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THE REFLECTION OF SLOW-MOVING ELECTRONS BY
COPPER.

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INTRODUCTION.

THE following is a report of experiments on the reflection by a rough copper surface—electrons having velocities from 0.4 volt to 50 volts. The copper was originally polished, but when taken out at the end of the experiment had a dull, smooth *gray* appearance, due probably to a deposit on it of copper distilled from hotter parts of the apparatus during the prolonged heating. This condition of the surface is to be especially borne in mind in interpreting the results.

The most striking result of the experiment is the smallness of the reflection observed—only 10 per cent. at 0.4 volt velocity, increasing to 40 per cent. at 50 volts velocity. Interesting evidence is also obtained regarding the cause of the spreading of a beam of slow-moving electrons, and the energy relations in the production of delta rays.

PREVIOUS EXPERIMENTS.

The best values of reflection at low velocities are those of von Baeyer¹ and Gehrts.² Their results may be summarized as follows: At about 2 volts velocity, the fraction of incident electrons reflected varies from 35 to 50 per cent., depending on the condition of the surface, rather than the material, of the reflector. The reflection increases with increase of velocity up to about 5 volts velocity, then decreases with further increase of velocity to 11 volts velocity, after which it increases rapidly and reaches 100 per cent. at about 30 volts velocity. At velocities above 30 volts more electrons leave the receiving plate than strike it. Hence in addition to reflection there must be a secondary emission of electrons by the bombarded plate, and this secondary emission is assumed to

¹ Ber. d. D. Phys. Ges., 10, 96–111, and 953–967, 1910.

² Ann. d. Phys., 36, 1000, 1911.

begin at 11 volts, the point where rapid increase in the apparent reflection begins.

The picture would be much simpler if, instead of an initial increase in reflection with increasing velocity, followed first by a decrease and then an increase, a constant decrease from the lowest velocities up to eleven volts had been observed, with subsequent increase. For the process could then be considered as the superposition of two, viz., a reflection, which is large for low velocities and decreases continually with increasing velocity, and a secondary emission which begins at about 11 volts and increases continually with the velocity. This view of the process von Baeyer actually adopts, considering, with good reason, that the smaller values of reflection which he observed at velocities below 8 volts could be explained by the poor vacuum which he had. Gehrts follows von Baeyer's example in drawing the same conclusion and discrediting his own data at low velocities, but with less justification, as his vacuum seems to have been excellent.

Some time ago the writer¹ had occasion, in the course of another research, to measure the reflection from a silver-plated brass surface, of photo-electrons produced by Shumann light, the velocities of which lie between 0 and 3 volts. The result was a surprise, as the reflection was found to be zero within the limits of measurement. Here again, however, it must be remembered that the surface was in a peculiar condition, namely that of an electrolytic deposit. For although the brass was

Fig. 1.

highly polished before being plated, and the silver plating was carried on very slowly, so that a fine mirror surface resulted, yet this surface was purposely not polished, and all electrolytic surfaces, however fine-grained, are undoubtedly granular. Such a surface, being composed of little Faraday cages, may reflect very differently from a polished or burnished surface.

The method used in this experiment may be briefly stated here, as it

¹ PHYS. REV., 35, 400, 1912.

differs from the methods that have been used by other experimenters and is free from the objection to which these methods are open.

An aluminum plate (Fig. 1) P , 4 cm. in diameter is supported by amber in a glass tube 5 cm. in diameter and can be illuminated by a carefully diaphragmed narrow beam of ultra-violet light through a fluorite window F , the reflected light passing out freely through a side tube T . Opposite P is the "reflector" R , in this case a silver plated brass disc, which is movable along the tube by means of a magnet. The inside of the tube is lined with copper foil covered by fine copper gauze, both being blackened with lamp black to diminish reflection.

When the reflector R is 4 mm. from P (in which position no light falls on it) all the electrons from P will strike R , since the beam of light is only 2 mm. in diameter and strikes accurately the center of P . Part of these will be reflected back to P , and after multiple reflections, assuming none escape to the cylinder walls, the net loss by P will be $N((1 - r)/(1 - rs))$ where r is the coefficient of reflection of the reflector R , *i. e.*, the fraction of the electrons striking it which it reflects back to P , s that of the plate P , and N the total number of electrons leaving P in the first place. Where R is drawn back to the farther end of the cylinder, 15 cm. from P , only a negligible fraction of the N electrons return to P . The number escaping is N . Hence if s is very small, as has usually been found true for aluminum, the ratio of these currents from P is $1 - r$; if s and r are equal the ratio is $1/(1 + r)$.

