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*On the construction of a
Table of Mortality*

BEING
THE INAUGURAL ADDRESS FOR
SESSION 1899-1900

BY
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On the construction of a Table of Mortality

GENTLEMEN,—Allow me to thank you very cordially for electing me to preside over your meetings during the ensuing Session. It is an honour very greatly appreciated by every one who has been selected to fill this chair. Indeed I feel doubly honoured, seeing that I am the first to address you in the new Hall of the Faculty of Actuaries under the gaze of the stony eyes of our own immortal Napier. Your election reminds me that I had the honour of presiding over your earliest meetings and that I also had the pleasure of reading the earliest papers that were presented for your consideration. These three papers I am proud to believe have been diligently perused by nearly every student of Actuarial Science, forming as they do the wicket opening into wider fields, the earliest lessons of the course. They have, as I have been told, been the guiding rays of light which induced some of our most eminent Actuarial stars, presently shining in our firmament, to follow in the pursuit of higher knowledge, and to join the profession. These simple papers have been printed three times, and are again out of print. Gentlemen, I do wish I were able to repeat that enthusiasm. It seems to me now to be a much more simple matter to find language to describe something which one is striving to understand for oneself, and labouring to make clear to one's own comprehension, than to write an essay upon something which appears so plain and so palpable that the wonder is how any one could fail to interpret the language of the symbols. These efforts were made long long ago, long before the mind was clouded by care or ruffled by the multitude of many inquietudes, and at a time when the energies were fresh and the heart was young; but as Tupper says:—

‘Light as flakes of falling snow,
Drop the silent-footed hours,
And the days—they come and go,
And the years—we scarcely know
How their frosts and fruits and flowers—

Transient crops of weal and woe—
 Change and pass and perish so.
 While we muse upon to-day,
 Lo! the dream has died away,
 And there lives what was to-morrow,
 With its present joy or sorrow.
 So on—so on—till years have sped
 By tens and twenties over head,
 And those flakes that fell unfelt
 Have changed to snows that never melt !'

Gentlemen, it has hitherto been the custom for the person whom you honour to fill this chair to open the Session by an address—touching lightly upon some of those points of our business that recent events have rendered of pressing, though it may be of only passing, importance. Such a theme is not beyond my ambition, but it is entirely out of the reach of my leisure and of my opportunities. My pen has been so much occupied plodding to the circumstances, and to the affairs of practical business, that I could not find the needful time to revel amongst these flowery fields. I have chosen another task. I have thought that I would be carrying out the objects and the aims of this Society by dwelling upon some subject on the level ground of our business, and treating it rather in an elementary and in an educative manner, than attempting to soar to higher and more hazardous themes. The results of such lofty ambitions might be that we may become

‘Blind unto the beautiful in common things that strew the common way.’

One special subject, however, has been engaging my attention for some time, and I have preferred, if you will be pleased to listen till the conclusion of my reflections, to dwell very much in detail upon the processes and upon the principles which to my mind ought to be kept in view in the congregation and collection of the necessary materials—in the combination of these materials, and in the concluding development and completion of these materials into a Table of Mortality. I feel very seriously the importance of my task, indeed, I have almost undertaken to do too much. A Table of Mortality is the instrument by which are performed all calculations depending upon the continuance of life—by its aid we lift the dark and obscure curtain of the future and measure longevity—parcelling out with some confidence the total number of years a multitude of persons are likely to live—by its aid we determine the value of engagements to pay annuities during life, or of capital sums payable at the termination of life, and of many other variations and combinations of life, and of death which the ingenuity of man has devised. In these circumstances you will be able to estimate the importance of the problem, and inasmuch

as this instrument, if it be false, may lead immensely astray, the construction of a Table of Mortality should receive the supreme consideration of our undivided efforts.

In the execution of these aspirations I propose to divide my subject into several well-defined sections, and, as has been suggested to me, to devote more than one evening to their consideration. Many points of considerable importance have to be referred to—and as some of my modes of treating these points are probably new to most of you, I hope you will deem them worthy of your attention.

If we consult the volumes wherein the experience of the seventeen Offices were published in 1839, or wherein the more matured results of the twenty Offices of the H^M. Table in 1868 were detailed, we feel that many links of much desired information are amissing. Upon these missing links I propose to dwell with correspondingly increased attention, or rather I shall so bestow my remarks upon the several sections and sub-sections of the compilation of a Table of Mortality as to give increased prominence to those details that have, with such apparent facility, been passed over in the two published tables based upon the experience of assured lives.

The first section of my subject is the processes to be employed in the collection of the facts preparatory to the ascertainment of the rate of mortality. The rate of mortality, at any age, is the ratio of the number of that age dying in one year, to the total number who enter upon that year of age and remain exposed to mortality during that year. Unquestionably the best method of collecting the material is to write the facts of each life, or of each policy on a card—not on loose cards, which have opportunities of becoming scattered—some here, some there—some in this desk, and some in that drawer—some complete and others incomplete—but written on sheets of thick paper, each sheet containing say 16 cards, and bound in volumes, say, of 160 leaves, so that each volume would contain the facts of 2560 policies. The printing is so arranged that the part of one sheet occupied by the print, ‘registers’ identically with the same part of all the other sheets. ‘Registering,’ as understood by printers, is the placing of each sheet of paper in exactly the same spot of the printing press, so that each impression registers or coincides with all the others. Thus a sheet is said to be correctly registered when the number of each page falls exactly on the number of the page on the other side of the sheet. A very slight attention attains this result. Previous to being bound each sheet has been passed through a numbering machine which mechanically prints numbers in arithmetical order on the top line of the card corresponding to the numbers of the Policies in the office records.

Being performed with mechanical accuracy, it is impossible for the facts of any policy to be omitted. A specimen of this sheet of card paper is shown. When seen in sheet the material seems almost too limp for the purpose—but when the sheet is cut up into 16 separate cards, each card is abundantly firm for all handling to which it is to be subjected. These pliant cards better facilitate ascertaining their numbers than firmer cards, and being free from artificial surface-sizing, pass through the fingers without risk of adhering. Their cost is also very materially less. I now call your attention to the data to be written on the card, and to the disposal of the spacing of that data. There is

- 1st. The number of the Policy.
- 2nd. The name of the Life assured.
- 3rd. The numbers of other Policies on the same life—usually recorded in the Office books.
- 4th. The duration of the risk.
- 5th. The date of exit—or the date of the close of the observations.
- 6th. The date of entry—that is the date when the Office entered upon the risk. The difference between these two dates being entry for No. 4.
- 7th. The date of birth. The difference between these two dates being the 6th and 7th.
- 8th. The age at entry.
- 9th. The age at exit—being the difference between the first and the last dates—No. 5 and No. 7—which is also checked by being the addition of the age at entry in No. 8, to the duration of the risk in No. 4.

The months of these dates are all tabulated at their numerical value—January one, February two, etc., so that any subtractions or additions are readily performed. Underneath the words ‘age at entry’ is noted the Office age next birthday, taken from the books, which affords a check upon the accuracy of the exact age derived from the tabulated dates of birth and of entry.

