On the Use of Variable Mutual Inductances

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1907 Proc. Phys. Soc. London 21 69

(<http://iopscience.iop.org/1478-7814/21/1/305>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 129.96.252.188

This content was downloaded on 03/10/2015 at 16:03

Please note that [terms and conditions apply](#).

to those described for magnetic space telegraphy, and had found that the closed circuit method was inferior to that in which earth-connected vertical coils were used. The ratio between the received currents when using an earth-connected and a closed circuit was often as great as eighty to one, and he thought the improvements in the closed-circuit method would have to be very great to make it equivalent to the earthed-circuit method.

V. *On the Use of Variable Mutual Inductances.* By ALBERT CAMPBELL, B.A.* (From the National Physical Laboratory.)

[Plate IV.]

CONTENTS.

1. Introductory.
2. Mutual versus Self Inductance.
- 3 & 4. Construction and Adjustment of Variable Mutual Inductances.
- 5, 6, 7 & 8. Uses of the same for measuring unknown Mutual Inductance, Self Inductance, and Capacity.
9. Appendix. The Scale of the Excentric-Coil Mutual Inductance.

1. *Introductory.*

THE accurate measurement of small self inductances and capacities has of late years assumed an increased importance owing to its direct practical application in connexion with wireless telegraphy; and much important and interesting work has been done on the subject especially as regards the calculation and measurement of self inductance. For some time past I have found that suitably designed mutual inductances are of the greatest assistance in such measurements, and I incline to think that by their use some of the sources of error inherent in other methods are avoided. The chief method (B) here described was designed for the measurement of the small self inductances (1 to 200 microhenries) used in wave telegraphy, and it has proved most useful in such work. Before I proceed to describe the designs and the methods of use which I have found valuable, let us consider and compare mutual and self inductances in relation to their capabilities in practical use.

* Read October 25, 1907.

2. *Mutual versus Self Inductance.*

For a number of reasons mutual inductances are more easily dealt with than self inductances. Those of invariable value, when properly designed, have the following advantages:—

(a) The absolute values can be calculated with much more certainty from the geometrical dimensions, since the formulas for mutual inductance are of high theoretical accuracy while those for self inductance are much less satisfactory*.

(b) Unless the conductors are highly stranded, the current distribution varies with frequency and in general the self inductance will also vary. By keeping the two circuits at a relatively large distance from one another the mutual inductance is practically free from this effect.

(c) The effects of distributed capacity are probably less in mutual than in self inductances. In all cases the distributed capacity of one of the two coils can be made very small by sufficiently decreasing the number of turns in it (or opening them apart) while increasing the number in the other coil to keep the M constant.

When the mutual inductance is of the variable type, it can always be designed so that its value can be varied continuously from *negative to positive through zero*. This is a very great advantage, for with variable self inductance standards the incapability of reaching a zero value is a distinct drawback.

3. *Construction of Variable Mutual Inductances.*

For short range standards the construction used by Lord Rayleigh may be employed†. In this the primary coil is inside the secondary and can be rotated round a common diameter as in the Ayrton-Perry variable self inductances, and the whole positive range is comprised within 90° of turning, which gives a scale not nearly open enough for high accuracy. I have therefore designed a different and

* For example see Rosa, *Bull. Bur. Stands.* vol. ii. p. 161 (1906); and Strasser, *Ann. der Phys.* vol. xviii. p. 763 (1905).

† Phil. Mag. vol. xxii. pp. 469–500 (1886); Collected Papers, vol. ii. p. 155.

more elaborate arrangement which gives a much more extended scale. I proceed to describe the first model, which has been in use for a considerable time with most satisfactory results.

The general arrangement of the apparatus is shown diagrammatically in figs. 1 and 2, which are plan and side view respectively.

Fig. 1.