Since P , R and the black coating on the cylinder walls were all of different material, it was necessary to compensate their contact difference of potential. This was determined by the Kelvin capacity method, as follows: When R was only 1 or 2 mm. from P the capacity of P was influenced only by R . By small movements of R it was easy to determine within .01 volt the potential it must have in order that the electrometer connected to P should not be disturbed. If R was now moved back several centimeters the electrometer was again disturbed, since P was no longer screened by R , and its capacity was influenced by the blackened walls of the cylinder. Hence by *large* movements of R it was possible to ascertain when the cylinder was at the same potential as P , and this adjustment could be made within about 0.1 volt.

When the potentials of the three surfaces had been thus equalized, it was found that the electron current from P was the same whether R was near or far away, in other words, there was no reflection.

As a check upon the adjustment of the potentials, the whole enclosure was coated with silver by making the silver reflector cathode in a high tension discharge for two hours. A determination of the surface poten-

tials by the method given above showed that they were all equal within 0.1 volt. Measurements on reflection again gave the same result, the current from P was independent of the position of R . In this case, since both surfaces were silver, we have $r = s$, and the ratio of the currents from P when R is near and far away respectively should be $1/(1 + r)$. The value actually found differed from unity by less than 1 per cent.

APPARATUS.

In the construction of apparatus two objects were especially borne in mind: (1) To obtain a beam of electrons as nearly as possible homogeneous with respect to speed and direction. The admixture in the beam of reflected and secondary electrons from the walls or edges of diaphragms was most to be feared, but it was also anticipated that the

Fig. 2.

shape of the electric field would have a marked effect. To avoid the first of these errors, sharp edged diaphragms (D , Fig. 2) in a wide tube were used instead of a narrow tube to limit the beam; to study the second a "focus plate" (F , Fig. 2) was placed between the filament and diaphragms. By charging this plate to different potentials, both positive and negative, the electric field near the diaphragms could be varied over a wide range. A copper ring r , concentric with the face of the filament and connected to its negative terminal, improved the focusing and enabled large currents to be obtained even at low voltages. (2) To avoid all substances capable of yielding condensible vapors or oils into the vacuum. For such oily films not only change the reflecting surface from metal to oil, thereby changing entirely its reflecting power, as Norman Campbell¹ has clearly demonstrated, but when bombarded by electrons these surfaces assume potentials from 10 to 50 volts above the potential of the metal on which they are deposited.² For this reason the apparatus was constructed entirely of glass and copper, and after being evacuated

¹ Phil. Mag., 28, 286-302, 1914.

² Stuhlmann and Compton, Phys. Rev., 2, 199, 1913; Seeliger, Phys. Z., 14, 1237, 1913.

was surrounded by an electric furnace and heated to 350° C. for forty-eight hours, with the Gaedé pump working continually. A tube immersed in solid carbon di-oxide and ether prevented mercury and oil vapors from the pump and McLeod gauge from diffusing into the apparatus.

The apparatus is shown in Fig. 2. Electrons from a filament f passed through a 6 mm. hole in a copper "focus plate" F , through a series of 4 mm. holes in the copper diaphragms D , into a copper cylinder C , 5 cm. in diameter and 10 cm. long, which was lined with fine copper gauze to decrease the reflection of electrons from it.¹ Here they fell upon the reflector R , a polished² copper disc, 4 cm. in diameter, which was movable along the axis of the cylinder by means of a magnet. A potentiometer P enabled filament and focus plate to be charged to any desired potentials, and galvanometers G_1 and G_2 measured the currents from reflectors and cylinder to earth. For convenience they were adjusted to the same sensitiveness by means of shunt resistance.

METHOD OF EXPERIMENT.

The procedure was to connect the negative end of the filament to a definite negative potential, which is taken as a measure of the velocity of the electrons when they entered the cylinder; charge the focus plate

TABLE I.