The two all-important factors in my method are the exact ages at entry and at exit, written in strong blue ink in the form of a fraction thus—

$$\frac{32.5}{48.9}$$

It ought to be stated that when a Policy had *completed* its experience the date of exit and the age at exit were written in

numbers, and altogether the whole of the work up to this point is all that could be desired, and is much superior to that resulting from the adoption of loose cards. We now possess 2560 cards from each volume, complete and ready for manipulating their facts—changing them from a general chaos into something of law and order for the purpose of ascertaining the rate of mortality at each of the ages of life. These cards are then to be separated into those of healthy home risks, and into other classes. The next step is to classify the names in order to gather the facts of Policies that referred to the same life, and to incorporate these cards as the facts of one entry if the risks have been continuous. If continuous, the cards of the continuous risks were put into envelopes specially constructed, of which I show specimens, and the facts of the continuous risk were written on the envelope thus—

$$\begin{array}{ccc} \frac{32.5}{48.9} & \frac{32.5}{48.9} & \text{or } \frac{32.5}{48.9} \\ L & W & D \end{array}$$

We now possess the cards of the experience ready for tabulation and it will now be our efforts to arrange them so as to ascertain the numbers who are of the same age. This tabulation falls naturally to be divided into three sections—

	One section for the Living,
Another	„ „ Withdrawals,
And a third	„ „ Died,

and each of these into two sub-sections—their ages at entry and their ages at exit.

You will have observed that the ages recorded on the cards were expressed in the form of a fraction, thus—

$$\begin{array}{ll} \text{Age at entry} & = 37.5 \\ \text{Age at exit} & = 43.8 \\ \text{Duration} & = 6\frac{3}{12} \text{ years.} \end{array}$$

That is to say, there were seven months remaining of the 37-38th year of life exposed to mortality, thereafter the life was at risk for five complete years, from 38 to 43, and there were eight months of the year of life 43-44 exposed, in all six years and three months as in the last column of the accompanying chart. When these facts are tabulated according to my process of tabulation they appear as follows, and first let the tabulation have reference to a single life, thus—

Year of life.	ENTRANTS.			EXITANTS.			Difference being years of life exposed to mortality.
	Number.	Net Value.	Progressive summation.	Number.	Net Value.	Progressive summation.	
37-38	1· -5	·7	·7	...	...	...	·7
38-39	·5	·5	1·	...	...	...	1·
39-40	...	...	1·	...	...	...	1·
40-41	...	...	1·	...	...	...	1·
41-42	...	...	1·	...	...	...	1·
42-43	...	...	1·	...	...	...	1·
43-44	...	...	1·	1· -8	·4	·4	·8
44-45	...	...	1·	·8	·8	1·	...
	...	...	1·	...	...	1·	...
Sums	1·	1·	8·7	1·	1·	2·4	6·3

It will be seen that the total number of years of life exposed to mortality ($6\frac{3}{12}$) is equal to the difference between the separate progressive summations of the Entrants and of the Exitants ($8\cdot7 - 2\cdot4 = 6\cdot3$).

If there were six cards, thus :

37·5	38·6	39·7	40·8	41·9	42·10
43·8	43·8	43·8	43·8	43·8	43·8
<hr/>					
Durations = 6·3	5·2	4·1	3·	1·11	·10 = 21·3

the first step in the tabulation would have reference to the Entrants and to their ages at entry, thus :

Age.	Number.
37-38	1· - 5
38-39	1· - 6
39-40	1· - 7
40-41	1· - 8
41-42	1· - 9
42-43	1· - 10
43-44	...

These are all the entries obtained from the cards, but bearing in mind that the first life entered upon the risk at age $37\frac{5}{12}$, there accordingly remained only seven months of that year of life (37-38) exposed to risk, and inasmuch as the risk was continued into subsequent years there were five months of the following year of life (38-39) entered upon, together making up a whole year of

life at that age, and so on for other years. Similarly with the life entering at age $38\frac{6}{12}$, there were six months exposed in the year of life 38-39 and other six months in the year of life 39-40, which together make up a whole year of life, and so on. The tabulation of the numbers entering thus becomes:

Age.	Number.	Net Number.	Progressive Summation.
37-38	$1\cdot -^5$	$\cdot 7$	$\cdot 7$
38-39	$1\cdot ^5 - ^6$	$\cdot 11$	$1\cdot 6$
39-40	$1\cdot ^6 - ^7$	$\cdot 11$	$2\cdot 5$
40-41	$1\cdot ^7 - ^8$	$\cdot 11$	$3\cdot 4$
41-42	$1\cdot ^8 - ^9$	$\cdot 11$	$4\cdot 3$
42-43	$1\cdot ^9 - ^{10}$	$\cdot 11$	$5\cdot 2$
43-44	10	$\cdot 10$	$6\cdot$
Sums—	$6\cdot$	$6\cdot$	$23\cdot 3$

It does not matter how long the progressive sum for the entrants is continued, provided a like progressive summation for the exitants is similarly continued. Accordingly up to the close of the observations these six lives passed through $23\frac{3}{12}$ years. When now we perform the same calculation for the six who all leave the observations at age $43\frac{8}{12}$, thus:

Age.	Number.	Net Number.	Progressive Summation.
37-38			
38-39			
39-40			
40-41			
41-42			
42-43			
43-44	$6\cdot - ^{48}$	$2\cdot -$	$2\cdot$
44-45	$+ ^{48}$	$4\cdot -$	$6\cdot$
	$6\cdot$	$6\cdot$	$6\cdot$

The net number of years of life will be the difference between the two progressive summations:

Age.	PROGRESSIVE SUMS.		DIFFERENCE.
	For the Entrants.	For the Exitants.	Years of Life exposed to Mortality.
37-38	·7		·7
38-39	1·6		1·6
39-40	2·5		2·5
40-41	3·4		3·4
41-42	4·3		4·3
42-43	5·2		5·2
43-44	6·	2·—	4·
44-45	6·	6·—	...
	6·	6·—	...
Sums—	35·3	— 14·	= 21·3

= Net number exposed, as previously shown.

If there be a promiscuous lot of cards of various ages at entry, it is sometimes convenient to arrange the facts in the following manner and to sum up the months while the cards at each age are passed through the fingers:

Age.	Number of Lives.	Sum of Months in excess of Integral Age.
14	2	— 21
15	4	— 10
16	11	— 74
17	13	— 72
18	21	— 135
19	45	— 263
20	69	— 423
21	132	— 796
22	162	— 1035
23	189	— 1176
24	233	— 1360
etc.	etc.	

and completing the calculation of the net number of years and months entering upon and passing through each year of life, thus :

Year of Life.	Number of Lives.	Sum of Months in excess of Integral Age.	Net Number entering upon each Year of Life.
14-15	2	— 21	= .3
15-16	4	21— 10	= 4.11
16-17	11	10— 74	= 5.8
17-18	13	74— 72	= 13.2
18-19	21	72— 135	= 15.9
19-20	45	135— 263	= 34.4
20-21	69	263— 423	= 55.8
21-22	132	423— 796	=100.11
22-23	162	796—1035	=142.1
23-24	189	1035—1176	=177.3
24-25	233	1176—1360	=217.8
etc.	etc.		

