt employed.	Sp. gr. at $20^\circ/4^\circ$.
2.4589	2.9551
1.8953	2.9559
3.8114	2.9535
3.8658	2.9561

Mean 2.9552

$$\text{Molecular Volume.} \quad \frac{M}{d} = \frac{677.4}{2.955} = 229.24.$$

Distance Ratios.—The axial angle β was previously shown to be $72^\circ 49'$, and the ratio of the axes

$$a : b : c = 0.7259 : 1 : 0.4906.$$

Combination of these with the molecular volume yields the following distance ratios.

$$\chi : \psi : w = 6.3638 : 8.7668 : 4.3010.$$

Optics.

Orientation of Axes of Optical Ellipsoid.—Transparent sections of this salt were successfully obtained by cementing the first ground face with solution of balsam upon one of the small glass discs furnished for use with the "parallel surface" crystal holder of the grinding instrument, covering the whole of the crystal with the balsam in order to retard efflorescence, allowing to stand all night to harden, and grinding the second surface next morning. Two section-plates parallel to the symmetry plane were thus obtained, and they each exhibited the extinction angle of $9^{\circ} 45'$ in front of the normal to $c\{001\}$, as represented in Fig. 22.

FIG. 22.

The *sign of double refraction* is positive, as demonstrated by means of a quarter undulation mica and proved by the relations of the three refractive indices. The morphological axial angle being $72^{\circ} 49'$, the orientation is as follows.

The first median line c lies in the acute angle of the morphological axes ac , and is inclined $9^{\circ} 45'$ to the axis a .

The second median line a lies in the obtuse angle ac , and is inclined $26^{\circ} 56'$ to the vertical axis c .

The first median line is also inclined $99^{\circ} 45'$ to the normal to $c\{001\}$, and $26^{\circ} 56'$ to the normal to $a\{100\}$.

Optic Axial Angle.—Three pairs of section-plates were obtained in the manner just described, and afforded the measurements given in the accompanying tables. Section 2 did not show the axes in air, being narrow in comparison with the considerable thickness required to afford fine brushes and small rings on account of the feeble double refraction.

Determination of the apparent Angle in Air of Cæsium Cadmium Sulphate.

Nature of light.	Section 1.	Section 3.	Mean 2E.
Li.....	114° 20'	114° 45'	114° 32'
C.....	114 15	114 40	114 27
Na.....	113 51	114 18	114 4
Tl.....	113 27	113 50	113 38
F.....	112 55	113 20	113 7

Determination of true Optic Axial Angle of Cæsium Cadmium Sulphate.

Nature of light.	No. of section perp. 1st median line	Observed values of 2Ha.	No. of section perp. 2nd median line.	Observed values of 2Ho.	Calculated values of 2Va.	Mean 2Va.
Li.....	{ 1	60° 58'	1a	97° 21'	68° 4'	68° 4'
	{ 2	60 36	2a	97 8	67 53	
	{ 3	60 55	3a	96 55	68 14	
C.....	{ 1	60 53	1a	97 17	68 2	68 2
	{ 2	60 32	2a	97 4	67 51	
	{ 3	60 50	3a	96 48	68 12	
Na.....	{ 1	60 30	1a	96 57	67 52	67 53
	{ 2	60 10	2a	96 45	67 42	
	{ 3	60 33	3a	96 30	68 6	
Tl.....	{ 1	60 4	1a	96 37	67 40	67 44
	{ 2	59 49	2a	96 20	67 35	
	{ 3	60 12	3a	96 11	67 57	
F.....	{ 1	59 25	1a	95 57	67 26	67 28
	{ 2	59 15	2a	95 50	67 20	
	{ 3	59 35	3a	95 43	67 39	

Monobromonaphthalene was used as immersion liquid for the determination of 2Ha and 2Ho. The order of dispersion was confirmed by observations in benzene, whose refractive index is near the mean index of the crystals, the angle being over half a degree larger for red than for blue.

Dispersion of the Median Lines.—The observations in benzene further indicated that the first median line was dispersed about 10–15', being nearer the morphological axis *a* for blue than for red. Both optic axes showed dispersion, the axis which emerges near the normal to *a*{100} much less than the other. The order is

red | blue blue | red.

No observations could be carried out at higher temperatures on account of the rapidity of decomposition.

Refractive Indices.—Six transparent prisms were successfully prepared on six different crystals by attaching the cementing wax of the crystal holder only to parts of the crystals removed from the portions where the surfaces were to be ground, and coating the first ground face thickly with balsam while the second surface was being

Determination of Refractive Indices of Caesium Cadmium Sulphate.

Index.	Nature of light.	Crystal 1.	Crystal 2.	Crystal 3.	Crystal 4.	Crystal 5.	Crystal 6.	Mean refractive index.
α vibrations parallel 2nd median line, a.	Li	—	1.4947	1.4944	1.4950	1.4953	1.4943	1.4947
	C	—	1.4950	1.4946	1.4952	1.4957	1.4949	1.4951
	Na	—	1.4973	1.4970	1.4977	1.4982	1.4973	1.4975
	Tl	—	1.5000	1.4995	1.5001	1.5007	1.4998	1.5000
	F	—	1.5032	1.5027	1.5036	1.5039	1.5033	1.5033
	G	—	1.5080	1.5076	1.5083	1.5086	—	1.5081
β vibrations parallel symmetry axis b (b).	Li	1.4967	—	—	1.4977	—	1.4971	1.4972
	C	1.4971	—	—	1.4981	—	1.4976	1.4976
	Na	1.4997	—	—	1.5003	—	1.4999	1.5000
	Tl	1.5022	—	—	1.5030	—	1.5026	1.5026
	F	1.5054	—	—	1.5062	—	1.5057	1.5058
	G	—	—	—	1.5109	—	1.5103	1.5106
γ vibrations parallel 1st median line, c.	Li	1.5029	1.5035	1.5030	—	1.5041	—	1.5034
	C	1.5033	1.5038	1.5034	—	1.5045	—	1.5038
	Na	1.5057	1.5064	1.5058	—	1.5068	—	1.5062
	Tl	1.5081	1.5091	1.5084	—	1.5096	—	1.5088
	F	1.5122	1.5124	1.5118	—	1.5129	—	1.5023
	G	—	1.5170	1.5168	—	1.5178	—	1.5172

ground. The prism faces were fitted with cover glasses as soon as the prisms were finished. The results are given in the accompanying table.

The β indices, corrected for a vacuum, are accurately expressed for any wave-length λ as far as F by the following formula.

$$\beta = 1.4876 + \frac{462\ 311}{\lambda^2} - \frac{621\ 000\ 000\ 000}{\lambda^4} + \dots$$

The α indices are very closely expressed by the formula if the constant 1.4876 is diminished by 0.0025, and the γ indices if it is increased by 0.0062. The former being the smaller difference from β , determines the sign of double refraction to be positive.

No observations at higher temperatures could be made on account of the rapid decomposition.

Ratio of Axes of Optical Velocity Ellipsoid.—The indices afford the following ratio.

$$a : b : c = 1.0017 : 1 : 0.9959.$$

Molecular Optical Constants.—The indices, combined with the density and molecular weight, give the following constants.

Axis of optical ellipsoid.	α .	b .	c .
Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n \dots$	{ C. 0.0987	0.0992	0.1002
	{ G. 0.1010	0.1014	0.1025
Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m \dots$	{ C. 66.91	67.19	67.91
	{ G. 68.39	68.67	69.42
Specific dispersion, $n_G - n_C \dots$	0.0023	0.0022	0.0023
Molecular dispersion, $m_G - m_C \dots$	1.48	1.48	1.51
Molecular refraction, $\frac{n - 1}{d} M \dots$	C. 113.59	114.16	115.58

COMPARISON OF RESULTS.

It will be most convenient and instructive to make the comparison of the results, afforded for each property, for the whole of the 22 salts at once, in order that a general supervision of the changes brought about by the replacement of potassium by rubidium and the latter by caesium in each of the eight sets may be obtained, and any modifications of such changes due to the influence of the second metal clearly exhibited.

Cleavage.

It has been found that—

The whole of the salts of the series exhibit a common cleavage direction, parallel to the orthodome $r'\{201\}$. The fracture is a very perfect one, exhibiting well-defined images of the goniometer signal at exactly 180° to r' .

Results of the Volume of Investigation.

Relative Density.—A comparative table of the densities found for the various salts is appended. The salts are arranged so that those containing the same alkali metal are vertically under each other, and with the second metals which they contain arranged in the order of their atomic weight. The differences between corresponding potassium and rubidium salts and rubidium and caesium salts are also shown.

Relative Densities of the Salts of the Series.

Second metal M contained.	Atomic weight of M.	Density of K salt.	Difference between K and Rb salts.	Density of Rb salt.	Difference between Rb and Cs salts.	Density of Cs salt.
Mg	24.0	2.028	0.354	2.382	0.288	2.670
Mn	55.0	—	—	2.459	0.279	2.738
Fe	56.0	2.169	0.347	2.516	0.275	2.791
Ni	58.9	2.233	0.350	2.583	0.282	2.865
Co	(Winkler.) 59.6	2.212	0.345	2.557	0.280	2.837
Cu	(Winkler.) 63.0	2.224	0.346	2.570	0.284	2.854
Zn	65.0	2.241	0.343	2.584	0.283	2.867
Cd	112.0	—	—	2.672	0.283	2.955

The following conclusions may be drawn from this table.

The increase in density brought about by the replacement of potassium by rubidium, and rubidium by caesium, is approximately constant for the particular change throughout the series. The difference evoked by the former change is larger than that produced by the latter in the proportion of almost exactly 5 : 4.

An increase in density usually accompanies the replacement of the second metal M by one of higher atomic weight. The nickel salts, however, do not fall in with this rule, being higher than the corresponding cobalt salts; this cannot be due to impurity in the nickel sulphate, for two different samples were employed in the preparation of each of the three salts, but no appreciable difference of density was detected; moreover, analyses of the double salts yielded almost theoretical results. A precisely similar fact is shown by the corresponding selenates containing potassium investigated by Topsøe and Christiansen. The sp. gr. of the potassium magnesium salt is given as 2.336, the potassium nickel salt 2.539, the potassium cobalt salt 2.514, the potassium copper salt 2.527, and the potassium zinc salt 2.538. Hence it must be a real property of the nickel salts. All these

differences due to the replacement of the second metal are much less in amount than those due to interchanges of the alkali metals.

The fact that the difference between the potassium and rubidium salts containing copper is about the average for the series, affords most satisfactory confirmation of the accuracy of the determinations of the density of the potassium salt, in connection with which unusual difficulties were experienced.

Molecular Volume.—A table is next given in which the molecular volumes are similarly compared.

Molecular Volumes of the Salts of the Series.

Second metal M contained.	Mol. vol. of K salt.	Difference between K and Rb salts.	Mol. vol. of Rb salt.	Difference between Rb and Cs salts.	Mol. vol. of Cs salt.	Difference between K and Cs salts.
Mg	198·23	9·33	207·56	13·19	220·75	22·52
Mn	—	—	213·66	12·93	226·59	—
Fe	200·09	9·13	209·22	13·42	222·64	22·55
Ni	195·65	9·27	204·92	12·99	217·91	22·26
Co	197·83	9·45	207·28	13·02	220·30	22·47
Cu	198·29	9·26	207·55	12·64	220·19	21·90
Zn	197·68	9·52	207·20	12·68	219·88	22·20
Cd	—	—	217·96	11·28	229·24	—

The most striking facts demonstrated by this table are the following.