Reflection of 8-Volt Electrons.

Potential of focus plate, + 18 volts. Pressure of gas .0006 mm.

Distance of R from End of Cylinder.	i_R Amp. $\times 10^{-7}$.	i_o Amp. $\times 10^{-7}$.	i Total Amp. $\times 10^{-7}$.	Scattering $\frac{i_{R, .1} - i_{R, d}}{i_{R, .1}}$.	Coefficient of Reflection. $\frac{i \text{ Total} - i_{R, .1}}{i \text{ Total}}$.
.1	2.00	.25	2.25	0.0	.23
6.0	1.59	1.01	2.60	.20	
.15	2.00	.27	2.27	0.0	
.25	2.00	.34	2.34	0.0	
.50	2.00	.43	2.43	0.0	
.75	1.98	.51	2.49	.01	
1.00	1.95	.60	2.55	.02	
2.00	1.87	.72	2.59	.07	
3.0	1.79	.83	2.62	.10	
4.0	1.71	.91	2.62	.15	
5.0	1.64	1.00	2.64	.18	
6.0	1.58	1.06	2.64	.21	
.1	2.02	.22	2.24		

¹ Lamp black was avoided on account of contact potential difference.

² At the end of the experiment this disc had a dull "gray" appearance, probably due to a deposit of copper distilled from hotter parts of the apparatus during the long heating.

to a carefully determined potential;¹ and measure the currents from cylinder C and reflector R to earth for a series of positions of R , ranging from 1 mm. to 6 cm. from the end of the cylinder. The difference between the total current $i_{C,d} + i_{R,d}$ at any distance, d , greater than 4 cm., and the current $i_{R,1}$ to R at the distance .1 cm., divided by the total current, gives the fraction of the electrons reflected, or the "coefficient of reflection."² Tables I. and II. give typical sets of readings which are shown graphically in Figs. 3 and 4.

TABLE II.

Reflection of 8-Volt Electrons.

Potential of focus plate, - 7 volts. Pressure of gas .0002 mm.

Distance of R from End of Cylinder.	i_R Amp. $\times 10^{-7}$.	i_C Amp. $\times 10^{-7}$.	i Total Amp. $\times 10^{-7}$.	Scattering $\frac{i_{R,1} - i_{R,d}}{i_{R,1}}$.	Coefficient of Reflection. $\frac{i_{Total} - i_{R,1}}{i_{Total}}$.
.1	1.35	.17	1.52	0.0	.24
6.0	1.20	.58	1.78	.11	
.1	1.35	.18	1.53	0.0	
1.0	1.32	.38	1.70	.02	
2.0	1.30	.45	1.75	.03	
3.0	1.27	.50	1.77	.06	
4.0	1.24	.52	1.76	.08	
5.0	1.21	.56	1.77	.10

Fig. 3.

Fig. 4.

EXPERIMENTAL RESULTS. COEFFICIENT OF REFLECTION.

The final values obtained for the coefficient of reflection are collected in Table II., together with the conditions of experiment. The values

¹ The optimum potential for the focus plate had to be determined by a long series of measurements on the divergence of the electron stream. This is discussed below under "scattering."

² It is here assumed that all the electrons reflected from R in the 1 mm. position escape. The validity of this assumption is discussed below under "scattering."

for velocities below 50 volts were taken at different times and in varying sequences, and represent the maximum variations observed. The low values were invariably observed at the beginning of a series, after the apparatus had been lying idle over night, and are to be attributed to gas adsorbed by the reflecting plate. As this gas is gradually removed by the electron bombardment the reflection increases. This effect, generally much more marked, has been observed by all experimenters in this field. The readings for velocities above 50 volts were taken somewhat hastily, as gas was generally evolved by the metal parts of the apparatus under the bombardment, and no great accuracy is claimed for them. The last

Fig. 5.

Variation of reflection with velocity.

column gives the amount of "scattering" of the beam, *i. e.*, the fraction of it which fails to strike the plate when the latter is 6 cm. from the last diaphragm. Those experiments in which the scattering was small are the more reliable. This is discussed below under "scattering."