It will generally be found convenient to arrange the tabulation in a more general form, considerably diminishing the labour and reducing the risk of error. The first part is extracted from the cards.

TABULATION OF THE CARDS AS FOR THE AGES ON ENTERING UPON THE RISK.

PRELIMINARY TABULATION OF THE NUMBER ENTERING THE RISK AT EACH MONTH OF AGE.

Age.	0	1	2	3	4	5	6	7	8	9	10	11	Total.
14	...	...	...	...	...	...	...	...	...	...	1	1	2
15	2	...	1	...	...	...	...	...	1	...	...	...	4
16	3	...	...	...	1	...	...	1	...	1	1	4	11
17	2	1	...	1	...	2	2	1	...	2	1	1	13
18	1	1	...	3	...	4	2	1	3	1	2	3	21
19	9	2	1	2	2	1	6	4	2	5	6	5	45
20	12	3	...	5	2	5	6	5	6	9	4	12	69
21	24	10	6	6	4	6	7	11	8	4	15	31	132
22	23	4	3	16	10	9	9	10	17	12	15	34	162
23	27	8	6	14	14	10	14	14	10	11	24	37	189
24	50	10	8	8	6	19	19	16	18	15	20	44	233
etc.				etc.					etc.				etc.

The subsequent calculation of the number of months when these lives entered upon the risk at ages, in excess of the integral ages on the margin of the above, follows as a matter of course, and is of the simplest description. If the above be tabulated on alternate lines the total number of months may be entered upon the intermediate line and written in red ink, as shown here in darker type. Thus:—

COMPLETE CALCULATION OF THE NUMBER OF MONTHS THE LIVES
ENTERED IN EXCESS OF THEIR INTEGRAL AGE.

Age.	0	1	2	3	4	5	6	7	8	9	10	11	Total number enter- ing.	Total months in ex- cess of integral age.
14	...	...	...	...	...	...	...	...	...	...	1 10	1 11	2	21
15	2	...	1 2	...	...	...	...	...	1 8	...	...	...	4	10
16	3	...	...	...	1 4	...	...	1 7	...	1 9	1 10	4 44	11	74
17	2	1 1	...	1 3	...	2 10	2 12	1 7	...	2 18	1 10	1 11	13	72
18	1	1 1	...	3 9	...	4 20	2 12	1 7	3 24	1 9	2 20	3 33	21	135
19	9	2 2	1 2	2 6	2 8	1 5	6 36	4 28	2 16	5 45	6 60	5 55	45	263
20	12	3 3	...	5 15	2 8	5 25	6 36	5 35	6 48	9 81	4 40	12 132	69	423
21	24	10 10	6 12	6 18	4 16	6 30	7 42	11 77	8 64	4 36	15 150	31 341	132	796
22	23	4 4	3 6	16 48	10 40	9 45	9 54	10 70	17 136	12 108	15 150	34 374	162	1035
23	27	8 8	6 12	14 42	14 56	10 50	14 84	14 98	10 80	11 99	24 240	37 407	189	1176
24	50	10 10	8 16	6 18	8 32	19 95	19 114	16 112	18 144	15 135	20 200	44 484	233	1360
Sums	153	39	25	53	41	56	65	63	65	60	89	172	= 881	
	..	39	50	159	164	280	390	441	520	540	890	1892	=	5365

The checks and cross checks will be apparent without remark. The natural process of reducing the above number of months into their equivalent value in years and months may appear troublesome and attendant with risk of error. The first entry is

2.-21

That is the two lives were together 21 months in excess of age 14, so that the net portion of their lives exposed to risk in that year of life was 3 months, the remaining 21 months were at risk in the following year of life. The second entry is four persons in the year of life 15—16, who together at entry exceeded age 15 by 10 months, and as 21 months of the entrants at the previous age entered in the 15-16 year, they thus complete the full number of entrants. The difference between the 21 months additive and the 10 months deductive leaves 11 months positive, making the net number entering the year of life 15—16 at 4 and $\frac{11}{12}$ and so on for other ages. The summation of this column gives the total entrants, as has already been explained.

Now the calculation of the net value of these total months will be found to be somewhat irritating and to be attendant with trifling errors. A smarter process is the following. Take the differences down in pencil—placing the additive sums in one column and the subtractive sums in another column. The totals of these columns should agree. Thus:—

Age.	Number of Lives.	Sum of months in excess of Integral age.	Additive.	Subtractive.
14	2	— 21	...	21
15	4	21— 10	11	...
16	11	10— 74	...	64
17	13	74— 72	2	...
18	21	72— 135	...	63
19	45	135— 263	...	128
20	69	263— 423	...	160
21	132	423— 796	...	373
22	162	796—1035	...	239
23	189	1035—1176	...	141
24	239	1176—1360	...	184
		1360	1360	...
		Sums . . .	1373	1373

By the aid of a table¹ the equivalent years and months for any number of months are readily obtained.

Having now obtained the exact number entering the risk of mortality in each year of age, the total number passing through each higher year of age is found by a progressive summation down the column, and having pursued the same process for the number exiting from the risk, the difference between the sums at each age in these progressive summations will give the net numbers exposed to mortality in each year of life, as in the following example of the number exposed to mortality from among those who are alive at the close of the observations :—

ENTRANTS.				EXITANTS.			
Age at Entry.	Number of Entrants.	Entrants re-allocated to exact ages at entry.	Progressive sum of these Entrants.	Number of Exitants.	Exitants re-allocated to exact ages at exit.	Progressive sum of these Exitants.	Years of life exposed to Mortality.
14	2	·3	·3	...	...	...	·3
15	4	4·11	5·2	...	...	...	5·2
16	11	5·8	10·10	...	...	...	10·10
17	13	13·2	24·	...	...	...	24·
18	21	15·9	39·9	...	...	...	39·9
19	45	34·4	74·1	...	...	...	74·1
20	69	55·8	129·9	...	...	...	129·9
21	132	100·11	230·8	...	...	...	230·8
22	162	142·1	372·9	...	...	...	372·9
23	189	177·3	550·	...	...	...	550·
24	233	217·8	767·8	...	...	...	767·8
25	266	257·11	1025·7	...	...	...	1025·7
26	276	262·7	1288·2	...	...	...	1288·2
27	286	291·9	1579·11	...	...	...	1579·11

¹ TABLE OF THE EQUIVALENT YEARS AND MONTHS FOR ANY NUMBER OF MONTHS.

Months	100	200	300	400	500	etc.
1	8·5	16·9	25·1	33·5	41·9	
2	8·6	16·10	25·2	etc.		
3	8·7	16·11	etc.			
4	8·8	17·				
5	8·9	17·1				
6	8·10	17·2*				
7	8·11	17·3				
8	9·	17·4				
9	9·1	17·5				
10	9·2	17·6				
etc.	etc.	etc.				
100—	16·8	25·	33·4	41·8		

* Thus 206 months are equivalent to 17 years and two months.