An increase in volume occurs when potassium is replaced by rubidium or the latter metal by cæsium. The amount of this increase is approximately constant throughout the series, the replacement of potassium by rubidium being invariably accompanied by an increase of about 9·3 units of volume, whilst the replacement of rubidium by cæsium results in the relatively larger increase of about 13 units.

A similar relation is shown by the atomic volumes of the alkali metals themselves; thus the difference between the volume of rubidium (56·3) and potassium (45·5) is 10·8, whilst the difference between that of cæsium (70·7) and of rubidium is 14·4.

The effect of the replacement of the second metal by another of different atomic weight cannot be expressed according to any such definite law. In the cases of the double salts containing magnesium, zinc, cobalt and copper the effect is almost nil, the molecular volumes of each set of salts containing the same alkali metal being almost the same. The salts containing iron and nickel, however, invariably exhibit molecular volumes about two units higher and lower respectively than the values for the four groups of salts just mentioned.

Topsøe and Christiansen also found that the corresponding potassium selenates containing magnesium, zinc, cobalt, and copper possessed almost identical molecular volumes, whilst the volume of the nickel salt was two units lower. In the cases of the members of the series containing manganese and cadmium, of which the double salts with potassium have not yet been prepared, considerably higher molecular volumes have been found. Indeed, it is not unlikely that this fact has something to do with the impracticability of bringing about the formation of the potassium manganese and potassium cadmium salts. The following statement expresses generally the facts regarding the influence of the second metal on the molecular volume.

The determination of the molecular volume is pre-eminently a function of the alkali metal R, change of the second metal M being usually accompanied by a relatively small change of volume. Thus, although the difference between the atomic weights of magnesium and either zinc or copper is nearly the same as that between either potassium and rubidium or rubidium and caesium, yet the maximum amount of alteration produced by interchange among the first three is less than 1 unit, whilst the amounts produced by interchange of the latter are as stated in the last rule, and when potassium is replaced by caesium the change of volume amounts to no less than 22 units.

The three nickel salts exhibit slightly lower and the iron salts slightly higher molecular volumes than do the salts containing magnesium, zinc, cobalt, and copper, whilst the salts containing manganese and cadmium exhibit abnormally large volumes, an observation which is probably connected with the fact that potassium manganous and potassium cadmium sulphate have not yet been found capable of preparation.

Distance Ratios.—We now approach the question of the relative distances apart, from centre to centre, of the structural units or groups of units of each set of three salts, along the directions of the three morphological axes.

In the accompanying table of comparison, in addition to the distance ratios actually afforded by the formulæ given in the introduction, the numbers for which were given in the description of each salt, there are also appended, on the right hand side, the numbers obtained by dividing out the values for all three salts by the value of ψ for the potassium salt, as was done in the case of the simple normal sulphates. The simpler numbers thus obtained will, of course, express the same relationship as the larger numbers.

It was shown in the communication concerning the morphology of the series that the morphological axial ratios are not capable of exhibiting very clearly the change of geometrical form on replacing one alkali metal by another, because of simultaneous and mutually neutralising changes in the facial angles which determine those

Comparative Table of Distance Ratios.

Salt.	χ	:	ψ	:	w	or	χ	:	ψ	:	w	Diff.
	Diff.		Diff.		Diff.		Diff.		Diff.		Diff.	
KMg	6·0881	:	8·2128	:	4·1006		0·7413	:	1	:	0·4993	
	1060		1577		637		129		192		78	
RbMg	6·1941	:	8·3705	:	4·1643		0·7542	:	1·0192	:	0·5071	
	837		2541		1014		102		309		123	
CsMg	6·2778	:	8·6246	:	4·2657		0·7644	:	1·0501	:	0·5194	
	837		2541		1014		102		309		123	
Total diff. ...	1897		4118		1651		231		501		201	
KZn	6·0618	:	8·1773	:	4·1247		0·7413	:	1	:	0·5044	
	978		1770		616		120		216		75	
RbZn	6·1596	:	8·3543	:	4·1863		0·7533	:	1·0216	:	0·5119	
	1002		2515		821		122		308		101	
CsZn	6·2598	:	8·6058	:	4·2684		0·7655	:	1·0524	:	0·5220	
	1002		2515		821		122		308		101	
Total diff. ...	1980		4285		1437		242		524		176	
KFe	6·0739	:	8·2336	:	4·1332		0·7377	:	1	:	0·5020	
	1093		1481		610		133		180		74	
RbFe	6·1832	:	8·3817	:	4·1942		0·7510	:	1·0180	:	0·5094	
	974		2680		900		118		325		109	
CsFe	6·2806	:	8·6497	:	4·2842		0·7628	:	1·0505	:	0·5203	
	974		2680		900		118		325		109	
Total diff. ...	2067		4161		1510		251		505		183	
KNi	6·0342	:	8·1775	:	4·1051		0·7379	:	1	:	0·5020	
	872		1509		774		107		185		95	
RbNi	6·1214	:	8·3284	:	4·1825		0·7486	:	1·0185	:	0·5115	
	1075		2396		878		131		293		107	
CsNi	6·2289	:	8·5680	:	4·2703		0·7617	:	1·0478	:	0·5222	
	1075		2396		878		131		293		107	
Total diff. ...	1947		3905		1652		238		478		202	
KCo	6·0625	:	8·1881	:	4·1243		0·7404	:	1	:	0·5037	
	1093		1624		601		134		198		73	
RbCo	6·1718	:	8·3505	:	4·1844		0·7538	:	1·0198	:	0·5110	
	877		2596		931		107		318		114	
CsCo	6·2595	:	8·6101	:	4·2775		0·7645	:	1·0516	:	0·5224	
	877		2596		931		107		318		114	
Total diff. ...	1970		4220		1532		241		516		187	
KCu	6·0893	:	8·1300	:	4·1366		0·7490	:	1	:	0·5088	
	1254		1674		361		154		206		45	
RbCu	6·2147	:	8·2974	:	4·1727		0·7644	:	1·0206	:	0·5133	
	1334		2476		536		164		304		65	
CsCu	6·3481	:	8·5450	:	4·2263		0·7808	:	1·0510	:	0·5198	
	1334		2476		536		164		304		65	
Total diff. ...	2588		4150		897		318		510		110	
RbMn	6·2542	:	8·4723	:	4·1938							
	899		2494		912							
CsMn	6·3441	:	8·7217	:	4·2850							
	899		2494		912							
RbCd	6·2828	:	8·5526	:	4·2174							
	810		2142		826							
CsCd	6·3638	:	8·7668	:	4·3010							
	810		2142		826							

ratios. It was observed, however, that in each set of three salts the ratios for the rubidium salt lay between those of the corresponding potassium and caesium salts. When, however, not only the change of geometrical form is considered, but also the change in the volume produced by the introduction of an alkali metal of greater atomic weight in the place of one of lower atomic weight, the differences exhibited are of so pronounced a character, as expressed by the numbers just tabulated, that the following important rules are at once recognised.

The replacement of potassium by rubidium or rubidium by caesium is accompanied by a marked increase in the separation of the centres of the structural units or groups of units along each of the axial directions, corresponding to the increase in volume. The distance ratios of any rubidium salt in the series are consequently intermediate between those of the potassium and caesium salts containing the same second metal. The amounts of increase when rubidium replaces potassium on the one hand, and when caesium replaces rubidium on the other, are approximately equal along the forward inclined axis a, but are considerably greater along both the lateral axis of symmetry b and the vertical axis c when caesium replaces rubidium than when the latter replaces potassium. As regards the relative amounts of increase along the directions of the three axes, the extension is invariably much the greatest along the axis of symmetry b, and least along the vertical axis c. Usually the extension along the latter direction does not fall much behind that along the inclined axis, but in the copper salts the individuality of that metal manifests itself in a considerable augmentation of the extension along the inclined axis at the expense of a corresponding decline of extension in the direction of the vertical axis.

If the relative amounts of extension of the axes are considered in relation to the initial relative lengths of the axes, which are approximately $0.75 : 1 : 0.5$, the amount along the axis of symmetry *b* is still found to predominate, being more in proportion to its length than in the cases of the other two directions.

Results of the Optical Investigation.

Orientation of the Axes of the Optical Ellipsoid.—The most convenient method of comparing the positions of the variable axes of the optical ellipsoid, which lie at right angles to each other in the symmetry plane, will be to tabulate the angles of inclination to the vertical morphological axis *c* of that axis of the ellipsoid which is not far removed from the normal to the basal plane $c\{001\}$. This direction has been shown throughout the whole series to be the maximum axis of the ellipsoid *a*, and it is the second median line in every case but that of caesium nickel sulphate, and the reason for that exception will presently be shown to be of a most interesting cha-

racter. It is not so convenient to consider the inclinations of the first median line to the nearest morphological axis *a*, as it is sometimes on one side of that axis and sometimes on the other.

Table of Inclinations of the Maximum Axis of the Optical Ellipsoid a to the Vertical Axis c.

			Diff.				Diff.
KMg	sulphate .	13° 38'	7° 36'	KZn	sulphate..	10° 18'	6° 25'
RbMg	" ..	21 14	25 29	RbZn	" ..	16 43	13 33
CsMg	" ..	46 43		CsZn	" ..	30 16	
KFe	" ..	11 57	5 12	KNi	" ..	8 42	3 56
RbFe	" ..	17 9	11 8	RbNi	" ..	12 38	11 29
CsFe	" ..	28 17		CsNi	" ..	24 7	
KCo	" ..	10 5	5 56	KCu	" ..	18 33	7 55
RbCo	" ..	16 1	10 7	RbCu	" ..	26 28	16 29
CsCo	" ..	26 8		CsCu	" ..	42 57	
RbMn	" ..	16 57	8 30	RbCd	" ..	15 53	11 3
CsMn	" ..	25 27		CsCd	" ..	26 56	

It will be apparent from this table that

The orientation of the variable axes of the optical ellipsoid which lie in the symmetry plane, for every rubidium salt of the series, is intermediate between that for the potassium and that for the caesium salt containing the same second metal. The maximum axis of the optical ellipsoid a lies nearest to the vertical morphological axis c in the case of the potassium salt of any group, and is furthest removed for the caesium salt. The intermediate position for the rubidium salt is always considerably nearer to the position for the potassium salt than to that for the caesium salt, the replacement of rubidium by caesium being attended by a much greater change of orientation than in mere arithmetical proportion to the change in atomic weight.

This important rule will be clearly demonstrated by the graphical representations of the orientations of the three salts of each group, given in Figs. 23—28 (p. 462).

The rule may be stated in a more interesting manner as follows.

The optical ellipsoid rotates about its intermediate axis b, which is co-incident with the morphological symmetry axis b, when one alkali metal is replaced by another. For any potassium salt of the series it is situated so that its major axis a is but slightly inclined to the vertical morphological axis c, in the obtuse axial angle ac; for the corresponding rubidium salt the inclination is greater in the same direction, and for the caesium salt greater still, the difference of inclination in the latter case being much larger than in the former. The amount of rotation is thus much greater when caesium replaces rubidium than when the latter metal replaces potassium.