The variation of reflection with velocity, as given in Table III., is shown graphically in Fig. 5. The curve is almost identical with that obtained by other investigators for lamp black¹ and platinum black,² and suggests very strongly that the small reflectivity in all these cases is due to the microscopic roughness of the surface. The fact that aluminum reflects less than other metals³ would be accounted for in the same way, and the irrecoverable loss in the reflecting power of a platinum strip by prolonged heating, observed by Campbell,⁴ would be satisfactorily explained as the result of crystallization of the platinum. The small reflectivity of electrolytic silver observed by the writer (p. 2, above) is another instance, and indicates, in harmony with chemical data, the extreme ultra-microscopic roughness of such surfaces compared with burnished surfaces. It is to be anticipated that for receiving

¹ Gehrts, I. c., p. (1005).

² Von Baeyer, I. c., p. (965).

³ Von Baeyer, I. c., p. (965).

⁴ N. Campbell, Phil. Mag., 28, 286, 1914.

TABLE III.

Variation of Reflection with Velocity.

Gas Pressure, Mm.	Velocity in Volts.	Potential of Focus Plate, Volts.	i_R at 1 Cm. Amp. $\times 10^{-8}$.	i (Total) at 6 Cm. Amp. $\times 10^{-8}$.	Coefficient of Reflection.	Scattering at 6 Cm.
.00015	0.4	+ 5	2.93	3.29	.106	
.00013	0.4	+ 5	2.85	3.19	.107	.63
.00015	0.4	+ 5	3.29	3.64	.098	
					Mean .104	
.00013	1.0	+ 5	2.45	2.90	.155	.62
.00035	1.0	+ 5	3.30	3.85	.160	.45
.00035	1.0	+ 5	3.40	4.10	.170	.45
.00038	1.0	+ 3	1.82	2.12	.147	.23
.00038	1.0	+ 3	1.86	2.20	.154	.22
					Mean .157	
.00050	3.0	- 2	3.22	4.17	.225	.37
.00052	3.0	- 2	3.01	3.90	.227	.38
.00052	3.0	- 2	3.10	4.00	.225	.23
					Mean .226	
.00052	5.0	- 3.5	7.96	10.39	.230	.14
.00023	5.0	- 3.5	8.20	10.65	.230	
.00023	5.0	- 4.0	6.35	8.10	.216	.26
.00060	5.0	+12	8.8	11.1	.203	.27
					Mean .220	
.00023	8.0	- 7	8.00	11.23	.26	.13
.00016	8.0	- 7	14.2	18.6	.25	.10
.00018	8.0	- 7	14.1	18.7	.25	.09
.00020	8.0	- 7	13.5	17.8	.245	.11
.00026	8.0	- 7	13.0	16.7	.22	.15
.00060	8.0	- 7	14.9	19.4	.23	.17
.00060	8.0	+18	20.2	26.4	.235	.21
.00060	8.0	+18	20.0	26.0	.230	.20
					Mean .240	
.00018	15	-14	17.0	22.8	.26	.16
.00018	20	-19	18.0	25.7	.28	.17
.00018	30	-29	19.1	29.8	.36	.20
.00018	40	-35	117	185	.37	.26
.00018	50	-43	159	269	.41	.27
.00016	50	0	21.6	36.3	.40	.31
.00016	50	0	21.8	37.9	.42	.35
					Mean .41	
.00016	60	0	53.0	103	.49	.38
.00016	80	0	36.9	74.5	.51	.43
.00016	100	0	20.9	46.1	.55	.47
—	150	0	8.6	27.0	.68	.59
—	150	0	7.8	19.5	.60	.62
					Mean .64	
—	200	0	6.1	14.6	.59	.67
—	200	0	11.2	30.7	.63	.67
					Mean .61	

electrodes in photo-electric and other similar experiments a surface formed by vacuum distillation or a fine-grained electrolytic deposit would be quite as good a "black body" for absorbing electrons as soot or platinum black.

VELOCITY DISTRIBUTION.

It is important to know what fraction of electrons in the beam denominated "1 volt" in the above table actually had that velocity. This was investigated in the usual manner by charging the reflecting plate to different negative potentials and observing how many electrons

Fig. 6.

Velocity distribution of 1-volt electrons.

Fig. 7.

Velocity distribution of 3-volt electrons.

were able to reach it. For most of these measurements the plate was brought close to the end of the cylinder at which the electrons entered (the "1 mm." position). But for low velocities these measurements

Fig. 8.