ENTRANTS.				EXITANTS.			
Age at Entry.	Number of Entrants.	Entrants re-allocated to exact ages at entry.	Progressive sum of these Entrants.	Number of Exitants.	Exitants re-allocated to exact ages at exit.	Progressive sum of these Exitants.	Years of life exposed to Mortality.
28	304	299·6	1879·5	...	..	...	1879·5
29	312	313·2	2192·7	...	...	...	2192·7
30	306	299·6	2492·1	...	...	...	2492·1
31	264	292·7	2784·8	...	...	...	2784·8
32	216	234·2	3018·10	...	...	...	3018·10
33	188	202·	3220·10	...	...	...	3220·10
34	201	187·4	3408·2	...	...	...	3408·2
35	175	195·9	3603·11	...	...	...	3603·11
36	125	151·6	3755·5	...	...	...	3755·5
37	132	113·10	3869·3	...	...	...	3869·3
38	119	131·9	4001·	...	...	...	4001·
39	90	104·2	4105·2	...	...	...	4105·2
40	87	90·9	4195·11	...	...	...	4195·11
41	68	73·1	4269·	...	...	...	4269·
42	46	54·11	4323·11	3	1·3	1·3	4322·8
43	42	43·6	4367·5	4	4·5	5·8	4361·9
44	42	45·	4412·5	3	2·7	8·3	4404·2
45	20	24·9	4437·2	6	4·6	12·9	4424·5
46	21	21·3	4458·5	12	9·8	22·5	4436·
47	19	21·3	4479·8	43	24·3	46·8	4433·
48	16	16·11	4496·7	42	48·5	95·1	4401·6
49	12	14·1	4510·8	56	45·9	140·10	4369·10
50	13	11·10	4522·6	73	65·4	206·2	4316·4
51	7	8·	4530·6	99	83·1	289·3	4241·3
52	5	6·3	4536·9	114	114·3	403·6	4133·3
53	6	5·9	4542·6	136	119·	522·6	4020·
54	5	5·4	4547·10	139	141·3	663·9	3884·1
55	2	3·	4550·10	163	154·	817·9	3733·1
56	2	2·10	4553·8	190	178·4	996·1	3557·7
57	1	1·3	4554·11	195	190·10	1186·11	3358·
58	...	·1	4555·	185	190·2	1377·1	3177·11
59	...	...	4555·	179	174·2	1551·3	3003·9
60	...	...	4555·	173	175·3	1726·6	2828·6
61	...	...	...	186	183·10	1910·4	2644·8
62	...	...	...	164	176·6	2086·10	2468·2
63	...	...	...	186	178·11	2265·9	2289·3
64	...	...	...	167	169·7	2435·4	2119·8
65	...	...	4555·	193	191·9	2627·1	1927·11
66	...	...	...	173	175·11	2803·	1752·
67	...	...	...	186	177·9	2980·9	1574·3
68	...	...	...	153	164·7	3145·4	1409·8
69	...	...	...	157	161·7	3306·11	1248·1
70	...	...	4555·	150	155·5	3462·4	1092·8
71	...	...	...	117	134·7	3596·11	958·1
72	...	...	...	121	111·3	3708·2	846·10
73	...	...	...	107	121·	3829·2	725·10
74	...	...	...	97	100·7	3929·9	625·3
75	...	...	4555·	98	97·2	4026·11	528·1
76	...	...	...	87	82·11	4109·10	445·2
77	...	...	...	78	89·3	4199·1	355·11
78	...	...	...	56	61·	4260·1	294·11

ENTRANTS.				EXITANTS.			
Age at Entry.	Number of Entrants.	Entrants re-allocated to exact ages at entry.	Progressive sum of these Entrants.	Number of Exitants.	Exitants re-allocated to exact ages at exit.	Progressive sum of these Exitants.	Years of life exposed to Mortality.
79	...	...	...	58	60·11	4321·	234·
80	...	...	4555·	46	54·1	4375·1	179·II
81	...	...	...	37	38·5	4413·6	141·6
82	...	...	...	25	33·	4446·6	108·6
83	...	...	...	22	23·7	4470·1	84·II
84	...	...	...	21	18·8	4488·9	66·3
85	...	...	4555·	14	19·1	4507·10	47·2
86	...	...	...	15	14·6	4522·4	32·8
87	...	...	...	7	11·8	4534·	21·
88	...	...	...	9	6·3	4540·3	14·9
89	...	...	...	1	5·4	4545·7	9·5
90	...	...	4555·	4	3·	4548·7	6·5
91	...	...	...	2	2·6	4551·1	3·II
92	...	...	...	2	1·6	4552·7	2·5
93	...	...	...	...	1·5	4554·	1·
94	...	...	...	...	...	4554·	1·
95	...	...	4555·	...	...	4554·	1·
96	...	...	4555·	1	·2	4554·2	·10
97	...	...	4555·	...	·10	4555·	...

Exactly the same process would be performed with the cards of those who have withdrawn from the risk during the course of the observations which need not be repeated.

The cards of the Died would be treated differently in the year of death. Since we are determining the rate of mortality per annum, the Died would be assumed to be on the risk up to the end of the year of age in which they died. That is, they would be taken off the risk then, being careful, however, to remember that they would be on the risk during that year. An extract from the calculations for the same lives will illustrate the point.

Age.	Number of Entrants.	Entrants reallocated to exact ages at entry.	Progressive sum of these Entrants.	Number dying.	Progressive sum of number of deaths.	Years of life exposed to Mortality.
14	4	·6	·6	...	...	·6
15	4	4·4	4·10	...	...	4·10
16	5	4·2	9·	...	...	9·
17	6	6·7	15·7	...	...	15·7
18	10	8·4	23·11	1·	1	22·II
19	26	13·11	37·10	...	1	36·10
20	36	34·9	72·7	1·	2	70·7
21	73	56·	128·7	...	2	126·7
22			208·6	4·	6	202·6
23			321·5	6·	12	309·5
24			484·10	3·	15	469·10
25			688·7	14·	29	659·7 etc.

And now finally I produce the total number exposed to Mortality from the three Sections—the Living—the Withdrawn—and the Died.

YEARS OF LIFE EXPOSED TO MORTALITY.

Age.	From the Living.	From the Withdrawn	From the Died.	Total.	Deaths.
9	...	·8	...	·8	...
10	...	1·	...	1·	...
11	..	1·	...	1·	...
12	...	1·	...	1·	...
13	...	2·4	...	2·4	...
14	·3	3·5	·6	4·2	...
15	5·2	4·4	4·10	14·4	...
16	10·10	10·9	9·	30·7	...
17	24·	17·1	15·7	56·8	1
18	39·9	31·10	22·11	94·6	...
19	74·1	60·3	36·10	171·2	1
20	129·9	96·8	70·7	297·	...
21	230·8	155·8	126·7	512·11	4
22	372·9	238·1	202·6	813·4	6
23	550·	335·1	309·5	1194·6	3
24	767·8	436·9	469·10	1674·3	16
25	1025·7	560·11	659·7	2246·1	14
26	1288·2	660·8	882·8	2831·6	15
27	1579·11	771·1	1124·9	3475·9	27
28	1879·5	875·3	1388·7	4143·3	23
29	2192·7	964·	1693·5	4850·	29
30	2492·1	1044·10	2019·1	5556·	44
31	2784·8	1129·10	2286·7	6201·1	40
32	3018·10	1176·5	2583·7	6778·10	47
33	3220·10	1211·5	2860·9	7293·	59
34	3408·2	1240·6	3129·9	7778·5	57
35	3603·11	1284·11	3403·1	8291·11	70
36	3755·5	1282·	3645·5	8682·10	64
37	3869·3	1278·5	3893·9	9041·5	88
38	4001·	1267·7	4079·	9347·7	79
39	4105·2	1244·5	4276·6	9626·1	79
40	4195·11	1215·7	4489·4	9900·10	80
41	4269·	1170·8	4623·3	10062·11	96
42	4322·8	1146·11	4753·8	10223·3	97
43	4361·9	1097·4	4853·6	10312·7	99
44	4404·2	1040·7	4935·6	10380·3	114
45	4424·5	969·8	4997·10	10391·11	111