It is also observed that

The different chemical nature of copper, as compared with the other

FIG. 23.

Mg.

FIG. 24.

Zn.

FIG. 25.

Fe.

FIG. 26.

Ni.

FIG. 27.

Co.

FIG. 28.

Cu.

second metals contained in the salts of the series, is again manifested by the fact that in the case of potassium copper sulphate the major axis a of the optical ellipsoid is inclined at so exceptionally large an angle to the vertical axis c as to be in front of the normal to the basal plane, whereas it is behind that normal in the other potassium salts of the series.

It will be most convenient to consider the refraction data before the optic axial angle phenomena in order that the latter may be more perfectly understood in the light of the refraction results.

Refractive Indices.—The accompanying tables (pp. 464 and 465) afford a comparison of the refractive indices of each of the eight sets of salts. It will be immediately observed that

The refractive indices of any rubidium salt of the series are, without exception, intermediate between those of the potassium and caesium salts containing the same second metal, and nearer to the values for the potassium salt.

The most accurate idea of the refraction relationships will be afforded by a comparison of the β indices. For the direction of vibration for the β indices is constant throughout the series, being the invariable axis of monoclinic symmetry b , whereas the directions within the crystals corresponding to the α and γ indices vary in accordance with the rule just formulated. As the difference in dispersion in the three salts of any set is so slight, it will be ample to compare the β values for sodium light, and this will be found to be more satisfactory than the consideration of the values for a so-called ray of infinite wave-length, afforded by the first constants of the formulæ expressing the refraction for any wave-length.

Table of β Indices for Sodium Light.

Salt.	β_{Na} .	Difference.	Salt.	β_{Na} .	Difference.
KMg	1.4629	60 } 229	KZn	1.4833	51 } 215
RbMg....	1.4689		RbZn....	1.4884	
CsMg....	1.4858		CsZn	1.5048	
KFe	1.4821	53 } 214	KNi	1.4916	45 } 213
RbFe	1.4874		RbNi	1.4961	
CsFe	1.5035		CsNi	1.5129	
KCo	1.4865	51 } 220	KCu	1.4864	42 } 197
RbCo	1.4916		RbCu	1.4906	
CsCo	1.5085		CsCu	1.5061	
RbMn....	1.4807	159	RbCd....	1.4848	152
CsMn	1.4966		CsCd	1.5000	

The following further conclusions may be drawn concerning the refractive indices after an inspection of the above table.

Comparison Table of Refractive Indices.

Index.	Nature of light.	Salts containing magnesium.			Salts containing zinc.			Salts containing iron.			Salts containing manganese.	
		K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.	Rb.	Cs.
<i>a.</i>	Li	1.4581	1.4646	1.4828	1.4748	1.4307	1.4994	1.4731	1.4789	1.4976	1.4741	1.4918
	O	1.4585	1.4650	1.4832	1.4752	1.4811	1.4998	1.4735	1.4793	1.4980	1.4745	1.4922
	Na	1.4607	1.4672	1.4857	1.4775	1.4833	1.5022	1.4759	1.4815	1.5003	1.4767	1.4946
	Tl	1.4631	1.4695	1.4880	1.4797	1.4857	1.5047	1.4782	1.4839	1.5028	1.4791	1.4872
	F	1.4658	1.4724	1.4912	1.4826	1.4886	1.5079	1.4811	1.4870	1.5061	1.4821	1.5003
<i>β.</i>	G	1.4699	1.4762	1.4956	1.4866	1.4929	1.5125	1.4852	1.4916	1.5105	1.4864	1.5046
	Li	1.4603	1.4664	1.4830	1.4805	1.4856	1.5020	1.4795	1.4847	1.5007	1.4781	1.4936
	C	1.4607	1.4668	1.4834	1.4809	1.4860	1.5024	1.4799	1.4851	1.5011	1.4785	1.4940
	Na	1.4629	1.4689	1.4858	1.4833	1.4884	1.5048	1.4821	1.4874	1.5035	1.4807	1.4966
	Tl	1.4652	1.4713	1.4881	1.4857	1.4908	1.5073	1.4847	1.4898	1.5061	1.4831	1.4991
<i>γ.</i>	F	1.4678	1.4743	1.4912	1.4889	1.4938	1.5104	1.4877	1.4929	1.5093	1.4860	1.5022
	G	1.4720	1.4782	1.4957	1.4929	1.4980	1.5151	1.4920	1.4973	1.5137	1.4907	1.5066
	Li	1.4727	1.4755	1.4888	1.4938	1.4947	1.5064	1.4941	1.4949	1.5065	1.4880	1.4995
	C	1.4731	1.4759	1.4892	1.4942	1.4951	1.5068	1.4945	1.4953	1.5069	1.4884	1.4999
	Na	1.4755	1.4779	1.4916	1.4969	1.4975	1.5093	1.4969	1.4977	1.5094	1.4907	1.5025
<i>δ.</i>	Tl	1.4778	1.4805	1.4940	1.4994	1.5001	1.5119	1.4995	1.5003	1.5121	1.4933	1.5051
	F	1.4810	1.4835	1.4970	1.5027	1.5033	1.5152	1.5028	1.5034	1.5153	1.4965	1.5083
	G	1.4853	1.4876	1.5015	1.5067	1.5078	1.5199	1.5071	1.5080	1.5198	1.5015	1.5129

Comparison Table of Refractive Indices.

Index.	Nature of light.	Salts containing nickel.			Salts containing cobalt.			Salts containing copper.			Salts containing cadmium.	
		K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.	Rb.	Cs.
α .	Li	1.4809	1.4868	1.5061	1.4780	1.4833	1.5028	1.4807	1.4858	1.5017	1.4773	1.4947
	C	1.4813	1.4872	1.5065	1.4784	1.4837	1.5032	1.4811	1.4862	1.5021	1.4777	1.4951
	Na	1.4836	1.4895	1.5087	1.4807	1.4859	1.5057	1.4836	1.4886	1.5048	1.4798	1.4975
	Tl	1.4860	1.4920	1.5112	1.4830	1.4882	1.5079	1.4861	1.4912	1.5074	1.4823	1.5000
	F	1.4889	1.4949	1.5146	1.4861	1.4910	1.5112	1.4893	1.4943	1.5108	1.4856	1.5033
β .	G	1.4933	1.4996	1.5192	1.4904	1.4954	1.5159	1.4944	1.4991	1.5159	1.4906	1.5081
	Li	1.4889	1.4933	1.5100	1.4838	1.4889	1.5057	1.4834	1.4878	1.5032	1.4820	1.4972
	C	1.4893	1.4937	1.5104	1.4842	1.4893	1.5061	1.4838	1.4882	1.5036	1.4824	1.4976
	Na	1.4916	1.4961	1.5129	1.4865	1.4916	1.5085	1.4864	1.4906	1.5061	1.4848	1.5000
	Tl	1.4941	1.4987	1.5154	1.4889	1.4940	1.5110	1.4893	1.4933	1.5089	1.4872	1.5026
γ .	F	1.4972	1.5017	1.5187	1.4919	1.4968	1.5142	1.4922	1.4966	1.5123	1.4905	1.5058
	G	1.5015	1.5062	1.5235	1.4961	1.5011	1.5188	1.4975	1.5013	1.5174	1.4955	1.5106
	Li	1.5022	1.5023	1.5133	1.4973	1.4985	1.5102	1.4990	1.5007	1.5122	1.4919	1.5034
	C	1.5026	1.5027	1.5137	1.4977	1.4989	1.5106	1.4994	1.5011	1.5126	1.4923	1.5038
	Na	1.5051	1.5052	1.5162	1.5004	1.5014	1.5132	1.5020	1.5036	1.5153	1.4948	1.5062
δ .	Tl	1.5077	1.5078	1.5189	1.5028	1.5038	1.5156	1.5047	1.5064	1.5180	1.4972	1.5088
	F	1.5109	1.5110	1.5221	1.5059	1.5068	1.5187	1.5081	1.5098	1.5216	1.5007	1.5023
	G	1.5153	1.5156	1.5266	1.5105	1.5114	1.5237	1.5134	1.5148	1.5266	1.5061	1.5172

An increase in the atomic weight of the contained alkali metal is accompanied by an increase in the refractive power of the crystal, and the increase in refraction becomes relatively greater, as the atomic weight becomes higher, than in mere numerical proportion to the increase of atomic weight. When the β indices are considered, whose corresponding direction of vibration is constant throughout the series, being the invariable symmetry axis b , the amounts of increase due respectively to the replacement of potassium by rubidium, and of rubidium by caesium, vary slightly in the different sets, according to the nature of the second metal present, from 1 : 3 to 1 : 4.

The difference between the indices for the potassium and caesium salts of any group is not so large as was observed between those of the simple sulphates of potassium and caesium. In the latter case the difference of the β indices for sodium light amounted to 0.0697 whereas the largest difference in the series under consideration is 0.0229.

No relation can be traced between the atomic weight of the second metal present and the refractive indices. For instance, although there is a rise in refraction when magnesium is replaced by zinc, yet there is a fall again when the latter is replaced by cadmium.

Ratio of Axes of Optical Velocity Ellipsoid.—The difference of refraction along the three axes of the optical ellipsoid in any one salt, the modification of that difference on replacement of the alkali metal by another, and the actual change of refraction along any one direction brought about by the same cause, are perhaps most clearly rendered evident by a consideration of the axial ratios of the optical ellipsoid, which are relative measures of the velocity of light vibration along the three rectangular axes a , b , c , of the ellipsoid. The ratios given in the description of each salt are calculated with reference to the value along the b axis as unity. Such ratios, therefore, only express the relative facility for light vibration along the three axial directions of the particular salt in question. It was shown in the memoir concerning the simple normal sulphates (*loc. cit.*, 699) that by taking the value along the intermediate axis of the ellipsoid of the potassium salt as unity, and expressing all the other values for the set of three salts proportionately, the actual changes of velocity along all three axes on passing from a potassium to a rubidium salt, and from a rubidium to a caesium salt, are exhibited. In the ratios thus obtained for any set of three salts, those for the potassium salt are, of course, identical with those given in the descriptions, and those for the rubidium and caesium salts are obtained in each case by dividing the refractive index for sodium light, corresponding to the axis in question in that particular salt, into the β_{Na} index of the potassium salt.

The two series of ratios are given side by side in the accompanying table.

Ratios of the Axes of the Optical Velocity Ellipsoids of the Series.