Velocity distribution of 8-volt electrons.

Fig. 9.

Velocity distribution of 50-volt electrons. Curve I., reflector alone at retarding potential. Curve II., both reflector and cylinder at retarding potential.

were checked by others in which the plate was at the other end of the cylinder, and the total current to plate and cylinder observed. The two methods gave identical results. For high velocities the latter method could not be used on account of the insufficient insulation of the cylinder.

The results are shown in Figs. 6 to 9. The homogeneity of the beams

is very satisfactory, especially at the lower velocities. This is in striking contrast to the velocity distribution (Fig. 12) in the preliminary experiments when mercury vapor (about .001 mm.) was present, which is discussed below under "Delta Rays." The curves for 50-volt electrons are very remarkable. They show the paradoxical fact that a plate upon which electrons fall receives more electrons the more negatively it is charged with respect to its surroundings. The same is evident in Fig. 7 for 3-volt electrons, and is still more strikingly shown in the curves

Fig. 10.

Velocity distribution of 50-volt electrons. Curve I., reflecting plate alone charged to retarding potential; Curve II., both reflector and cylinder at retarding potential.

of Fig. 10, where the plate was at the far end of the cylinder. It is readily explained by the change of "reflection coefficient" with velocity, and is a striking demonstration of the reality of this change. Curve 2, Fig. 10, is especially convincing, as here all influences, including surface leakage over the glass insulation, which becomes considerable at high negative potentials of the plate, combine to decrease the number of electrons striking the plate. Yet when the plate is 47 volts negative it still receives more electrons than when it is at zero potential.

SCATTERING.

The method employed in these experiments assumes a nearly cylindrical beam, and it is important to know to what extent this condition was fulfilled. If the electrons move in straight lines after entering the system of diaphragms the cone of rays emerging should have an angle of only 11° . Its maximum diameter, after travelling 6 cm. in the chamber, would be 1.5 cm. Thus the whole beam ought to strike the 4 cm. plate at all times, and the current received by the plate should be constant. The data given in the last column of Table III. show that this was never the case, a fact which must now be accounted for.

This "scattering" of a beam of slow-moving electrons has been

observed by all experimenters, and is generally attributed to the effect of gas-molecules. As the calculated mean free path is often large compared with the dimensions of the apparatus, it has been necessary to assume a special attraction between molecule and electron, which can be effective at distances much greater than the "radius of molecular action." There are, however, three other very plausible explanations of this scattering, viz., (2) mutual repulsion of the electrons in the beam; (3) divergence of the beam caused by the strong electrostatic field near the edges of the diaphragms; (4) multiple reflections of electrons between the reflector and cylinder.

Two facts speak very strongly against the first explanation, that of scattering by gas molecules, namely, that the observed scattering was independent of gas pressure from .12 to .65 μ , and that it was greater for fast electrons than for slow ones (Table III., columns 1, 2 and 7).

The scattering due to mutual repulsion can be calculated approximately. Assume a cylindrical beam of radius R and uniform density ρ . The electric intensity at the edge of the beam is given by Poisson's equation,

$$\frac{d}{dr} \left(r \frac{dV}{dr} \right) = - 4\pi\rho r;$$

which gives

$$\left(\frac{dV}{dr} \right)_R = - 2\pi\rho R = \frac{2i}{Rv}$$

since

$$-\rho = \frac{i}{\pi R^2 v},$$

where i is the total current, and v the velocity of the electrons. For small displacements δr , this force will be sensibly constant, so that

$$\begin{aligned} \delta r &= \frac{1}{2} \frac{dV}{dr} \frac{e}{m} \frac{x^2}{v^2} \\ &= \frac{1}{2} \frac{2i}{Rv} \frac{e}{m} \frac{x^2}{v^2} \\ &= \frac{i}{R} \frac{e}{m} \frac{x^2}{v^3}, \end{aligned}$$

where x is the distance travelled parallel to the axis of the beam, and δr the radial displacement of an electron on the edge of the beam. This gives an upper limit for δr , since an increase in the diameter of the beam and a decrease in its density near the edge, as compared with the center, will both have the effect of decreasing dV/dr at the edge.