YEARS OF LIFE EXPOSED TO MORTALITY—*continued.*

Age.	From the Living.	From the Withdrawn.	From the Died.	Total.	Deaths.
46	4436·	918·6	5030·1	10384·7	103
47	4433·	861·11	5040·5	10335·4	124
48	4401·6	800·1	5031·1	10232·8	121
49	4369·10	740·8	5001·2	10111·8	111
50	4316·4	688·11	4974·9	9980·	154
51	4241·3	624·3	4889·5	9754·11	162
52	4133·3	566·7	4787·1	9486·11	158
53	4020·	512·2	4678·2	9210·4	147
54	3884·1	455·10	4587·3	8927·2	158
55	3733·1	405·11	4479·8	8618·8	170
56	3557·7	359·1	4339·9	8256·5	166
57	3368·	326·8	4212·7	7907·3	152
58	3177·11	289·10	4097·7	7565·4	165
59	3003·9	261·5	3960·1	7225·3	177
60	2828·6	232·9	3800·7	6861·10	203
61	2644·8	206·7	3612·2	6463·5	191
62	2468·2	190·1	3438·6	6096·9	189
63	2289·3	167·1	3257·3	5713·7	204
64	2119·8	141·10	3061·	5322·6	197
65	1927·11	124·10	2874·1	4926·10	197
66	1752·	106·6	2684·	4542·6	183
67	1574·3	87·10	2505·8	4167·9	177
68	1409·8	75·	2331·8	3816·4	196
69	1248·1	62·4	2140·1	3450·6	172
70	1092·8	53·	1972·6	3118·2	192
71	958·1	44·3	1781·2	2783·6	182
72	846·10	36·1	1601·11	2484·10	190
73	725·10	33·5	1314·9	2074·	167
74	625·3	29·2	1250·	1904·5	168
75	528·1	23·8	1082·	1633·9	169
76	445·2	21·7	913·	1379·9	136
77	355·11	17·5	777·	1150·4	132
78	294·11	13·1	645·	953·	102
79	234·	10·2	543·8	787·10	110
80	179·11	9·7	434·	623·6	92
81	141·6	9·	342·	492·6	78
82	108·6	8·	264·	380·6	46
83	84·11	6·9	218·	309·8	53
84	66·3	5·	165·	236·3	36
85	47·2	3·1	129·	179·3	37
86	32·8	2·8	92·	127·4	27
87	21·	1·3	65·	87·3	18

YEARS OF LIFE EXPOSED TO MORTALITY—*continued.*

Age.	From the Living.	From the Withdrawn.	From the Died.	Total.	Deaths.
88	14·9	·11	47·	62·8	17
89	9·5	...	30·	39·5	10
90	6·5	...	20	26·5	8
91	3·11	...	12·	15·11	4
92	2·5	...	8·	10·5	3
93	1·	...	5·	6·	2
94	1·	...	3·	4·	2
95	1·	...	1·	2·	...
96	·10	...	1·	1·10	...
97	...	...	1·	1·	1
	152,575·	33,563·8	176,380·3	362,518·11	7322

The foregoing method of ascertaining the years of life exposed to Mortality is strictly correct, and, when properly gone about, is not attended with any greater trouble than the methods where integral years only are adopted. One of these methods is to assume that the life entered at his nearest birthday and thereafter to follow the exposure through policy years. This method assumes policy years to be all integral years, which they certainly are not. Many policy years are in half-years or in quarter-years. Many of those that are yearly are withdrawn from risk at intermediate periods of the year. It occurred to me to test whether this method tends to gravitate towards one side. If all the lives entered at intervals under six months of their ages at entry and they were assumed to have entered at their nearest integral age, the integral ages at entry would all be *understated*. If all the lives entered at intervals over the six months of their ages at entry, and were taken to have entered at their next nearest birthday, the ages would all be *overstated*. And if those who entered under $x + \frac{6}{12}$ were taken as of age x , and those who entered over $x + \frac{6}{12}$ were taken as of $x + 1$, and if all who entered at exactly $x + \frac{6}{12}$ were taken, one half at age x , and the other half at age $x + 1$, probably a sufficient adjustment may have been made. From the jotting made with the materials in my possession, I formed the opinion that the above method of integral years slightly overstates the rate of mortality. This method has also been officially pronounced unsuitable for the experience of Endowment Assurances, which alone is a sufficient condemnation of the method. When applied to Sickness experience the method is exceedingly interesting.

Before referring to the methods followed in producing the Seventeen Offices and the H^M tables, allow me to show you a method

of tabulation, when integral ages are employed, of exhibiting the years of life exposed to mortality of the three classes separately, the Living, the Discontinued, and the Died, which was followed in the development of the experience of the Scotch Offices in 1863.

I simply ran up "broad sheet" tables of the entrants and the exits of each class. Thus:

I. THE EXISTING.

Age at Entry.	Ages at Exit											Total No. of Entrants of the Living.
	1	2	3	4	5	6	7	8	9	10	11 etc.	
1	5	3	1	3	1	3	2	1	...	2	etc.	28
2	5	...	1	1	1	1	...	2	3	1		16
3	5	3	...	1	...	2	...	1	...	...		9
4	5	3	2	2	3	...	...	1	3	...		12
5	5	3	2	1	1	1	1	1	1	...		16
6	5	3	7	3	1	...	1	...	1	...		17
7	5	3	6	6	3	2	2	1	1	...		24
8	5	3	10	7	3	2	2	1	3	1		35
9	5	3	7	7	7	1	3	1	3	etc.		37
10	5	3	9	9	9	14	4	etc.	12	etc.		46

II. THE DISCONTINUED.

Age at Entry.	Ages at Exit.										Total No. of Entrants of the Discon- tinued.
	1	2	3	4	5	6	7	8	9	10 etc.	
1	...	2	1	2	...	...	2	2	2	...	14
2	...	...	1	...	...	...	...	1	...	...	7
3	...	...	4	1	...	...	1	...	1		12
4	...	2	...	...	...	1	1	...	...		7
5	...	2	2	...	3	...	2	1	...		13
6	...	6	1	...	2	1	3	...	...		17
7	...	1	...	...	...	4	3	1	...		25
8	...	3	...	...	...	...	2	2	...		24
9	...	5	...	...	...	12	...	2	...		44
10	...	11	...	etc.	...	6	etc.	...	...		48

III. THE DIED.

Age at Entry.	Ages at Exit.											Total Entrants of the Died.
	1	2	3	4	5	6	7	8	9	10	etc.	
1	...	...	...	...	...	...	...	...	...	...		...
2		...	...	...	...	...	...	...	...	...		...
3			1	...	...	...	...	...	...	...		1
4			1	...	...	...	...	...	...	...		1
5			1		...	...	...	...	...	...		1
6						1	1	...	...	...		3
7						1	...	...	...	...		1
8						1	1	...	...	...		2
9							1		...	1		5
												5

It will be observed that by this process only one numbering or counting of the cards is required—viz. those who exit. The whole of the entrants at each age in each section are gathered together without being numbered, these are then separated according to their ages at exit and numbered—the horizontal summation producing the number of entrants at each age, and the perpendicular summations the number of exitants of each age. When all these numbers are corrected for the half-year of entry, and the discontinued are corrected for the half-year of exit, the following would be the years of life exposed to mortality by each of these sections.