Salt.	a.	b.	c.	or	a.	b.	c.
KMg.....	1·0015 3	: 1 :	0·9915 24		1·0015 : 44	1 : 41	0·9915 17
RbMg....	1·0012 11	: 1 :	0·9939 22		0·9971 : 124	0·9959 : 113	0·9898 90
CsMg....	1·0001	: 1 :	0·9961		0·9847	0·9846	0·9803
KZn.....	1·0039 5	: 1 :	0·9903 30		1·0039 : 39	1 : 34	0·9909 4
RbZn....	1·0034 17	: 1 :	0·9939 31		1·0000 : 126	0·9966 : 109	0·9905 77
CsZn....	1·0017	: 1 :	0·9970		0·9874	0·9857	0·9828
KFe.....	1·0042 2	: 1 :	0·9901 30		1·0042 : 38	1 : 36	0·9901 5
RbFe....	1·0040 19	: 1 :	0·9931 30		1·0004 : 125	0·9964 : 106	0·9896 77
CsFe....	1·0021	: 1 :	0·9961		0·9879	0·9858	0·9819
KNi.....	1·0054 10	: 1 :	0·9910 30		1·0054 : 49	1 : 30	0·9910 1
RbNi....	1·0044 16	: 1 :	0·9940 38		1·0014 : 127	0·9970 : 111	0·9909 71
CsNi....	1·0028	: 1 :	0·9978		0·9887	0·9859	0·9838
KCo.....	1·0039 1	: 1 :	0·9907 28		1·0039 : 35	1 : 34	0·9907 6
RbCo....	1·0038 19	: 1 :	0·9935 34		1·0004 : 132	0·9966 : 112	0·9901 78
CsCo....	1·0019	: 1 :	0·9969		0·9872	0·9854	0·9823
KCu.....	1·0019 6	: 1 :	0·9896 18		1·0019 : 34	1 : 28	0·9896 10
RbCu....	1·0013 4	: 1 :	0·9914 25		0·9985 : 107	0·9972 : 103	0·9886 77
CsCu....	1·0009	: 1 :	0·9939		0·9878	0·9869	0·9809
RbMn...	1·0027 14	: 1 :	0·9933 28				
CsMn....	1·0013	: 1 :	0·9961				
RbCd....	1·0034 17	: 1 :	0·9933 26				
CsCd....	1·0017	: 1 :	0·9959				

In discussing the results afforded by the comparison of these ratios, it will be convenient to consider first the total change represented by the second series of ratios, and subsequently to revert to the effect of this change in modifying the mutual relations of the three axes of the optical ellipsoid of each salt, as exhibited by the first series of ratios.

FIG. 29. Mg.

FIG. 30. Zn.

FIG. 31. Fe.

FIG. 33. Co.

FIG. 34. Cu.

The second series reveal the following facts.

The relative velocity of light vibration along each of the three axes of the optical ellipsoid of the crystals of every rubidium salt of the series is intermediate between the velocities along the corresponding axial directions in the crystals of the potassium and caesium salts containing the same second metal. Increase in the atomic weight of the alkali metal is accompanied by a decrease in the velocity of light vibrations, or, in other words, by an increase of resistance to the vibration of light, along each of the axes of the ellipsoid, and this alteration is relatively much greater when caesium replaces rubidium than when potassium is replaced by rubidium. In the case of the more strictly comparable direction of the invariable axis b , which is identical with the axis of morphological symmetry b , the relative amounts are as about 3 : 1. The amounts of change along the axis a are invariably slightly larger than those along the axis b , and in the same proportion; those along the axis c are always much less, and the difference between the effects respectively of replacing potassium by rubidium, and the latter by caesium, is still more pronounced.

It must be borne in mind that in the case of these monoclinic double sulphates the optical ellipsoid is not fixed as regards its axial directions, as in the case of the rhombic simple sulphates, but is rotatable about its intermediate axis b , coincident with the morphological symmetry axis b , as stated in one of the preceding rules. Hence the ellipsoid for a caesium salt does not contain entirely within it the ellipsoid for the corresponding rubidium salt, and the latter in turn the ellipsoid for the analogous potassium salt, as was the case with the simple sulphates.

The relations of the various salts, as regards the vibration of light along the directions of the principal axes of the optical ellipsoid, are best exhibited graphically by means of curves. These are given in Figs. 29 to 34 for the six sets of salts whose three members are all capable of preparation. The atomic weight of the contained alkali metals is taken for ordinates and relative velocity of light vibration for abscissæ.

What is most apparent is the striking similarity of the curves, indicating unmistakably how little is the influence of the second metal in modifying the optical properties. The second point which attracts attention is the fact that in every case the curve takes a more or less steep course between the potassium and the rubidium salt, indicating the relatively small amount of change in the velocity on passing from one to the other, then rapidly curves to the left with diminishing steepness as it passes to the caesium salt, indicating, in accordance with the rule above given, a much greater change when rubidium is replaced by caesium. A third property common to the series is the

fact that the three curves of each set converge towards each other on passing from the potassium salt through the rubidium salt to the cæsium salt, indicating a progressive diminution in the amount of double refraction. Before enlarging on this last property, which leads to consequences of great interest, and only observing in passing how it ends in the case of the magnesium salts in the two nearest curves, those for the *a* and *b* axes, all but meeting, it will be advisable to discuss the first series of ratios.

The first series represent the relative velocities of light vibration along the three principal axes of the optical ellipsoid of each salt, compared with the value along the intermediate axis (the invariable axis of morphological symmetry) taken as unity. They further represent the net effect on the mutual relations of the axes of the progressive changes indicated by the second series. Now these mutual relations of the maximum, intermediate, and minimum velocities exclusively dominate the amount and the sign of the double refraction, and the optic axial angle, and it is for this reason that the discussion of the results of the observations of the latter have been deferred to a later stage, when these relations will have been made clear. In order that relations of such importance should be exhibited in the most striking manner, which is necessarily not achieved by the curves of Figs. 29 to 34 intended to demonstrate most clearly the properties referred to in preceding paragraphs, another series of curves are given in Figs. 35 to 40. In these curves the velocity along the intermediate axis is naturally always a vertical straight line representing unity. In every set the convergence of the curves for the two other axes is apparent. The curves corresponding to the axis *c* are in every case very nearly straight lines; those for the *a* axis exhibit more curvature, the amount of which varies somewhat according to the second metal present, but still the position of the rubidium salt is invariably an intermediate one. These observations lead to the following rule.

An increase in the atomic weight of the alkali metal is accompanied by convergence of the values of the velocity along the three axial directions abc towards unity, and consequently by a diminution in the amount of the double refraction. This property is already a very feeble one in the potassium salts, and is extremely weak in the cæsium salts, the rubidium salts occupying an intermediate position in this respect. The amounts of change in the velocity along the directions of the axes a and b, when potassium is replaced by rubidium or the latter metal by cæsium, being but slightly different, the relations of these two velocities are only slightly altered; the slower rate of change along the c axis, however, has the effect of introducing more considerable changes in the relations of the velocity along this direction to the velocities along the other two directions. Both the former slight relative alteration and the latter more consider-

FIG. 35. Mg.

FIG. 36. Zn.

FIG. 37. Fe.

FIG. 38. Ni.

FIG. 39. Co.

FIG. 40. Cu.

able one are progressive in character, the rubidium salts invariably occupying intermediate positions.

The bearing of this important rule on the individual refraction phenomena of the various salts of the series must next be discussed. It has been shown that the whole of the salts, with one exception, namely, caesium nickel sulphate, exhibit positive double refraction, that is, their β indices of refraction are nearer to the α values than to the γ values, and consequently the intermediate velocity ratios for the b axis approach nearer to the maximum a values than to the minimum c values. This is evident from the table of ratios, and also from the curves of both series, the a and b curves lying closer together than the b and c curves.

In the exceptional case of caesium nickel sulphate, the β and γ refractive indices and the b and c ratios and curves approximate more closely to each other than do the α and β indices and the a and b ratios and curves, and therefore the sign of double refraction is negative. This apparent anomaly, instead of being an inexplicable exception, is a direct consequence of the operation of the rule last laid down, and is one of the most remarkable proofs of the validity of the rule which could be adduced. On inspection of the curves of either series, Figs. 32 or 38, for the salts containing nickel, it will be observed that for the potassium salt the a and b values, forming the starting points of their respective curves, are nearer together than the b and c values, and that at the rubidium salt the progress of the convergence of the curves still leaves them bearing similar relations, corresponding to a positive sign of double refraction. Now the potassium nickel salt presents greater divergence of the a and b values than any other salt of the series, as will be perhaps best seen by referring to the first series in the table of ratios, a being here at its maximum with a value of 1.0054. Remembering that the relative change is more considerable along the c axis, represented by more rapid convergence of its curve in Fig. 38, it is not surprising that, as the a curve starts in the potassium salt further from b than usual, it should, owing to its lesser rate of convergence, arrive at the caesium salt with a value further removed from b than does the rapidly converging c curve. This is indeed the case, and the reversal of the double refraction is due entirely to the operation of the rule. It may therefore be stated that

The preceding rule receives somewhat remarkable confirmation in the case of caesium nickel sulphate. With the exception of this salt, the whole of the salts of the series exhibit positive double refraction. Owing to the fact that in the potassium nickel salt the a and b values are at their maximum amount of removal for the series, although still nearest together of all the three values in accordance with positive double refraction.

tion, the precise operation of the rule causes it to happen that in the caesium salt the ϵ value, in consequence of its more rapid rate of convergence, has approached nearest the intermediate value $\mathfrak{b}$, and so this salt ought to exhibit negative double refraction, a presumption in complete accord with actual observation.

Just as the salts containing nickel exhibit a remarkable effect of the operation of the rule on account of the initial potassium salt exhibiting an unusually large divergence of the velocities along $\mathfrak{a}$ and $\mathfrak{b}$, so the magnesium salts present another extraordinary effect of the rule, on account of the potassium salt presenting a particularly low difference of the values along the same two directions. For the effect of still further diminution in this difference, in accordance with the rule of convergence towards unity, results in the values of the velocities along the $\mathfrak{a}$ and $\mathfrak{b}$ axes actually arriving at unity in the caesium salt, as is graphically exhibited by the curves, Figs. 29 and 35.

It has been shown in the description of the caesium magnesium salt that the α and β refractive indices afforded by light vibrating parallel to $\mathfrak{a}$ and $\mathfrak{b}$ are almost identical; they differ by about two units in the fourth place of decimals for the red end of the spectrum, by one unit for sodium light, corresponding to the difference of the same amount shown in the ratios, which were calculated for sodium light, but are identical when expressed to four places for F light, and are absolutely identical for the wave-length 450 in the blue. Beyond this they again diverge, so that for G light they differ by one unit in the fourth decimal place, but the direction in the crystal which was formerly the maximum $\mathfrak{a}$ axis is now the intermediate $\mathfrak{b}$ axis, and yields the intermediate refractive index β . Hence the crystals of caesium magnesium sulphate yield only two refractive indices in blue light, simulating uniaxial optical properties; and for other colours it requires a spectrometer eye-piece of high power to detect any separation of the refracted images of the slit furnished by prisms affording vibrations parallel to the $\mathfrak{a}$ and $\mathfrak{b}$ axes. When such an eye-piece is employed, the images are observed to approach each other as the illuminating light changes from red towards the other end of the spectrum; they actually coincide when blue is reached, then pass each other, and again separate slightly on opposite sides of each other as their colour changes to violet. Moreover, the operation of the rule of convergence causes the third value ϵ to be comparatively near to the other values, thus rendering the double refraction extremely feeble. These remarkable effects of the rule in the case of caesium magnesium sulphate will be further followed when the optic axial angle phenomena, which are bound to be correspondingly extraordinary, are discussed. The facts as regards the refraction phenomena may be summarised as follows.