For the 3 volt electrons of Table III., $i = 10^{-7}$ amp., $v = 10^8$ cm./sec.,

$R \geq .2$, $x = 6$, which gives $\delta r \leq .028$ cm. For the 1 volt electrons, $v = 6 \times 10^7$, $i = 4 \times 10^{-8}$ amp., hence $\delta r \leq .053$ cm. In the case of the .4 volt electrons the reflecting plate was charged to $-.6$ volt. The electrons entered the chamber at 1 volt velocity, and travelled the greater part of its length at this velocity so that the spreading would be not very different from that calculated for the 1-volt electrons. Thus in all the above experiments the scattering due to this cause was negligible.

The effect of the electric field near the edges of the diaphragms is hard to calculate, but it can be studied experimentally by varying the potential of the focus plate. When the focus plate is at zero or negative potential, the electric field near the edges of the diaphragms, which are always earthed, is an accelerating one for electrons; when the focus plate is positive this field is a retarding one for electrons. In the first case the field will tend to make the beam divergent, in the second case convergent. Hence if there is any scattering due to this cause it should be greater in

TABLE IV.

Effect of Focusing Upon Reflection and Scattering.

Velocity of Electrons. Volts.	Potential of Focus Plate. Volts.	Distance of R from End of Cylinder, Cm.	i_R Amp. $\times 10^{-7}$.	i_c Amp. $\times 10^{-7}$.	i Total Amp. $\times 10^{-7}$.	Per- centage Reflection.	Per- centage Scattering
50	0	6	1.60	2.25	3.85		
"	"	.1	2.34	.69	3.03		
"	"	5	1.67	2.20	3.87		
						39.3	32
50	-2	5	1.61	2.72	4.33		
"	"	.1	2.34	.63	2.97		
"	"	6	1.51	2.80	4.31		
"	"	.1	2.32	.60	2.92		
						46	35
50	0	.1	2.35	.60	2.95		
"	"	6	1.60	2.30	3.90		
"	"	.1	2.36	.59	2.95		
						39.7	32
50	0	.1	2.47	.60	3.07		
"	"	6	1.64	2.39	4.03		
"	"	.1	2.42	.70	3.12		
						39.3	33
50	-2	.1	2.40	.60	3.00		
"	"	6	1.50	2.94	4.50		
						46.6	35

the first case than in the second. This was found to be true. When the scattering was large both it and the calculated coefficient of reflection depended very much on the potential of the focus plate. Table IV.

gives a series of readings with 50-volt electrons, where a change of 2 volts in the potential of the focus plate made a change from 32 per cent. to 35 per cent. in the scattering, and from 39 per cent. to 46 per cent. in the reflection coefficient. This is an extreme case and shows how serious the scattering due to this cause can be. In the final experiments this source of error was largely avoided by adjusting the potential of the focus plate until the scattering was a minimum, and in most cases quite small. There were in general two potentials of the focus plate, one positive and one negative, which satisfied this condition, and both gave the same value for the coefficient of reflection. Tables I. and II. are excellent examples, where for 8-volt electrons, changing the focus potential from +18 to -7 had no effect on the value found for reflection. In Tables I., II. and III. the values of scattering, measured by the percentage difference $(i_{.1} - i_6)/i_{.1}$ in the current to the plate at 1 mm. and 6 cm. distance respectively, is given as a measure of the reliability of the results.

The fourth explanation mentioned above, multiple reflection between the plate and cylinder, is probably responsible for the residual 10 per cent.

Fig. 11.

apparent scattering at low velocities which was the minimum that could be obtained by focusing, and for the large values at high velocities. For if there is reflection at all velocities, then some of the electrons reflected from the plate to the cylinder walls will be re-reflected, and the fraction of these twice-reflected electrons that strikes the plate will be greater when the plate is near the closed end of the cylinder than when it is 6 cm. back. An estimate of the difference to be expected can be easily obtained if we assume, as has been shown to be at least very nearly true, that the reflected electrons leave the plate with random directions, that is, are distributed, as regards direction, uniformly over the whole hemisphere.