Age.	Years exposed to Mortality by the			Total years of life exposed to Mortality.	Deaths.
	Living.	Withdrawn.	Died.		
1	14.	7.	...	21.	...
2	31.	16.5	...	47.5	...
3	40.5	24.	5	65.	1
4	49.	29.5	.5	79.	...
5	56.	36.	1.5	93.5	...
6	66.5	49.	3.5	119.	1
7	77.	66	4.5	147.5	1
8	99.5	82.	5.	186.5	...
9	126.5	104.5	8.5	239.5	...
10	154.	142.	13.5	309.5	1
	etc.	etc.	etc.	etc.	
Totals,	761,118.	162,211.	277,071.5	1,200,400.5	20521

The total coincides with the figures given on page 24 of the H^M Mortality Experience.

I now call your attention to the method followed in developing the H^M Experience and in obtaining the numbers exposed to the risk of Mortality in each year of age. It is described on page 18 of the preface to the first volume. As a reference is there made to the Tables of Observations I give the observations with reference to the current age at entry of 30.

Current age at Entry.

30

Total Entrants.

5791

Current age at Exit.	Existing.	Discontinued.	Died.
30	319	75	4
31	252	365	28
32	230	220	35
33	235	153	49
etc.	etc.	etc.	etc.
	3677	1491	623
	5791		

It was assumed, as stated on page 3, that lives entering at these current ages would attain to those ages, as their actual age, at the end of their years of entry. All lives thus entered are six months younger than their real age, and only attained that age, as their actual age, at the end of the year in which they entered.

‘The number exposed to risk was obtained in the first year of assurance by taking half the number who entered in the year of age less half the number who discontinued in the same year. In the second year of assurance it was found by deducting from the number who entered on the second year of assurance half the number who discontinued during that year. Thus in Table of Observations in current age 30 :—

1. *Number exposed during first year—*

The total number of entrants was 5791 half of which
= 2895.5

Discontinued the first year 75 half of which
= 37.5

Number exposed to risk in the first year, — 2858

2. *Number exposed during the second year—*

Number who entered on the first year,
5791

Deduct those leaving during the
first year

Existing, . . . 319

Discontinued, . . . 75

Died, . . . 4

— 398

Leaving the number who entered
on the second year, . . . 5393

Deduct half of the Discontinued
in the second year, . . . 182.5

Number exposed to risk in the second year, . . . = 5210.5

3. *Number exposed during the third year—*

Number who entered on the
second year, . . . 5393

Deduct those leaving during the
second year,

Existing, . . . 252

Discontinued, . . . 365

Died, . . . 28

— 645

Leaving the number who entered
on third year, . . . 4748

Deduct half of the Discontinued
in third year, . . . 110

Number exposed to risk in the third year, . . . = 4638

I have taken the liberty of amplifying the expressions by adding words that seemed to me required to make the meaning more apparent.

It will be seen that the number exposed to risk during six months of the first year of assurance is half of the number (5791) who entered upon that year less half of the number (75) who discontinued in that year, that is to say the number 2858 so computed, is assumed to be exposed to risk for twelve months and of whom 4 died. The number exposed to risk during the second year of assurance is the number 5393 who entered upon the second year, less half of those (365) who discontinued in that year. This

number (5210·5) is assumed to have been on the risk for twelve months, and of whom 28 died, and so on for other years.

That is to say, the Mortality is from birthday to birthday, but the first year's rate of mortality is the ratio of the total deaths during the average period of six months to the number exposed during that same average period of six months. The method proposed by me is to ascertain the total years of life exposed to risk from the exact age at entry expressed in years and months up to the attainment of the end of that year of age, or up to the date of withdrawal in that year of age. The ratio of the total deaths in that period to the total years of life exposed is the rate of mortality in the first year of life after entry. For those who die before completing to the end of that year of age, the period of exposure will not be the fractional period of the year, but twelve months experience will be set apart for them—seeing that we are calculating the rate of mortality for “year-units.”

Another method of arriving at the same results as those of the H^M table consists of a slightly different arrangement—but which appears to me to be much more expeditious. It is the method followed by me in 1863 as you may observe from the volume laid before you.

Current age at entry 30.					Total of these Numbers W _{x+n} D _{x+n} L _{x+n}	Sum of these (upwards)	Half of the number en- tering less half the Withdrawn.	Number exposed to Mortality.
Current age at- tained in year of exit.	Total number of lives entered on risk.	Of whom						
		With- drawn.	Died.	Living at close of Observations.				
		At age in col. 1.						
<i>x+n</i>		W _{<i>x+n</i>}	D _{<i>x+n</i>}	L _{<i>x+n</i>}				
30	5791	75	4	319	398	...	2858·	2858·
31	...	365	28	252	645	5393	182·5	5210·5
32	...	220	35	230	485	4748	110·	4638·
33	...	153	49	235	437	4263	76·5	4186·5
34	...	147	51	198	396	3826	73·5	3752·5
*		*	*	*	*	*	*	
	Sums	1491	623	3677	5791			etc. as in page 147

There is still another arrangement of the data of Entrants of each age, and which affords information more closely related to the method I have suggested. It ascertains the quantum of the years of life derived from the three sections—the Withdrawn, the Died, and the Living. It embraces :—

- 1st. The years of life exposed from the Withdrawn,
- 2nd. The years of life exposed from the Living,
- 3rd. The years of life exposed from the Died, and finally,
- 4th. The total years of life.

But this method can only be obtained from the H^M experience by a re-arrangement of all the statistics.

These examples refer to the entrants at any one age—and the table showing the experience of the whole number may either be derived from the addition of the sectional experiences. Thus:—

	30	31	32	33	34	35
30	2858·4	5210·528	4638·35			
31		2438·10	4492·36			
32			2345·12			
			etc.			
33						
	27112·5(224)	29213·(215)	31232·(260)			

as in the H^M table, or as in the MS. volume before you, or the aggregate may be derived at once as follows:—

(The figures of the following illustrations as taken from some of the Scottish Experience in 1863.)

AGGREGATE NUMBER EXPOSED TO RISK OF MORTALITY AT EACH YEAR OF LIFE.