Fig. 2.

view respectively. The primary* circuit consists of two equal coaxial coils C and C', which are connected in series, their windings being in the same direction of turning. The secondary consists of the coils D and F in series. Of these coils, D is movable, being mounted on an excentric axis Q so as to be free to turn in a plane parallel to those of C and C' and midway between them. Rigidly connected with the movable coil is a pointer H which moves over a circular scale of about 180° in extent and graduated to read directly. The coil F is subdivided into ten sections, which are in series, each of them being 0.1 millihenry, and their junctions are brought to a set of separate terminals or studs with a turning head. The range of the moving coil extends from -0.002 to $+0.11$ millihenry. This gives a continuous range from 0 up to 1 millihenry, readable near zero to 0.02 microhenry, to 1 part in 500 at 0.1 millihenry, and to 1 in 5000 at 1 millihenry. The subdivision of the coil F is easily carried out by the following artifice. The coil is wound with uniformly stranded wire of ten insulated strands, all the strands are connected in series, and the whole adjusted

* It is only for convenience of description that the circuits are here distinguished as *primary* and *secondary*. They are really quite interchangeable.

to give 1 millihenry *. If the stranding has been properly done, it will be found that no one of the sections differs from its neighbours by more than 1 part in 1000, and each is 0.1 millihenry. The placing of the movable secondary coil midway between the planes of the two primary ones ensures that small axial displacements shall have very little effect on the mutual inductance.

4. *Adjustment and Calibration of Variable Mutual Inductances.*

The equality of any pair of sections can be tested by connecting them in series with their windings in opposition in circuit with a ballistic galvanometer and reversing the current in the primary. It should be noticed here that, if a primary coil have any number of secondary circuits, the mutual inductance to all the secondaries in series is equal to the algebraic sum of their separate mutual inductances (+ or - according to the direction of the winding). Owing to this very important property we can build up and step down in the values as easily as if we were dealing with resistances, and there is the further simplification that we can subtract as well as add the values. The marking of the scale and the setting of the coil F are done by comparison with a fixed standard mutual inductance such as I have described elsewhere †. The comparison may be made by Maxwell's method ‡, using a sensitive ballistic galvanometer or a vibration galvanometer as detector. When a vibration galvanometer is used as in fig. 3, it should be remembered that, for a balance, *two* conditions must be satisfied, viz.,

$$\begin{aligned} M_1/M_2 &= R_1/R_2 \\ \text{and} \quad L_1/L_2 &= R_1/R_2, \end{aligned}$$

where R_1 and R_2 include the resistances of the secondary coils. In order that the second condition may hold, it is

* Mutual Inductances with stranded wires were used many years ago by Prof. Brillouin (*Thèses présentées à la Faculté des Sciences de Paris*, 1882).

† Phys. Soc. May 1907; Phil. Mag. [6] vol. xiv. p. 494, Oct. 1907. Also see Proc. Roy. Soc. A. vol. lxxix. p. 428, June 5, 1907.

‡ Maxwell, *Elect. and Mag.* 2nd edition, ii. § 755.

necessary to introduce into one of the secondary branches a coil a whose self inductance can be continuously varied ; by alternate adjustments of R_1/R_2 and the self inductance of this coil, a balance is easily obtained. The fact that R_1 and R_2 are partly of copper coils is apt to introduce some inaccuracy. The copper resistance, however, can usually be largely swamped without losing too much of the sensitivity.

Fig. 3.

In connexion with this method I may mention that it is perfectly applicable to the case of comparing the mutual inductances between one primary circuit and two separate secondaries. This case is shown in fig. 4.

Fig. 4.

By this means a thorough intercomparison (or adjustment) can be made between the various sections of a subdivided mutual inductance. By further sets of secondary coils for higher multiples (10, 100, and so on) any desired range may be obtained. By stranding (or otherwise subdividing) the primary coils as well as the secondary ones a multiple-range

inductance is the result. In all cases the subdivision can be effected by other means than stranding the wires, which is merely used to avoid the separate adjustment of the sections. The main objection to the subdivision by stranding is that it increases the distributed capacity of the coils. With coils of many turns this might be a serious drawback, but with the relatively few turns required for 1 millihenry the effect can only be very small and I have not yet noticed it in practice. It should be kept in mind that, as in all standard inductances, eddy currents should be prevented by using only highly stranded wire for the winding of every section and by avoiding the use of any metal parts near the coils.

By varying the relative dimensions and positions of the two fixed primary coils CC' (fig. 2) and the movable secondary D , the scale can be considerably modified, and can be designed to suit special purposes. The most generally useful type of scale becomes more and more open as the readings approach zero, for this gives good accuracy of reading even at very low values. Fortunately in the present instance this type of scale can readily be attained. Further discussion of this point will be found in an Appendix.