Another extraordinary effect of the operation of the rule is afforded by the refraction phenomena of *cæsium magnesium sulphate*. The differences between the *a* and *b* values is at its minimum for the potassium salts of the series in the case of *potassium magnesium sulphate*, and the rule of convergence towards unity results in the ratio *a* : *b* actually arriving at unity for the *cæsium* salt. Owing to the existence of the usual slight difference in dispersion along the axial directions, the coincidence is only absolute for wave-length 450 in the blue. The difference of the two velocities, or of the corresponding refractive indices, is about 0·0001 for Na and 0·0002 for Li light, the order being as usual throughout the series; for G light the values also differ by about 0·0001, but the order is reversed. These amounts are so slight that the pair of images of the spectrometer slit refracted by a 60° prism formed by faces having the directions theoretically required to yield these indices, are apparently coincident unless an eye-piece of very high power is employed. Hence for wave-length 450 *cæsium magnesium sulphate* exhibits apparent uniaxial optical properties.

It is further to be remarked that the influence of the second metal is only appreciably perceptible in the ratio *a* : *b*, best seen in the first series of ratios, the ratio *c* : *b* remaining substantially constant throughout the whole series of salts. No definite relation is remarked between the values of *a* : *b* in the groups of salts containing the same alkali metal, just as was remarked concerning the refractive indices themselves. As has just been shown, however, in the cases of the salts containing magnesium and nickel, although the specific effects on the ratio *a* : *b*, due to the second metal, are not connected by any rule involving only the atomic weight of that metal, those specific modifications are of very considerable importance in determining the optical phenomena presented by the crystals.

Molecular Optical Constants.—In the accompanying tables (pp. 478, *et seq.*), the optical constants, calculated by use of the formulæ of Lorenz and of Gladstone and Dale, are compared. The following facts are indicated by them.

The whole of the specific and molecular optical constants for any rubidium salt of the series are intermediate between those of the potassium and *cæsium* salts containing the same second metal.

The replacement of potassium by rubidium, and of the latter metal by *cæsium*, is accompanied by a diminution of specific refraction and specific dispersion, the amount of which is about twice as great when the former chemical change occurs as when the latter substitution is effected. The maximum amount of diminution in specific refraction occurs in the direction of the axis *c* of the optical ellipsoid; the somewhat smaller amounts of diminution along the directions *a* and *b* are almost identical, the

amount along the axis *b* being invariably, however, slightly the larger of the two.

The same chemical changes, involving an increase in the atomic weight of the alkali metal, are attended by an increase of molecular refraction and dispersion, the amount of which is greatest when cæsium replaces rubidium, the proportions for the two changes in molecular refraction being as 3 : 5. The amounts of increase of molecular refraction are nearly the same in the directions of the axes *a* and *b*, that along the former direction is slightly the greater, and is the maximum increase; the minimum increase occurs in the direction of the axis *c*. This rule is equally valid whatever may be the wave-length for which the comparison is made, and whether the constants are calculated by the formulæ of Lorenz or of Gladstone and Dale; moreover, as refraction and density are similarly affected by change of temperature within the limits between which the salts are capable of existing, it is independent of temperature.

With regard to the latter statement, it may be remarked that the main object of the determinations of refractive index at higher temperatures, recorded in the descriptions of the various salts, was to prove this point. In every case a rise of temperature was found to be accompanied by a lowering of the refractive index, the amount for 50° rise of temperature varying from 0·0012 to 0·0020.

The relations just established concerning the molecular refraction are in complete accordance with those set forth in a previous rule regarding the relative velocity of light vibration, as deduced from the second series of velocity ratios.

It is to be remarked that similar relations are exhibited by the refraction equivalents of the alkali metals themselves. The difference between the refraction equivalents of rubidium (10·5) and potassium (7·8) is 2·7, whilst that between the equivalents of cæsium (15·3) and rubidium is 4·8. The values taken for the equivalents are those deduced by the author from the comparison of the results of the two investigations concerning the simple and double sulphates, as described in the next communication.

The influence of the second metal in bringing about changes in the molecular optical constants is but slight compared with that due to the alkali metal. It is, however, perceptible in the case of the three more intimately related metals magnesium, zinc, and cadmium. If the rubidium or the cæsium salts containing respectively these three second metals are compared, it will be observed that: *The specific refraction diminishes when magnesium is replaced by zinc, and again when zinc is replaced by cadmium, whilst the molecular refraction increases under the same circumstances. The amount of the change is greatest, in the case of the specific refraction, when magnesium is replaced by zinc, and, in the case of the*

Table of Specific Refractions and Dispersions (Lorenz).

Salt.	Specific refraction, $\frac{n^2 - 1}{(n^2 + 2)d} = n$.						Specific dispersion, $n_D - n_C$.		
	For ray C (H α).			For ray G (H γ).			a.	b.	c.
	a.	b.	c.	a.	b.	c.			
KMg	0·1348 187	0·1353 188	0·1385 200	0·1377 192	0·1382 192	0·1415 205	0·0029	0·0029	0·0030
RbMg	·1161 90	·1165 94	·1185 103	·1185 91	·1190 96	·1210 105	·0024	·0025	·0025
CsMg	·1071	·1071	·1082	·1094	·1094	·1105	·0023	·0023	·0023
KZn	·1258 156	·1271 159	·1301 171	·1283 158	·1298 163	·1328 174	·0025	·0027	·0027
RbZn	·1102 76	·1112 81	·1130 92	·1125 77	·1135 82	·1154 93	·0023	·0023	·0024
CsZn	·1026	·1031	·1038	·1048	·1053	·1061	·0022	·0022	·0023
KFe	·1296 168	·1310 170	·1344 184	·1323 170	·1339 175	·1373 187	·0027	·0029	·0029
RbFe	·1128 77	·1140 83	·1160 93	·1153 80	·1164 85	·1186 96	·0025	·0024	·0026
CsFe	·1051	·1057	·1067	·1073	·1079	·1090	·0022	·0022	·0023
KNi	·1276 161	·1294 167	·1324 179	·1303 164	·1321 170	·1352 183	·0027	·0027	·0028
RbNi	·1115 76	·1127 82	·1145 94	·1139 78	·1151 83	·1169 96	·0024	·0024	·0024
CsNi	·1039	·1045	·1051	·1061	·1068	·1073	·0022	·0023	·0022
KCo	·1282 163	·1295 165	·1325 176	·1309 167	·1322 169	·1354 181	·0027	·0027	·0029
RbCo	·1119 76	·1130 82	·1149 93	·1142 77	·1153 83	·1173 94	·0023	·0023	·0024
CsCo	·1043	·1048	·1056	·1065	·1070	·1079	·0022	·0022	·0023
KCu	·1281 163	·1287 165	·1322 175	·1311 168	·1318 170	·1353 179	·0030	·0031	·0031
RbCu	·1118 83	·1122 85	·1147 94	·1143 84	·1148 87	·1174 97	·0025	·0026	·0027
CsCu	·1035	·1037	·1053	·1059	·1061	·1077	·0024	·0024	·0024
RbMn	·1145 84	·1153 89	·1173 98	·1169 86	·1178 91	·1200 102	·0024	·0025	·0027
CsMn	·1061	·1064	·1075	·1083	·1087	·1098	·0022	·0023	·0023
RbCd	·1060 73	·1068 76	·1087 85	·1084 74	·1093 79	·1113 88	·0024	·0025	·0026
CsCd	·0987	·0992	·1002	·1010	·1014	·1025	·0023	·0022	·0023

Table of Molecular Refractions and Dispersions (Lorenz).

Salt.	Molecular refraction, $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{M}{d} = m.$						Molecular dispersion. $m_D - m_C.$		
	For ray C (Ha).			For ray G (Hγ).			a.	b.	c.
	a.	b.	c.	a.	b.	c.			
KMg	54·18 3·24	54·40 3·21	55·66 2·91	55·34 3·26	55·55 3·27	56·88 2·92	1·16	1·15	1·22
RbMg	57·42 5·69	57·61 5·54	58·57 5·21	58·60 5·89	58·82 5·68	59·80 5·34	1·18	1·21	1·23
CsMg	63·11	63·15	63·78	64·49	64·50	65·14	1·38	1·35	1·36
KZn	55·72 3·30	56·29 3·24	57·61 2·87	56·86 3·39	57·48 3·30	58·84 2·95	1·14	1·19	1·23
RbZn	59·02 5·67	59·53 5·45	60·48 4·98	60·25 5·83	60·78 5·58	61·79 5·09	1·23	1·25	1·31
CsZn	64·69	64·98	65·46	66·08	66·36	66·88	1·39	1·38	1·42
KFe	56·22 3·18	56·87 3·15	58·34 2·75	57·41 3·29	58·09 3·21	59·60 2·81	1·19	1·22	1·26
RbFe	59·40 5·91	60·02 5·63	61·09 5·20	60·70 5·99	61·30 5·74	62·41 5·30	1·30	1·28	1·32
CsFe	65·31	65·65	66·29	66·69	67·04	67·71	1·38	1·39	1·42
KNi	55·75 3·25	56·54 3·13	57·84 2·75	56·93 3·34	57·73 3·21	59·06 2·84	1·18	1·19	1·22
RbNi	59·00 5·84	59·67 5·59	60·59 5·02	60·27 5·93	60·94 5·72	61·90 5·09	1·27	1·27	1·31
CsNi	64·84	65·26	65·61	66·20	66·66	66·99	1·36	1·40	1·38
KCo	56·08 3·23	56·66 3·24	58·00 2·89	57·28 3·25	57·84 3·28	59·26 2·92	1·20	1·18	1·26
RbCo	59·31 5·88	59·90 5·61	60·89 5·11	60·53 6·04	61·12 5·77	62·18 5·23	1·22	1·22	1·29
CsCo	65·19	65·51	66·00	66·57	66·89	67·41	1·38	1·38	1·41
KCu	56·48 3·17	56·75 3·11	58·30 2·90	57·81 3·18	58·11 3·11	59·68 2·93	1·33	1·36	1·38
RbCu	59·65 5·89	59·86 5·34	61·20 4·98	60·99 5·55	61·22 5·48	62·61 5·08	1·34	1·36	1·41
CsCu	65·04	65·20	66·18	66·54	66·70	67·69	1·50	1·50	1·51
RbMn	60·14 5·67	60·58 5·43	61·65 5·03	61·43 5·78	61·89 5·54	63·04 5·10	1·29	1·31	1·39
CsMn	65·81	66·01	66·68	67·21	67·43	68·14	1·40	1·42	1·46
RbCd	61·71 5·20	62·23 4·96	63·31 4·60	63·13 5·26	63·66 5·01	64·81 4·61	1·42	1·43	1·50
CsCd	66·91	67·19	67·91	68·39	68·67	69·42	1·48	1·48	1·51

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molecular refraction, when zinc is replaced by cadmium, just as was observed on replacement of the alkali metal.