Current Age.	Entrants.	Exitants.			Sum of these.	Total Entrants at all younger ages.	Total Exitants at all younger ages.	Differ-ence.	Allowance of half year for those entering and those Withdrawing at each age.	NUMBER EXPOSED TO RISK OF MORTALITY	Completed age.
		With-drawn.	Died.	Living.							
1	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...
4	2	...	...	...	...	...	...	...	1·	1·	3
5	4	...	...	1	1	2	...	2	2·	4·	4
6	8	...	...	2	2	6	1	5	4·	9·	5
7	11	...	...	3	3	14	3	11	5·5	16·5	6
8	17	2	...	3	5	25	6	19	7·5	26·5	7
9	25	2	...	2	4	42	11	31	11·5	42·5	8
10	31	...	...	3	3	67	15	52	15·5	67·5	9
11	34	3	1	5	9	98	18	80	15·5	95·5	10
21	875	76	13	213	302	1962	601	1361	399·5	1760·5	20
31	3169	492	138	1255	1885	26086	11059	15027	1338·5	16365·5	30
41	2015	402	224	1671	2297	53354	32648	21206	806·5	22012·5	40
51	726	169	235	1225	1629	67541	53285	14256	278·5	14534·5	50
61	140	51	190	619	860	71875	65588	6287	44·5	6331·5	60
71	8	6	93	160	259	72505	70866	1639	1·	1640·	70
81	...	...	28	225	53	72549	72323	226	...	226·	80
	...	...	1	...	1	72549	72544	5	...	5·	90
	72549	12421	8810	51318	72549					627845·	

A very interesting and independent check upon the correctness of the total number exposed may be obtained by ascertaining the total of the ages at entry of the three classes separately, and also the ages at exit. Thus I take from the figures of the Males with Profits of the Scottish Experience in 1863 :—

I. LIVING.—

Sum of the 51318 ages at entry	=1,773,792	
" " exit	=2,220,521	
Difference—years of life exposed	<u> </u>	=446,729

II. WITHDRAWN.—

Sum of the 12421 ages at entry	=	416,327	
" " exit	=	471,742	
Difference—years of life exposed	—	55,415	

III. DIED.—

Sum of the 8816 ages at entry	=	351,067	
" " exit	=	446,704	
Difference—years of life exposed		<u> </u>	= 95,637

597,781

Add—Half the difference between the total Entrants	=	72,549	
And the Withdrawn	=	12,421	
		<hr/>	
		2) 60,128	= 30,064

Total years of life—which coincides with the Total years when computed in the ordinary way (as in the previous page) = 627,845

and consequently we have

Average age at entry of the Living	34·565	
„ exit „	43·770	
Average term exposed of the Living	— — —	9·205
Average age at entry of the Withdrawn	33·518	
„ exit „	37·979	
Average term exposed of the With- drawn	— — —	4·461
Average age at entry of the Died	39·849	
„ exit „	51·204	
Average term exposed of the Died	— — —	11,355
Average age at entry of the whole number	$\frac{2,541,186}{72,555}$	= 35·024
Average age at exit of the whole number	$\frac{3,138,967}{72,555}$	= 43·264
Average duration of the whole risks		<u> = 8·240</u>

These very interesting facts are not usually supplied at Mortality Investigations. Indeed, there was no attempt whatever made to obtain them at either the experience of the Seventeen Offices or that of the H^M. table. It seems to me that they furnish a key to the maturity and to the trustworthiness of any experience, and when contrasted with the like results of other Investigations they afford a fair measure of their relative significance and weight.

We have now arrived at that very interesting stage of being able to determine the rate of mortality experience in each year of life. It is the ratio of the number dying in a year to the number of lives exposed in the same year—thus H^M table, p. 244 and p. 273 :—

Age.	Number exposed to risk.	Number of Deaths.	Rate of Mortality.
10	379·	3	·00792
11	434·	...	·00000·
12	491·5	2	·00407
13	578·	2	·00346
14	731·	3	·00410
15	908·	2	·00220
16	1129·	...	·00000
17	1421·	6	·00422
18	1810·5	11	·00608
19	2414·	17	·00704

and the number alive at the end of each year in a Table of Mortality is arrived at by the following additional columns :—

Age.	Probability of living a year p_x .	Log. of p_x .	Log. of Number living.	Number Living.	Deaths.
10	·99208	1·99655	4·00000	10000	79
11	1·00000	·00000	3·99655	9921	...
12	·99593	1·99823	·99655	9921	40
13	·99654	·99849	·99478	9881	35
14	·99590	·99822	·99327	9846	40
15	·99780		·99149	9806	22
16	1·00000		·99053	9784	...
17	·99578			9784	41
18	·99392			9743	59
19	·99296			9684	

A much more direct method of obtaining the number living in a Table of Mortality is the following:—

Age.	Number exposed to Mortality.	Number surviving the year.	Log. A.	Log. B.	Log. of p_x ($\pounds.B - \pounds.A$).	Log. l_x .	l_x .	$l_x - l_{x+1}$.
	A	B						
10	379.	376.	5786392	5751878	9965486	5.0000000	100000	792
11	434.	434.	6374897	6374897	0000000	4.9965486	99208	000
12	491.5	489.5	6915235	6897527	9982292	9965486	99208	403
13	578.	576.	7619278	7604225	9984947	9947778	98805	342
14	731.	728.	8639174	8621314	9982140	9932725	98463	404
15	908.	906.	9580858	9571282	9990424	9914865	98059	216
16	1129.	1129.	0526939	0526939	0000000	9905289	97843	000
17	1421.	1415.	1525941	1507564	9981623	9905289	97843	413
18	1810.5	1799.5	2577985	2551519	9973534	9886912	97430	592
19	2414.	2397.	3827373	3796680	9969307	9860446	96838	
20 etc.	Sums	and check	£3374072	£3203825	£9829753	9829753 etc.	96156	

The last two columns form the Table of Mortality of which it has been the object of this paper to describe the construction. That table embraces the probabilities of dying in each year of life derived from the several statistics applicable to these ages. But, by being continuous, that is, by making the number surviving one age to form the entrants upon the following age, Actuaries are enabled to ascertain, by one continuous stream of calculation, the number of years which persons of any age are likely to live, or the probability of one person of any age living to any other age, or the probability of one life of one age surviving another life of a different age, and many other complicate and delicate combinations of life and of death. When combined with the discount of money they are able to ascertain the capital sum which will provide an annuity during a lifetime, and the present value of a capital sum payable on surviving a given age, or on dying before attaining to that age, and thus to conduct the business of life assurance and annuities, which now constitutes so large a portion of the wealth of all well-conducted communities.

It only remains to be graduated from age to age, without interfering with its character, before being ready for practical use.

A check upon the correctness of the operations by summations is shown, which, in continuous calculations, is exceedingly valuable.

