137. Philo's Line

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To these add $b+b'$ terms of N where b is number of terms of N that must be taken to make the sum of $a+a'$ of P , and b of N just $< S$.

To these add the next $c+c'$ terms of P where c is the number of terms of P that must be taken to make the sum of the $a+a'$ of P , $b+b'$ of N , and c of P just $> S$, and so on.

Since, if the $q'+1$ terms are from N they will be all negative, at q^{th} stage the new series has a sum differing from S by less than the sum of $q'+1$ terms of P (or N) where q' is the integer of series $a', b', c' \dots$ selected for q^{th} stage. But the terms of P and N are eventually infinitely small and $\therefore$ sum of a finite number of them is.

Thus the new series converges to S , and since a', b', c' were arbitrary finite integers there is a infinite number of rearrangements. H. L. TRACHTENBURG.

136. [K. 8. f.] On the maximum quadrilateral of given sides.

$$BD=x, AC=y.$$

(1) $x \cdot AE$ to be a maximum (twice the area);

(2) while $x \cdot CE = \text{constant}$

$$= \frac{(a^2 - d^2) - (b^2 - c^2)}{2}.$$

Now, when (1) is a maximum, so is

$$(1)^2 + (2)^2 = x^2 y^2; \therefore xy \text{ is a maximum.}$$

But $xy < ac + bd$ unless the figure is concyclic;

$\therefore$ for maximum the figure must be concyclic. R. CHARTRES.

137. [K. 4.] Philo's Line.

$D = \text{the fixed point.}$

BDC , the base of the triangle OBC , is infinite when BDC is parallel to AB or to AC ;

$\therefore$ it must have a minimum value at the position of symmetry or when $OB = OC$;

$\therefore$ perp. OF bisects BC as well as DE ;

$$\therefore BD = CE.$$

Or, without using the position of symmetry, since BDC is the minimum line through D ;

$\therefore HLK$, the par^l tangent, is a minimum.

But the minimum tangent to arc PQ is bisected at L ;

$\therefore BC$ also is bisected at F ; $\therefore BD = CE$. R. CHARTRES.

138. [K. 2. b, c.] To prove that the 9-point circle touches the in- and ex-circles of a triangle.

Let D, E, F be the points of contact of the in-circle with BC, CA, AB . A', B', C' , the mid-points of the sides.

Bisect $\hat{A}$ by AU . From AC cut off $AH = AB$.

Join UH cutting $A'B', A'C'$ at b', c' .

By Euc. I. 4, $\triangle AHU = \triangle ABU$ and thus $\hat{AUH} = \hat{AUD'}$;

$\therefore UH$ touches the in-circle.

We will now prove

$$A'b' \cdot A'B' = A'D^2.$$

$$A'D = A'B - BD = \frac{1}{2}a - \frac{1}{2}(c + a - b) = \frac{1}{2}(b - c).$$