*Table of Molecular Refractions (Gladstone and Dale), $\frac{n-1}{d}$ M,
for Ray C.*

Salt.	a.	Diff.	b.	Diff.	c.	Diff.
KMg....	90·97	5·63	91·40	5·57	93·86	5·00
RbMg...	96·60	10·16	96·97	9·83	98·86	9·22
CsMg...	106·76		106·80		108·08	
KZn....	94·02	5·75	95·14	5·64	97·77	4·90
RbZn...	99·77	10·21	100·78	9·78	102·67	8·85
CsZn....	109·98		110·56		111·52	
KFe....	94·82	5·54	96·10	5·48	99·03	4·68
RbFe...	100·36	10·65	101·58	10·12	103·71	9·28
CsFe...	111·01		111·70		112·99	
KNi....	94·25	5·67	95·81	5·44	98·42	4·67
RbNi...	99·92	10·54	101·25	10·06	103·09	8·94
CsNi....	110·46		111·31		112·03	
KCo....	94·72	5·62	95·87	5·63	98·54	4·95
RbCo...	100·34	10·61	101·50	10·08	103·49	9·08
CsCo...	110·95		111·58		112·57	
KCu....	95·48	5·51	96·01	5·40	99·11	4·98
RbCu...	100·99	9·65	101·41	9·56	104·09	8·87
CsCu...	110·64		110·97		112·96	
RbMn...	101·47	10·15	102·32	9·71	104·44	8·92
CsMn...	111·62		112·03		113·36	
RbCd...	104·21	9·38	105·23	8·93	107·39	8·19
CsCd...	113·59		114·16		115·58	

Optic Axial Angle.—Having ascertained the facts concerning the refraction of the various salts of the series, and the relations of the velocity of light vibration along the principal axes of the optical ellipsoid of each salt, the study of the optic axial angle phenomena may now be approached with greatest advantage. In the accompanying table are set forth the values of the true optic axial angle, $2Va$, within the crystal, for the various salts of the series, arranged so that the potassium, rubidium, and caesium salts containing the same second metal may be readily compared. The following facts will be apparent.

Excluding the three salts containing magnesium, the caesium salt of which is not strictly comparable with the other two salts owing to the extraordinary conditions, previously described, following the operation of the rule regarding the velocity ratios, the optic axial angle of any rubidium salt of the series is intermediate between the optic axial angles of the potassium and caesium salts containing the same second metal, whatever wave-length of light is chosen for the comparison.

This rule is connected with the second rule established in connection with the velocities along the axial directions of the optical ellipsoid, the

progressive nature of the change there referred to leading to a progressive change in the optic axial angle, which varies in character in the different groups according to the relative rates of convergence of the extreme values of the velocity towards the intermediate value on passing from the potassium salt to the cæsium salt of any group.

This variation in the character of the progressive change in the optic axial angle is due to the influence of the second metal.

Comparison Table of true Optic Axial Angles, 2Va.

Nature of light.	CsMg salt (not compar- able, special circum- stances).			KMg salt.	RbMg salt.	KZn salt.	RbZn salt.	CsZn salt.
Li.	48° 0'	49° 6'	18° 10'			68° 17'	73° 42'	74° 30'
C.	47 59	49 2	18 0			68 16	73 40	74 27
Na.	47 54	48 46	16 25			68 14	73 33	74 11
Tl.	47 48	48 29	14 20			68 12	73 27	73 52
F.	47 40	48 10	11 15			68 9	73 18	73 31
	KFe salt.	RbFe salt.	CsFe salt.			KNi salt.	RbNi salt.	CsNi salt.
Li.	67° 1'	73° 24'	75° 2'			75° 21'	82° 5'	92° 45'
C.	67 2	73 23	75 0			75 19	82 4	92 43
Na.	67 7	73 21	74 51			75 16	82 0	92 39
Tl.	67 12	73 18	74 42			75 13	81 56	92 31
F.	67 19	73 13	74 31			75 9	81 48	92 20
	KCo salt.	RbCo salt.	CsCo salt.			KCu salt.	RbCu salt.	CsCu salt.
Li.	68° 38'	75° 15'	81° 42'			46° 1'	44° 26'	43° 6'
C.	68 39	75 14	81 40			46 6	44 29	43 9
Na.	68 41	75 11	81 34			46 32	44 42	43 24
Tl.	68 44	75 8	81 29			47 0	44 57	43 40
F.	68 48	75 3	81 22			47 33	45 15	44 3
		RbMn salt.	CsMn salt.				RbCd salt.	CsCd salt.
Li.	—	67° 10'	60° 10'			—	72° 20'	68° 4'
C.	—	67 8	60 7			—	72 21	68 2
Na.	—	67 5	59 57			—	72 26	67 53
Tl.	—	67 1	59 46			—	72 31	67 44
F.	—	66 55	59 28			—	72 37	67 28

It will be interesting to follow the variation just referred to with the aid of the curves of Figs. 36 to 40. In the case of the zinc salts (Fig. 36), the more rapid convergence of the ϵ curve compared with that of the a curve causes a progressive diminution of the relative separation of b and ϵ compared with a and b , and therefore an increase in the optic axial angle on passing from the potassium to the rubidium salt; the fact that the a curve converges more rapidly as the cæsium salt is approached tends to diminish the effect, and so the increase of angle exhibited by the cæsium salt is not so large as

was the former increase. The same applies to the iron salts (Fig. 37). In the case of the cobalt group (Fig. 39), the difference between the optic axial angles of the potassium and rubidium salts is almost the same as that between the rubidium and caesium salts, as the ϵ curve also converges more rapidly as the caesium salt is approached. The different chemical nature of copper is again illustrated by the much smaller optic axial angle, due to the fact that the ϵ velocity is at its maximum removal from the b value in each of the copper salts, as shown in Fig. 40, while the a value is exceptionally close to the intermediate value; moreover, the a curve is concave, whereas in all the other groups it is convex.

The nickel group is specially interesting, as exhibiting, in the case of the caesium salt, a reversal of the median lines. It will be observed in the table that the optic axial angle given is over 90° . This of course is the obtuse angle, but it is this angle which is comparable with the others because its median line (the second) is the analogous direction to the directions of the first median lines of the other two salts. The relations of the velocities along the axial directions of the optical ellipsoid which determine this have already been fully stated with reference to Fig. 38, and it only remains to point out that the optic axial angle phenomena are precisely what would be expected from those relations. The following statement concerning it should be included in the general conclusions to be drawn from this investigation.

The first median line is the minimum axis of the optical ellipsoid ϵ in every salt of the series except caesium nickel sulphate, and the sign of double refraction is consequently positive. In the case of caesium nickel sulphate a reversal of the median lines occurs, the first median line being the maximum axis a and the sign of double refraction negative. This is a direct consequence of the operation of the second rule concerning the velocity ratios as explained in the general statement concerning this salt which follows the rule. In accordance with these relationships, the optic axial angle is already unusually large in potassium nickel sulphate, and the operation of the rule of progressive change in the optic axial angle renders it so large when the caesium salt is reached that it exceeds 90° , and the median line which had hitherto been the first becomes the second.

It now remains to discuss the optic axial angle phenomena of the magnesium group of salts. Caesium magnesium sulphate exhibits one of the most interesting cases of crossed axial plane dispersion yet known in connection with monoclinic symmetry. Its extraordinary refraction phenomena have already been fully discussed, and have been shown to be another direct consequence of the operation of the rule concerning the velocity of light vibration. In view of the pre-

vious experience with normal rubidium sulphate, where extremely feeble double refraction was combined with the very near approach to coincidence of two of the velocity values, it appeared most probable that a similar state of matters in the case of cæsium magnesium sulphate would be accompanied, as in the former case, by great sensitiveness of the optic axial angle to change of wave-length of the illuminating light and to change of temperature. The results given in the detailed description of this salt show in how interesting a manner this supposition proves to be correct. The main facts may be briefly summarised in the following terms.

In the case of the three salts containing magnesium the progressive alteration of optic axial angle proceeds according to the rule from the potassium to the rubidium salt. Owing, however, to the remarkable relationships of the velocities along the three axial directions when the cæsium salt is reached, brought about by the operation of the rule previously enunciated and described in the general statement concerning this salt following the enunciation, quite extraordinary optic axial angle phenomena are exhibited by crystals of cæsium magnesium sulphate, including very large dispersion of the optic axes for different wave-lengths of light, a crossing of the optic axial planes, and great sensitiveness to change of temperature. Owing, moreover, to the exceptionally close approximation of the extreme values of the velocity, corresponding to extremely feeble double refraction, section-plates require to be 5 mm. thick in order to afford sharply defined interference figures. For Li light the optic axes are separated about 18° in the symmetry plane. As the illuminating light is changed through yellow towards the blue, the axes approach each other more and more until for blue light of the wave-length 450, they unite in the centre of the field and an apparently uniaxial figure is produced. On still further diminishing the wave-length, the axes again separate, but in the plane at right angles to their former one, and for G light the separation in this plane amounts to about 7° . These values refer to the true angle $2V_a$; the separation for $2E$, the apparent angle in air, is about 26° in the symmetry plane for Li light and 13° in the plane at right angles for G light. On warming the crystal, the axes in the symmetry plane also approach each other, and at 50° the uniaxial cross and circular rings are produced for F light, at 70° for Tl light, at 85° for Na light, at 100° for C light and 103° for Li light, when the dispersion occurs entirely in the plane at right angles to the symmetry plane. If allowed to cool immediately, the section-plate is not injured, and the phenomena recur in the reverse order. The separation of the axes, however, is permanently slightly reduced by the experiment. These beautiful phenomena are the direct result of the operation of the rule connecting the atomic weight of the contained alkali metal with the velocity of light vibration.

In connection with the alteration of the optic axial angle brought about by change of temperature, a further interesting fact has been revealed, connecting the nature of the change with the relative atomic weights of the contained alkali metals. The following tabular statement shows the order and the amounts of change observed in the apparent optic axial angle in air, produced by a rise of 50° of temperature, in the cases of those salts for which such observations were practicable.

Alteration of apparent Angle in Air, 2E, for 50° .

Second metal contained	Alkali metal contained		
	K.	Rb.	Cs.
Mg.....	Increase of $2\frac{1}{4}^\circ$	No appreciable alteration	Decrease of exceptional nature
Zn.....	„ $2\frac{1}{4}^\circ$	Increase of $1\frac{1}{4}^\circ$	Decrease of $2\frac{1}{2}^\circ$
Fe.....	„ 2°	„ 1°	„ $1\frac{1}{2}^\circ$
Ni.....	„ $2\frac{1}{2}^\circ$	„ $1\frac{1}{2}^\circ$	Angle too large to measure
Co.....	„ $2\frac{1}{4}^\circ$	„ 2°	„ „
Cu.....	„ 9°	„ $3\frac{1}{4}^\circ$	Decrease of $1\frac{1}{2}^\circ$
Mn.....	—	„ 1°	„ $2\frac{1}{2}^\circ$

It will at once be apparent that

Rise of temperature causes an increase in the optic axial angles of the potassium salts usually amounting to about $2\frac{1}{4}^\circ$ for a rise of 50° ; the effect of a similar change of temperature on the rubidium salts is to provoke a smaller increase of angle, inappreciable in the case of rubidium magnesium sulphate but usually amounting to about $1\frac{1}{2}^\circ$. In the case of the caesium salts, a decrease of optic axial angle is always observed under like circumstances, averaging 2° , but exceptionally large in the case of caesium magnesium sulphate as previously stated. The copper salts again exhibit the individuality of that metal in exhibiting larger increases, of 9° and $3\frac{1}{4}^\circ$ respectively, in the cases of the potassium and rubidium salts of that group. In all cases, however, it is observed that the rubidium salt of any group occupies an intermediate position with an optic axial angle exhibiting but slight change on rise of temperature between the corresponding potassium salt exhibiting a larger positive change and the caesium salt of the same group which exhibits a negative change.