The rate of mortality experienced during the year in which the lives were entering, at, say, equal intervals throughout the year, was, in the H^M table, determined by assuming that the whole number of entrants in that year were at risk for half a year—or what is very much the same thing, by assuming that half the entrants were at risk for the whole year, and that the quotient resulting from a division of the number who died in that semi-twelve months by that number of entrants was the rate of mortality for the first year. The period was really only six months; but the deaths and the years of life for the six months were so adjusted to each other as to give the probability of dying during twelve months. When put in the form of a fraction

$$\begin{aligned}
 q_{x:1} &= \frac{\text{Number dying during 6 months.}}{\text{Years of life exposed during 6 months.}} \\
 &= \frac{\text{Number dying during 6 months.}}{\text{Half the years of life exposed during 12 months.}} \\
 &\stackrel{\text{Multiplying numerator and denominator by 2.}}{=} \frac{\text{Double the number dying during 6 months.}}{\text{Years of life exposed during 12 months.}}
 \end{aligned}$$

There would not have been much objection to this assumption, for we have all been trained to believe that the value of a fraction is unaltered when numerator and denominator are multiplied by the same quantity.

I have however deduced the rate of mortality on this changed hypothesis, *i.e.* I have doubled the deaths in the first year, and I have not halved the number who entered upon the year. I have also calculated the number remaining alive at the end of each year out of a radix of 100,000,—ascertained the number who die at each year of age—graduated that number by Woolhouse's method, and I now exhibit the difference between the number of deaths according to the two hypotheses, from which it will be seen (barring ages 10, 11, 12, 13) that fewer die up to age 54, and more at older ages,—a character of table that exacts smaller premiums at ages under 54, and larger premiums thereafter—and that requires larger values of policies all through; a table of mortality essentially different from the H^M table.

DISTRIBUTION OF THE 100000 DEATHS BY THE TWO TABLES.

	Number of deaths.		Difference	
	Table based upon doubling the deaths. A	Table based upon halving the number exposed. B.	A greatest.	B greatest.
10-14	1991	1776	215	...
15-19	1824	2001	...	177
20-24	2852	3162	...	310
25-29	2975	3196	...	221
30-34	3493	3584	...	91
35-39	3911	3997	...	86
40-44	4290	4365	...	75
45-49	5161	5193	...	32
50-54	6193	6213	...	20
55-59	7669	7647	22	...
60-64	9608	9569	39	...
65-69	11270	11173	97	...
70-74	12611	12433	178	...
75-79	11943	11761	182	...
80-84	8672	8508	164	...
85-89	4034	3962	72	...
90-94	1346	1325	21	...
95, etc.	157	135	22	...
	100000	100000	1012	1012

You will thus observe that it is a very serious business tampering with a table of mortality. The change illustrates the superior advantages of my suggested method of finding the number exposed by taking the difference between the exact age at entry in years and months, and the exact age at exit in years and months. In many minor ways, that I can scarcely find language sufficiently exact to express, I observe its advantages.

Another example of tampering with numbers exposed may be found in the table of Seventeen Offices' Experience. It is stated that some of the experiences were multiplied by four, and that that four-folded experience was added to the others before deducing the general results. The grounds for the alteration are not given with such exactness as to enable me to re-explain them. It does not seem incorrect to multiply to any extent you like the numerator and denominator of the fraction that expresses the rate of mortality—the resulting quotient is unaffected by that multiplication. But when we have any total experience divided or separated into sections, and the total of these sections is the result to be aimed at, it does not appear to me to be correct to intensify any one section and leave the others alone, before finding the average of the aggregate. If we have three separate experiences:—

$$\begin{array}{cccc} \text{Office A.} & \text{Office B.} & \text{Office C.} & \text{Aggregate.} \\ q_x = \frac{1}{100} + \frac{1}{90} + \frac{1}{80} & = & \frac{3}{270} = \frac{1}{90} \end{array}$$

and if the experience of Office *A* is multiplied by 4 and the others left alone, the aggregate experience becomes less than $\frac{1}{90}$.

$$\text{Thus} \quad q_x = \frac{4}{400} + \frac{1}{90} + \frac{1}{80} = \frac{6}{570} = \frac{1}{95}$$

It seems clear that the multiplication of the experience of Office *A*—even although both numerator and denominator be multiplied by the same quantity—intensifies the aggregate result in the direction of the experience of Office *A*.

I do not enlarge any more on this point at present, further than to introduce to your notice the problem that I hope to have the pleasure of discussing another evening. I lay before you the complete observations upon which the H^M table was founded. There are the numbers exposed, emanating from the age at entry and passing through each subsequent age. The total deaths in each year divided by the totals passing through the same age gives the rate of mortality at each age. Thus, taking age 40, the complete observations are:—

				Number exposed.	Deaths.	Rate of Mortality.
Entrants from age 5 passing through age 40				1·	...	·000000
”	7	”	”	1·	...	·000000
”	8	”	”	1·	...	·000000
”	9	”	”	3·	...	·000000
”	10	”	”	7·	...	·000000
”	11	”	”	3·	...	·000000
”	12	”	”	5·	...	·000000
”	13	”	”	15·	...	·000000
”	14	”	”	16·	...	·000000
”	15	”	”	18·	...	·000000
”	16	”	”	22·	..	·000000
”	17	”	”	26·	...	·000000
”	18	”	”	31·	...	·000000
”	19	”	”	46·	1	·021739
”	20	”	”	85·5	...	·000000
”	21	”	”	188·5	1	·005305
”	22	”	”	286·5	4	·013962
”	23	”	”	416·	4	·009615
”	24	”	”	657·	9	·013698
”	25	”	”	806·5	10	·012399
”	26	”	”	1013·	13	·012833
”	27	”	”	1281·	26	·020297
”	28	”	”	1506·5	15	·009957
”	29	”	”	1921·	16	·008329
”	30	”	”	1937·	20	·010325
”	31	”	”	2172·	24	·011050
”	32	”	”	2475·5	24	·009695
”	33	”	”	2729·	33	·012092
”	34	”	”	2817·	31	·011005
”	35	”	”	2968·5	37	·012464
”	36	”	”	2828·	29	·010255
”	37	”	”	3227·	29	·008287
”	38	”	”	3182·5	21	·006599
”	39	”	”	3740·	21	·005615
”	40	”	”	1762·	9	·005108
				38195·	377	(·009870)
Average, $\frac{·230629}{35} = \underline{\underline{·006589}}$						

Of all these various rates of mortality of persons passing through the year of life 40–41, the question I have propounded to myself is—Which is the most probable? I do not think it is the $\frac{377}{38195} = \cdot009870$, because, however much the deaths and exposures be interchanged, the above result would remain the same. Thus, if the deaths among the last twenty-five exposures were reversed, the summations would still remain at $\frac{377}{38195}$, but to my mind the most probable rate of mortality would in these changed circumstances be very different. It may be that it is the mere numerical magnitude of the observations that affects the result. It may be that it is the force of selection that lessens the

average and creates the most probable. It may be that I have imported a small measure of heresy into the solution of the problem. But in a table which is to embody the several experiences and incorporate them into one average rate, for such a "going concern" as an Insurance Office involving new insurances as well as old insurances, it seems to me that due weight should be given to the magnitude of the experiences—the numbers exposed and the number of their deaths—and that we should not take the mere summations as the only factors for determining the rate of mortality in an aggregate table.

To this problem I hope to have the pleasure of drawing your attention on a subsequent evening.