Let N_0 be the number of electrons striking the reflecting plate, ρ the coefficient of reflection of the plate, and ρ' that of cylinder walls. If the primary beam is narrow the number of electrons which, after reflection from the plate, strikes the end of the cylinder (Fig. 11) in the circular zone between r and $r + dr$ is $(\rho N_0 \cos \theta r dr)/(r^2 + L^2)$. Of these the fraction that is re-reflected and strikes the plate again is $(\rho' A \cos \theta)/(2\pi(r^2 + L^2))$ approximately where A is the area of the plate. Hence the total number returned to the plate by the zone between r and $r + dr$ is

$$\frac{\rho\rho'N_0A\cos^2\theta r dr}{2\pi(r^2+L^2)^2} = \frac{\rho\rho'N_0AL^2r dr}{2\pi(r^2+L^2)^3}$$

and the total number returned by the end of the cylinder is

$$N_e' = \frac{\rho\rho'N_0AL^2}{2\pi} \int_0^R \frac{r dr}{(r^2+L^2)^3} = \frac{\rho\rho'N_0A}{8\pi} \left[\frac{1}{L^2} - \frac{L^2}{(L^2+R^2)^2} \right].$$

Similarly the number returned to the plate by the element of the cylindrical surface between l and $l+dl$ is

$$\frac{\rho N_0 2\pi R \sin\theta dl}{2\pi(R^2+l^2)} \frac{\rho'A \cos\theta}{2\pi(R^2+l^2)} = \frac{\rho\rho'N_0AR \sin\theta \cos\theta dl}{2\pi(R^2+l^2)^2} = \frac{\rho\rho'N_0AR^2 dl}{2\pi(R^2+l^2)^3}$$

and the total number returned to the plate by the cylindrical surface is

$$N_c' = \frac{\rho\rho'N_0AR^2}{2\pi} \int_0^L \frac{dl}{(R^2+l^2)^3} = \frac{\rho\rho'N_0A}{8\pi} \left[\frac{1}{R^2} - \frac{R^2}{(L^2+R^2)^2} \right].$$

The total number of electrons returned to the plate by reflection from both the end and the cylindrical surface is therefore

$$N' = N_e' + N_c' = \frac{\rho\rho'N_0A}{8\pi} \left(\frac{1}{L^2} + \frac{1}{R^2} - \frac{1}{L^2+R^2} \right).$$

Substituting the values of A and R from the dimensions of the apparatus, and taking $\rho = \rho' = .25$, we have

$$\text{for } L = 1 \text{ cm., } N_1' = .05 N_0;$$

$$\text{for } L = 6 \text{ cm., } N_6' = .008 N_0.$$

Hence the reflecting plate will show a current about 4 per cent. smaller when it is 6 cm. from the end of the cylinder than when it is 1 cm. from the end, which is not far from the value observed in the best experiments.

If $\rho = \rho' = .50$ then

$$N_1' = .12 N_0,$$

$$N_6' = .02 N_0.$$

In this case the current to the plate should decrease 10 per cent. in moving from 1 to 6 cm.¹

¹No account has been taken of multiple reflections of higher order, which, however, become very important when the reflection coefficient is large. If the plate is not too large in comparison with the cylinder, so that the solid angles subtended at a point on the plate by the elements of area of the cylinder are approximately the same for all points on the plate, then the number of electrons returned by the cylinder to the plate at the second reflection will obey the same law as for the first reflection, and we shall have, denoting the multiplier of N_0 in the equation for N' above by σ , $N' = \sigma N_0$; $N'' = \sigma N'$; $N''' = \sigma N''$, etc., and the net current received by the plate will be

$$N = N_0 + N' + N'' + N''' + \dots = N_0(1 + \sigma + \sigma^2 + \sigma^3 + \dots) = \frac{N_0}{1 - \sigma}.$$

DELTA RAYS.

In the preliminary experiments, in which mercury vapor was not excluded from the apparatus, large currents were obtained of electrons of "zero velocity," that is, when the receiving plate and cylinder were at the same potential as the most negative end of the filament, the focus plate alone being at a positive potential. The electrons from the filament were thus accelerated as they approached the focus plate, retarded as they left it, and reached the cylinder with only their initial velocity of thermal emission. When the velocity distribution of these electrons was measured, it was found that a very considerable fraction of them had velocities as high as 9 volts. Tables V. and VI. give the observed

TABLE V.