This fact, illustrating the perfection with which change of atomic weight of the alkali metal dominates even minutest details of the optical properties of the crystals, suggests the question as to what are the corresponding changes brought about by rise of temperature in the velocities of light vibration along the three axial directions.

For the reasons stated in the introduction and in connection with the refractive indices of the first salt described (KMg sulphate), the refractive indices could not be so accurately determined at 70° with this series of salts containing water of crystallisation, as to enable full reliance to be placed upon the individual values for the different wave-lengths and for the different axial directions, as indicative of slight changes in dispersion or in the relations of the three indices, the mean lowering of the refraction being all that could be determined with certainty. The changes in optic axial angle, however, indicate that the increase of velocity brought about by rise of temperature is greatest along the c axis and least along the a axis. The more rapid approach of c to b would account for the angle increasing in the potassium and rubidium salts, but, owing to the much closer approximation of the a and b values in the cæsium salts, the smaller change along the former axis would be relatively most important in effecting alteration in the relations of the three values, particularly in the case of cæsium magnesium sulphate, and hence a decrease of angle would occur. That this is so was clearly proved in the case of the salt just mentioned, for the two images of the slit refracted by the prisms affording the indices α and β were observed, on warming, to approach, coincide, pass each other, and separate on opposite sides, indicating the approach and interchange of the a and b values. It is quite evident that any large change of c short of actual approach to b would be incompetent to prevent the approach to apparent uniaxial properties, both as regards refraction and optic axial angle phenomena. The other groups undoubtedly behave similarly as regards relative changes of velocity, although their cases are not so extreme.

The order of dispersion of the median lines is the same throughout the series with the exception of two salts. These are the cæsium magnesium and cæsium copper salts, and both are cases in which there is very near approach to coincidence of the α and β refractive indices. The amount of the dispersion is very small, rarely approaching half a degree between Li and F, and usually not more than a quarter of a degree. Hence it is much too small to enable differences between the potassium, rubidium, and cæsium salts of any one set to be determined.

One further fact remains to be noticed, concerning the salts containing manganese and cadmium.

Although the potassium salts of the groups containing manganese and cadmium have not yet been prepared, the results afforded by the rubidium and cæsium salts containing manganese and cadmium are so similar to those yielded by the other rubidium and cæsium salts of the series that there can be no reasonable doubt that, if the potassium salts could be prepared, they would exhibit relationships to the respectively analogous

rubidium and caesium salts similar to those which have been found to exist in the six groups whose members have all been studied.

It may be worth while to indicate the probable morphological and physical constants of these two possible salts of the series, in case they should ever be prepared. Taking into consideration all the facts presented in the foregoing discussion of the results of this investigation, there can be little doubt that the following values will not be very far removed from the truth.

Potassium manganous sulphate, $K_2Mn(SO_4)_2 \cdot 6H_2O$ —

Axial angle β , $75^\circ 13'$.

Ratio of axes, $a : b : c$, $0.739 : 1 : 0.497$.

Relative density, 2.110 .

Orientation of axes of optical ellipsoid, $a : c$, 13° .

Refractive indices (Na), α 1.469 , β 1.475 , γ 1.488 .

Optic axial angle, 70° .

Potassium cadmium sulphate, $K_2Cd(SO_4)_2 \cdot 6H_2O$ —

Axial angle β , $75^\circ 20'$.

Ratio of axes, $a : b : c$, $0.737 : 1 : 0.495$.

Relative density, 2.325 .

Orientation of axes of optical ellipsoid, $a : c$, 12° .

Refractive indices (Na), α 1.474 , β 1.480 , γ 1.492 .

Optic axial angle, 74° .

SUMMARY OF CONCLUSIONS.

The foregoing communication has been confined exclusively, with the exception of the forecast of the properties of the two hitherto unprepared potassium salts of the series, to the actual observations made during the course of this work, and the results are consequently a collection of facts. Such theoretical considerations and speculations as may be deduced from them, bearing on the interesting question of the constitution of the structural unit, the so-called physical molecule or crystal element, and other problems, will be advanced in a separate communication.

In order that the whole work on this series of salts may be conveniently reviewed at once, the conclusions concerning their morphology, taken, in abridged form, from the previous communication, are included.

The complete series of conclusions now follows, those from the previous memoir being Nos. 1 to 6.

Morphology

1. The salts of the monoclinic series of double sulphates,

containing cæsium as the alkali metal, exhibit the greatest facility of formation and deposition in the form of crystals, and those containing potassium least; the salts containing rubidium occupy an intermediate position in this respect.

2. The characteristic habit of the crystals of the potassium salts of the series is widely different from that exhibited by the crystals of the cæsium salts, and the habit assumed by the crystals of the rubidium salts is intermediate between the two.

3. The morphological obtuse axial angle β increases with increase in the atomic weight of the alkali metal contained, and its magnitude in any rubidium salt of the series is approximately midway between that in the potassium and that in the cæsium salt containing the same second metal. The relative amounts of change brought about in the magnitude of the axial angle by replacing the alkali metal potassium by rubidium, and the latter by cæsium, are thus approximately in direct simple proportion to the relative differences between the atomic weights of the metals interchanged. The changes in the axial angle referred to are of considerable magnitude, usually exceeding a whole degree in each case, and are therefore far removed from the limits of error.

4. The magnitudes of all the angles between the faces of the crystals of the salts of this series containing rubidium as the alkali metal lie between the magnitudes of the corresponding angles upon the crystals of the salts containing potassium and cæsium respectively. The differences between corresponding angles, except in the case of the axial angle, are not generally in simple proportion to the differences between the atomic weights of the alkali metals; the maximum deviation from simple proportionality occurs in the prism zone, where the ratio of the differences, brought about respectively by the replacement of potassium by rubidium and rubidium by cæsium, varies according to the nature of the second metal present from 1 : 2 to 1 : 3. Hence, as the atomic weight becomes higher, it exercises an influence on the angles greater than in mere numerical proportion to its increase.

5. The alkali metals R exert a preponderating influence in determining the geometrical form of the crystals of the salts of this series, the angles being altered on replacing one such metal by the next higher or lower to an extent attaining a maximum of more than a whole degree in certain angles, whilst the replacement of the second metal M by any other of the same family group is unattended by any material change in the morphological angles.

The different chemical nature of copper, compared with the other second metals, is seen in the differences, sometimes amounting to a degree, between the absolute values of the angles of the salts contain-

ing this metal and those of the analogous salts containing the other second metals. The relations presented by the three salts containing copper, however, are in rigorous accordance with rules 3 and 4, a fact indicating that those rules are not merely fortuitous.

6. Although the importance of the morphological axial ratios, as indicative of the change of geometrical form on replacing one alkali metal by another, is greatly diminished by the occurrence of simultaneous mutually neutralising changes in the angles which determine them, it is generally observed that, in any group of three salts containing the same second metal, the ratios for the rubidium salt lie between those of the potassium and caesium salts, and somewhat nearer to those of the potassium salt.

7. The whole of the salts of the series exhibit a common cleavage direction parallel to the orthodome $r'\{201\}$.

Volume.

8. The relative density increases by an approximately constant amount for each of the two specific changes throughout the series, when potassium is replaced by rubidium or the latter by caesium. The difference evoked by the former change is larger than that produced by the latter in the proportion of 5 : 4.

9. An increase in molecular volume occurs when potassium is replaced by rubidium, or the latter metal by caesium. The amount of the increase is approximately constant for each of the two specific changes, the replacement of potassium by rubidium being invariably accompanied by an increase of about 9.3 units, whilst the interchange of caesium for rubidium results in the relatively larger increase of about 13 units.

10. The determination of the molecular volume is pre-eminently a function of the alkali metal R, change of the second metal M being accompanied by a relatively small change of volume. Thus, although the difference between the atomic weights of magnesium and either zinc or copper is nearly the same as that between either potassium and rubidium or rubidium and caesium, yet the maximum amount of alteration produced by interchange amongst the first three is less than one unit, whilst the amounts produced by the interchange of the latter are as stated in rule 9, and when potassium is replaced by caesium the change of volume amounts to no less than 22 units.

11. A comparison of the distance ratios, obtained by combining the morphological axial ratios with the molecular volume, indicates that the replacement of potassium by rubidium, or the latter by caesium, is accompanied by a marked increase in the separation of the centres of the structural units or groups of units along each of the axial directions, corresponding to the increase in volume. The distance

ratios of any rubidium salt of the series are consequently intermediate between those of the potassium and cæsium salts containing the same second metal. The relative amounts of increase when rubidium replaces potassium, on the one hand, and when cæsium replaces rubidium, on the other, are approximately equal along the inclined axis a , but are considerably greater along both the axis of symmetry b , and the vertical axis c , when cæsium replaces rubidium than when the latter replaces potassium. As regards the relative amounts of increase along the directions of the three axes, the extension is invariably much the greatest along the axis of symmetry b , and least along the vertical axis c . Usually the extension in the latter direction does not fall much behind that along the inclined axis; but in the copper salts the individuality of that metal manifests itself in a considerable augmentation of the extension along the inclined axis at the expense of a corresponding decline of extension in the direction of the vertical axis.

Optics.

12. The orientation of the variable axes lying in the symmetry plane of the optical velocity ellipsoid, the polar reciprocal of the optical indicatrix of Fletcher, of every rubidium salt of the series, is intermediate between that for the potassium and that for the cæsium salt containing the same second metal. The maximum axis of the optical ellipsoid a lies nearest to the vertical morphological axis c in the potassium salt of each group, and is furthest removed for the cæsium salt. The intermediate position for the rubidium salt is always considerably nearer to the position for the potassium salt than to that for the cæsium salt, the replacement of rubidium by cæsium being attended by a much greater change of orientation than in mere arithmetical proportion to the change in atomic weight. The optical ellipsoid thus rotates about its intermediate axis b , which is coincident with the morphological symmetry axis b , when one alkali metal is replaced by another. For any potassium salt of the series, it is situated so that its major axis, a , is but slightly inclined to the vertical morphological axis c , in the obtuse axial angle ac ; for the corresponding rubidium salt the inclination is greater, and for the cæsium salt greater still, and to a much larger extent.

The different chemical nature of copper, as compared with the other second metals, is again manifested by the fact that the major axis a , of the optical ellipsoid of potassium copper sulphate, is inclined at so exceptionally large an angle to the vertical axis c as to be in front of the normal to the basal plane, whereas it is behind that normal in the other potassium salts of the series.