5. *Employment of Variable Mutual Inductances.*

Variable inductances of this type may be used for several purposes. Their most obvious application is to the calibration of ballistic galvanometers for magnetic testing; and I have found them of the greatest value for quickly adjusting a galvanometer circuit to read B directly for any given iron ring. I need not dwell on this application at present, but proceed to describe several methods to which they are applicable.

6. *Measurement of Mutual Inductance (Method A).*

Any unknown mutual inductance, whose value lies within the range of the variable standard, can be at once determined by connecting the primaries of the unknown and the variable in series to B (fig. 5), a source of alternating or intermittent current, while the secondaries, with their windings in opposition, are connected in series to a vibration galvanometer G . The variable M is then adjusted to bring the deflexion to

zero, and the reading gives directly the value of the unknown M . This is an extremely simple method as it involves no knowledge of any resistances. A ballistic galvanometer and a commutated current may be used. This

Fig. 5.

method does not apply to mutual inductances higher than the maximum value of the variable standard; for these another method will be described below (see § 8).

7. Measurement of Self Inductance (Method B).

In fig. 6 let a variable mutual inductance M whose *primary* includes the subdivided coil be connected into a Wheatstone's network, as shown, along with a self inductance L_2 . Let the resistances of the arms be P , Q , R , and S respectively,

Fig. 6.

the self inductance of the arm P being L_1 (the secondary coils of M) and that of Q being L_2 . Let H be a source of periodic current and G a vibration galvanometer tuned to resonance with it, so that we may take the wave form of the currents to be a sine curve. Let the instantaneous potentials of the three upper corners be v_1 , 0 , and v_2 respectively, and

the instantaneous values of the currents into the upper corner be i_1 , i_2 , and i as marked. Let $p=2\pi n$ where n is the frequency, and for convenience of writing let $p\sqrt{-1}$ be denoted by α , so that $\alpha^2=-p^2$. The mutual inductance M may be made positive or negative according to the way in which the coils are connected; and in all that follows we might write $\pm M$ for M throughout. When the galvanometer shows a balance, $v_1=v_2$, and the instantaneous value of the current through G is zero.

Also

$$i = -i_1 - i_2.$$

Accordingly we may write

$$(P + L_1\alpha)i_1 - M\alpha i = (Q + L_2\alpha)i_2;$$

therefore

$$[P + (L_1 + M)\alpha]i_1 = [Q + (L_2 - M)\alpha]i_2$$

also

$$Ri_1 = Si_2.$$

Hence

$$S[P + (L_1 + M)\alpha] = R[Q + (L_2 - M)\alpha].$$

Equating the real and imaginary parts each to zero we have

$$SP = QR, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{and} \quad S(L_1 + M) = R(L_2 - M). \quad . \quad . \quad . \quad . \quad . \quad . \quad *(2)$$

Exactly the same equations hold when the positions of the source and the galvanometer are interchanged.

The most useful case is when the noninductive arms are made equal, *i. e.* $S=R$; then (1) and (2) become

$$P=Q,$$

$$\text{and} \quad L_2 - L_1 = 2M. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This case gives an extremely convenient way of measuring small self inductances, which is done as follows.

The arrangement is shown in fig. 7. The noninductive arms are equal (R, R). In the arm AB there is the secondary coil a of self inductance L in series with a practically non-

* (Nov. 22, 1907.) I find that a case slightly more complicated than that of Equation (2) has been worked out and applied in an ingenious manner by Grätz (Wied. *Ann.* vol. 50. p. 766, 1893).

inductive rheostat r . In the arm AC is placed a "balancing" coil b also of self inductance L and of resistance equal to or slightly greater than that of b . By adjusting r the bridge will balance when $M=0$. The small self inductance N to be measured is now inserted in series with coil b in arm AC, and a balance obtained by altering r and M . Then, by (3), $N=2M$. Thus N is found directly from the reading of M , and the range of values that can be measured runs from 0 up to twice the highest reading of the variable mutual inductance.

Fig. 7.

[For values of N above this range the more general case (equations (1) and (2)) may be used.] The L 's of the coils a and b should be adjusted to equality once for all by putting M at zero and setting one of the coils. An exact setting is convenient, but not necessary, for if L_a and L_b differ slightly, they can be balanced (without N) by a small reading M_0 . If M be the reading for balance when N is inserted, then $N=2(M-M_0)^*$.

Even if a and b are well matched, it is always well to begin by reading their difference if any.