Velocity Distribution of "Zero-Volt" Electrons.

Distance of reflector from end of cylinder 1 mm. Potential of focus plate 12.4 volts. Reflector alone charged to retarding potential. Gas pressure .00025 mm. + Hg at 16° C.

Retarding Potential of Reflector.	i_R Amp. $\times 10^{-7}$.	i_c Amp. $\times 10^{-7}$.	i_R Per Cent.
0	5.20	.50	1.00
1	4.75	.60	.91
2	4.12	.82	.79
3	3.32	1.13	.64
4	2.60	1.40	.50
5	1.90	1.68	.365
6	1.23	1.92	.236
7	.72	2.12	.138
8	.25	2.13	.048
9	.09	2.07	.017
10	0	1.95	0
0	5.23	.50	

distribution. In the first the retarding potential was applied through the galvanometer to the plate alone, which was placed close to the opening of the cylinder; in the second the plate was at the far end of the cylinder and connected to it, and the current to both measured at the corresponding potentials. Fig. 12 shows this distribution graphically.

As these velocities are very much higher than could be accounted for by thermal emission, great care was taken to make sure their reality. Many similar series of readings were taken under widely varying conditions. Currents were obtained with the negative end of the filaments at - 6 volts, all other parts of the apparatus except the focus plate being earthed, and the velocity distribution of these was similar to that of Fig. 12. The maximum velocity could be increased by increasing the potential of the focus plate, but was always smaller, measured in volts, than

TABLE VI.

Velocity Distribution of "Zero-Volt" Electrons.

Distance of reflector from end of cylinder 1 mm. Potential of focus plate 12.4 volts. Reflector and cylinder both charged to retarding potential. Gas pressure .00025 mm. + Hg at 16° C.

Retarding Potential of Reflector and Cylinder.	i Total Amp. $\times 10^{-7}$.	i Total Per Cent.
0	6.80	1.00
1	6.31	.93
2	5.62	.83
3	4.71	.69
4	3.70	.542
5	2.76	.406
6	1.90	.280
7	1.12	.164
8	.48	.070
9	.10	.015
10	-.09	-.014
0	6.80	1.00

that potential. Such sources of error as surface leakage were, of course, carefully investigated, and it was easily proven, by changing the temperature of the filament, that the electrons came, directly or indirectly, from the hot filament.

The best evidence regarding the nature, as well as the reality, of these high velocity electrons, is *that they entirely disappeared when the U tube*

Fig. 12.

Velocity distribution of "zero-volt" electrons. Curve I., reflecting plate alone at retarding potential. Curve II., both reflector and cylinder at retarding potential.

leading to the pump was immersed in solid carbon di-oxide and ether. They are evidently delta rays, produced at the filament by the bombardment of the mercury ions. There is no record, so far as I am aware, of the production of such relatively high speed delta rays by the impact of slow positive ions, though they have been observed by Dr. Langmuir in some experiments not yet published. The most striking characteristic of these rays is *that their maximum initial energy, 9 volts, is nearly the*

same as that (12.4 volts) of the ions which produce them, which suggests that the interchange of energy between ions and electrons, as well as that between electrons and radiation, may be governed by quantum relations. An initial energy greater than that of the ions, such as would violate this law, was never observed.

CONCLUSION.

The electron reflection from a copper surface roughened by prolonged heating at 350° C. in vacuum is, like that of electrolytic silver, very small, being only 10 per cent. for electrons having a velocity of 4/10 volt. This is explained by the extreme ultra-microscopic roughness of these surfaces. Such surfaces should make excellent electric "black bodies" for receiving electron currents.

The scattering of a beam of slow-moving electrons has been shown to be due to deflection by strong electric field, and multiple reflection between the plate and its surroundings, rather than to gas molecules or mutual repulsion of the electrons.

Delta rays have been produced by mercury ions which have fallen through only 12.4 volts. The energy of the fastest of these delta rays is approximately the same as that of the ions, suggesting that the energy exchanges between ion and electron are governed by quantum relations.

These experiments were made at the physical laboratory of the Worcester Polytechnic Institute during the winter of 1913-14. Further experiments are in progress at the research laboratory of the General Electric Company, and will be reported in the near future.