13. The refractive indices of any rubidium salt of the series are, without exception, intermediate between those of the potassium and cæsium salts containing the same second metal, and nearer to those of the potassium salt. An increase in the atomic weight of the contained alkali metal is accompanied by an increase in the refractive power of the crystal, and the increase in refraction becomes relatively greater as the atomic weight becomes higher. In the case of the intermediate β indices, whose corresponding direction of vibration is the invariable symmetry axis b , the amounts of increase due respectively to the replacement of potassium by rubidium and of rubidium by cæsium vary slightly, according to the second metal present, from 1 : 3 to 1 : 4.

14. The relative velocities of light vibration along the three axes of the optical ellipsoid of the crystals of every rubidium salt of the series are intermediate between the velocities along the corresponding axial directions in the crystals of the potassium and cæsium salts containing the same second metal. Increase in the atomic weight of the alkali metal is accompanied by an increase of resistance to the vibrations of light waves along each of the axes of the ellipsoid, and this alteration is relatively much greater when cæsium replaces rubidium, than when the latter replaces potassium. In the case of the invariable direction of the axis b , the relative amounts are as 3 : 1; the amounts along the axis a are always slightly larger than those along b , and in the same proportion, whilst those along the axis c are in every case much less, and the difference between the effects of the two replacements is more pronounced.

15. An increase in the atomic weight of the alkali metal is accompanied by convergence of the values of the velocity along the three axial directions abc towards unity, and, consequently, by a diminution in the amount of the double refraction. This property is already a very feeble one in the potassium salts, and it is extremely weak in the cæsium salts; the rubidium salts occupy an intermediate position in this respect. The amounts of change in the velocity along the two directions of the axes a and b , when potassium is replaced by rubidium, or the latter metal by cæsium, being but slightly different, the relations of these two velocities are only slightly changed; the slower rate of change along the c axis, however, has the effect of introducing more considerable changes, in the form of convergence, in the relations between the value for this axis and the other two axial values. Both the former slight relative alteration and the latter more considerable one are progressive in character, the rubidium salt invariably occupying an intermediate position.

16. The preceding rule receives somewhat remarkable confirmation in the case of cæsium nickel sulphate. With the exception of this

salt, the whole of the salts of the series exhibit positive double refraction. Owing to the fact that in the potassium nickel salt the a and b values are at their maximum separation for the series, although still nearest together of all the three values in accordance with positive double refraction, the operation of the rule causes it to happen that, in the cæsium salt, the c value, in consequence of its more rapid rate of convergence, has approached nearest the intermediate value b , and so this salt ought to exhibit negative double refraction, a presumption in complete accord with actual observation.

17. Another extraordinary effect of the operation of rule 15 is afforded by the refraction phenomena of cæsium magnesium sulphate. The difference between the a and b values is at its minimum for the potassium salts of the series in the case of potassium magnesium sulphate, and the rule of convergence towards unity results in the ratio $a : b$ actually arriving at unity for the cæsium salt. Owing to the existence of the usual slight difference in dispersion along the axial directions, the coincidence is only absolute for wave-length 450 in the blue. The difference of the two velocities, or of the corresponding refractive indices, is about 0.0001 for sodium light, and 0.0002 for lithium light, the order being as usual throughout the series; for violet G light the values also differ by about 0.0001, but the order is reversed. Hence for wave-length 450 cæsium magnesium sulphate exhibits apparent uniaxial properties. The differences just enumerated for other wave-lengths are so slight that a 60° prism affording these indices exhibits only one image of the spectrometer slit, even for lithium light, unless an eye-piece of very high power is employed in the telescope.

From this and the preceding conclusion 16, it will be evident that although the slight influence of the second metal, which is chiefly observed as regards the ratio $a : b$, cannot be expressed according to any rule involving the atomic weight of that metal, yet its specific nature is of considerable importance in determining the optical phenomena presented by the crystals.

18. The whole of the specific and molecular optical constants, calculated by means of the formulæ of Lorenz or of Gladstone and Dale, for every rubidium salt of the series, are intermediate between those of the potassium and cæsium salts containing the same second metal.

The replacement of potassium by rubidium, and of the latter by cæsium, is accompanied by a diminution of specific refraction and specific dispersion, the amount of which is twice as great when the former chemical change occurs as when the latter substitution is effected. The maximum diminution in specific refraction occurs along the axis c ; the somewhat smaller amounts of diminution along a and

b are almost identical, the amount along *b* being invariably, however, slightly the larger.

The same chemical changes, involving an increase in the atomic weight of the alkali metal, are attended by an increase of molecular refraction and molecular dispersion, the amount of which is greatest when cæsium replaces rubidium, the proportions for the two changes in molecular refraction being as 3 : 5. The amounts of increase of molecular refraction are nearly the same along *a* and *b*, that along the former direction is slightly the greater, and is the maximum increase; the minimum increase occurs along *c*.

These rules are equally valid, whether the constants are calculated by the formulæ of Lorenz, or of Gladstone and Dale, and whatever may be the wave-length for which the comparison of refraction constants is made. Moreover, as refraction and density are similarly affected by change of temperature, they are independent of temperature.

The influence of the second metal in bringing about changes in the molecular optical constants is but slight compared with that due to the alkali metal. It is, however, perceptible in the case of the more intimately related metals magnesium, zinc, and cadmium. The specific refraction diminishes when magnesium is replaced by zinc, and again when zinc is replaced by cadmium, whilst the molecular refraction increases under the same circumstances. The amount of the change is greatest, in the case of the specific refraction, when magnesium is replaced by zinc, and, in the case of the molecular refraction, when zinc is replaced by cadmium, a similar order as regards atomic weight as was observed on replacement of the alkali metal.

19. Excluding the salts containing magnesium, the cæsium salt of which is not strictly comparable with the other two salts, on account of the extraordinary conditions following the operation of rule 15, the optic axial angle (angle between the optic binormals of Fletcher) of any rubidium salt of the series is intermediate between the optic axial angles of the potassium and cæsium salts containing the same second metal, whatever wave-length is chosen for the comparison. This rule is in complete accordance with rule 15, the progressive change there referred to leading to a progressive change in the optic axial angle, which varies in character in the different groups, according to the relative rates of convergence of the extreme values of the velocity towards the intermediate value on replacing the alkali metal by one of higher atomic weight.

This variation in the character of the progressive change in the optic axial angle is due to the influence of the second metal.

20. The first median line is the minimum axis of the optical ellipsoid *c* in every salt of the series, except cæsium nickel sulphate,

in accordance with the positive sign of double refraction. In the cæsium nickel salt, a reversal of the median lines occurs, the first median line being the maximum axis a and the sign of double refraction negative, a direct consequence of rule 15, as described in the statement concerning this salt which follows the rule. In accordance with the velocity relationships there referred to, the optic axial angle is already unusually large in potassium nickel sulphate, and the operation of the rule of progressive change in the optic axial angle renders it so large when the cæsium salt is reached that it exceeds 90° , and the median line, which had hitherto been the first, becomes the second.

21. In the case of the three salts containing magnesium, the progressive alteration of optic axial angle proceeds, according to the rule, from the potassium to the rubidium salt. Owing, however, to the remarkable relationships of the velocities along the three axial directions when the cæsium salt is reached, brought about by the operation of rule 15, and described in paragraph 17, quite extraordinary optic axial angle phenomena are exhibited by the crystals of cæsium magnesium sulphate, including very large dispersion of the optic axes for different wave-lengths in crossed axial planes, and great sensitiveness to change of temperature. Indeed, cæsium magnesium sulphate exhibits one of the most interesting cases of crossed axial plane dispersion yet described in connection with crystals of monoclinic symmetry, and the whole phenomena are the direct result of rule 15, connecting the atomic weight of the contained metal with the velocity of light vibration.

22. Rise of temperature causes an increase in the optic axial angles of the potassium salts of the series, usually amounting to about $2\frac{1}{4}^\circ$ for a rise of 50° ; the effect of a similar change of temperature on the rubidium salts is to provoke a smaller increase of angle, inappreciable in the case of rubidium magnesium sulphate, but usually amounting to about $1\frac{1}{2}^\circ$. In the case of the cæsium salts, a decrease of optical axial angle is always observed under like circumstances, averaging 2° , but exceptionally large in the instance of cæsium magnesium sulphate from the cause just described. The copper salts again manifest the individuality of that metal in exhibiting larger increases than usual of 9° and $3\frac{3}{4}^\circ$ respectively, in the cases of the potassium and rubidium salts of that group. In all cases, however, it is observed that the rubidium salt of any group occupies an intermediate position, with an optic axial angle exhibiting but slight change on rise of temperature, between the corresponding potassium salt exhibiting a larger positive change, and the cæsium salt of the same group, which exhibits a negative change.

23. Although the potassium salts of the groups containing man-

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ganese and cadmium have not yet been prepared, the results afforded by the rubidium and cæsium salts containing manganese and cadmium are so similar to those yielded by the other rubidium and cæsium salts of the series, that there can be no reasonable doubt that if the potassium salts could be prepared, they would exhibit relationships to the respectively analogous rubidium and cæsium salts similar to those which have been found to exist in the six groups whose members have all been studied.

24. The foregoing results have clearly demonstrated that the alkali metal R exerts a predominating influence in determining the crystallographical characters of the double sulphates of the series $R_2M(SO_4)_2 \cdot 6H_2O$, the influence of the second metal M being comparatively slight. It has been further established that the salts of each group, containing the same second metal, exhibit a regular progression of crystallographical properties corresponding to the progression in the atomic weight of the alkali metal contained. With regard to every property, morphological or physical, the rubidium salt of every group stands intermediate between the potassium and the cæsium salt of that group. It has also been generally observed that the replacement of rubidium by cæsium is accompanied by a much greater modification of the crystallographical characters than the interchange of rubidium for potassium, indicating that change in the atomic weight of the alkali metal becomes more potent in its modifying influence as the atomic weight becomes higher, than in mere arithmetical proportion to that change. Moreover, this statement is rigorously correct as regards even the minutest details of the physical properties, and for all temperatures within whose limits the salts are capable of existing. Further, the extraordinary optical properties exhibited by one member of the series, cæsium magnesium sulphate, have been shown to be the direct result of the operation of this generalisation, and without it would have been inexplicable. So absolutely general, without exception, is it in its application to the 22 salts investigated, that it is now possible to predict in detail the properties which would be exhibited by the two hitherto unprepared salts of the series, potassium manganous sulphate, and potassium cadmium sulphate, if these salts should ever be isolated in the crystallised condition.

That the collection of rules now established is really entitled to be called a generalisation, and is not merely fortuitous, repeated in every group of the series, on account of the similar chemical nature of the second metals and the slight influence which they exert in modifying the properties of the crystals, is conclusively demonstrated by the results of the investigation of the copper group. The different chemical nature of this metal has been unmistakably emphasised, with regard to almost every property, by the exceptional character of

the absolute values of the various morphological and physical constants. Yet the relations of these values for the three salts containing copper are in every instance in rigorous accordance with the rules found for the rest of the series.

The final conclusion of this investigation, therefore, is that

The alkali metal R exerts a predominating influence in determining the characters of the crystals of the isomorphous monoclinic series of double sulphates, $R_2M(SO_4)_2 \cdot 6H_2O$, and the whole of the crystallographical properties of the potassium, rubidium, and caesium salts containing the same second metal M are, in the case of every such group throughout the series, functions of the atomic weight of the alkali metal which they contain.