It will be noticed that the method is really a differential one; when N is introduced into the arm AC no alteration has

* (Oct. 1st, 1907.) If, instead of the coil N , a condenser K with series (absorption) resistance q be added, then

$$1/p^2K=2M,$$

and q =change in r .

to be made in the other arms except to increase the resistance of AB by an amount equal to the resistance of the coil N. But although it has *all the advantages of differential measurement, the reading can be made to give N directly* without having to take a difference at all. This is due to the use of the inductive balancing coil b^* .

The method has the advantage that it does not require the knowledge of the absolute value of any resistance. The non-inductive bridge arms must be equal; to check the equality they can be interchanged. For the noninductive adjustable resistance r I usually employ a special rheostat consisting of two slightly flattened thin wires running parallel to one another at a few millimetres distance, with a sliding contact piece across them to complete the circuit. The inductance of such a rheostat can be approximately calculated, and may thus be allowed for when measuring very small self inductances. The inductance of the part added to compensate for the introduction of N has merely to be subtracted from the result.

In practice the method proves very convenient; with the variable mutual inductance described above, self inductances of any value from 0.1 microhenry up to 2000 microhenries can be measured directly without the bridge being altered in any way except in the rheostat r . In a later model shown in the accompanying Plate IV. the whole scale of the movable coil corresponds to 20 microhenries, and at this value it can be read to 1 or 2 in 1000—at 200 microhenries to 1 or 2 in 10,000. All the resistances of the coils are very low, and the sensitivity can be considerably increased by using M. Wien's method of connecting the vibration galvanometer to the bridge by means of a transformer of suitable ratio (n_1/n_2 small). The method will also give the difference between two unknown self inductances introduced into AB and AC.

The addition of a condenser to Method (B) gives an interesting corroborative method (Method C). If (fig. 8) the condenser K is in shunt with S,

* If coil b be made noninductive we revert to Maxwell's method of comparing the M of a pair of coils with the L of one of them. Equation (2) then reduces to $L/M = -(1 + R/S)$. Maxwell, 'Elect. & Mag.' 2nd edit. vol. ii. § 756.

it may be shown that, for a balance,

$$SP - RQ = p^2 RSK(L_2 - M), \quad (4)$$

and $S(L_1 - M) - R(L_2 + M) = QRSK. \quad (5)$

When $R = S$ and $L_1 = L_2 = L,$
 $P - Q = p^2 RK(L - M), \quad (6)$

and $2M = QRK. \quad (7)$

Fig. 8.

Equation (7) gives a check of K against M without having to measure the frequency. The adjustment is dependent on p , however, and therefore a steady frequency is necessary, which is not required in Carey Foster's Method.

From (6) and (7) we get

$$Q(P - Q) = 2p^2 M(L - M). \quad (8)$$

8. Measurement of Mutual Inductance greater than Standard (Method D).

When the unknown M is beyond the range of the variable standard the following method is applicable.

Let the primaries of the variable standard and the unknown M be connected up as in fig. 9, adding to the latter if necessary a coil c of sufficient self inductance to make N considerably greater than L . Let a balance be obtained (by Maxwell's Method) and hence

$$PS = RQ, \quad (9)$$

and $NS = LR. \quad (10)$

Now, without altering any other branches, let the secondary coils be introduced (in opposition) into the galvanometer circuit as shown in fig. 10.

Fig. 9.

Fig. 10.

If a balance be now obtained by adjusting the variable M , then it is easy to show that

$$(P + R)M_2 = (Q + S)M_1, \quad \dots \quad (11)$$

and $PS - p^2 M_2 (L - M_1) = QR - p^2 M_1 (N - M_2). \quad \dots \quad (12)$

Since (9) and (10) still hold, these equations both reduce to

$$RM_2 = SM_1 \quad \dots \quad (13)$$

which gives M_2 immediately in terms of the reading M_1 and the resistances R and S (neither of which include copper coils). The self inductance of the added coil c need not be known, but if R and S have not a slide wire between them, c ought to be slightly adjustable, which is easily arranged by having part of it a small coil of a few turns that can be shifted about over the remainder.

9. Measurement of Capacities (Method E).

As in fig. 11, let the primary P of the variable standard

Fig. 11.

be connected in series with a condenser K to a periodic

source H, while the secondary Q is connected to a vibration galvanometer as shown. Let M be adjusted until the galvanometer gives no deflexion, in which case we have

$$\pm M\alpha i = \frac{1}{K\alpha} i, \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\text{or} \quad \pm p^2 MK = -1.$$

The secondary must be connected in such a direction that the lower sign is taken, and thus

$$p^2 MK = 1, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where M is in henries and K in farads.

It is interesting to notice that this formula is similar in form to that for resonance with a condenser and self inductance, viz. $p^2 LK = 1$. With self inductance we usually tune to get a maximum current through the galvanometer; with mutual inductance, on the other hand, we tune for zero current. If M is in henries and K in microfarads, equation (15) gives

$$n \doteq \frac{159 \cdot 2}{\sqrt{MK}}, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and if M and K are known, the frequency can be determined.

If the frequency is not fairly high, the product MK becomes inconveniently high, *e. g.* M=1 henry, K=10 mfd. gives $n \doteq 50 \sim$ per sec. In contrast, $n \doteq 10,000$ for 256 microhenries and 1 mfd.

The following is a practical example of the method:—The source was a microphone hummer for $2000 \sim$ per sec. and the detecting instrument a tuned telephone. A balance was obtained for K=10·06 mfd. and M=0·626₈ millihenry, whence the frequency was found to be $2004 \sim$ per sec. With another condenser (a carefully calibrated subdivided one), capacities of 1·000, 0·900, 0·800, 0·701, 0·601, and 0·551 gave

$$n = 1999, 2005, 2009, 2001, 2004, \text{ and } 2001 \sim \text{per sec.}$$

respectively; the variable M was not, however, so accurately readable in this case.

In this method it is very important that the insulation

resistance S of the condenser shall be high. If the leakage is not negligible, equation (14) becomes

$$\begin{aligned}\pm M\alpha i &= \frac{S}{1 + SK\alpha} i \\ &= \frac{S(1 - SK\alpha)}{1 + p^2 S^2 K^2} i. \quad . \quad . \quad . \quad (17)\end{aligned}$$

Putting the real part equal to zero we have

$$1/(1 + p^2 S^2 K^2) = 0,$$

showing that an exact balance can only occur when $p^2 S^2 K^2 = \infty$. In practice the insulation resistance of a good mica condenser is quite high enough to give a good balance; even a good paraffined paper condenser will work fairly well, but an ordinary one gives only a minimum deflexion on the galvanometer*.

The application of this null method to the high frequencies used in wave telegraphy is made difficult by reason of the coexistence of two frequencies in the waves used.

APPENDIX.

Scale of Excentric-Coil Variable Mutual Inductance.

The problem of finding the mutual inductance between two circles (thin circular coils) not in the same plane and whose axes are parallel has not yet been solved, as far as I know. For any particular case the result may be calculated by quadratures as follows.

In fig. 12 let the two circles be H and G , of radii a and A respectively, b being the distance between their planes, and q the distance between their axes. Let the circle H' be the projection of H on the plane of G , and let BCD be an elementary strip concentric with H' of radial width ∂r , where $O'B = r$. Let $CO'B = \theta$. Then the M between circle H and strip BCD

$$\begin{aligned}&= \frac{\theta}{\pi} \cdot (M \text{ of circle } H \text{ and annulus } TBD) \\ &= \frac{\theta}{\pi} \cdot \frac{\partial}{\partial r} \left[-4\pi n_1 n_2 \sqrt{ar} \left\{ \left(k - \frac{2}{k} \right) F_1 + \frac{2}{k} E_1 \right\} 10^{-9} \right] \partial r,\end{aligned}$$

* This may, however, be due more to absorption than leakage.

or

$$\partial M = \frac{\theta}{\pi} \cdot \frac{2\pi n_1 n_2 10^{-9}}{\sqrt{(a+r)^2 + b^2}} \left\{ 2a(F_1 - E_1) - (r-a) \left(\frac{k}{k'} \right)^2 E_1 \right\} \partial r, \quad (18)$$

where

$$k = 2 \sqrt{ar} / \sqrt{(a+r)^2 + b^2} = \sin \gamma,$$

$$k' = \cos \gamma,$$

and F_1 and E_1 are complete elliptic integrals to modulus k .

Also

$$\theta = \cos^{-1} \frac{r^2 + q - A}{2rq}.$$

Fig. 12.

Thus when a , A , b , and q are given, by calculating ∂M by (18) for a series of equally spaced successive values of r , from $r=q \sim A$ to $q+A$, and adding in the usual way, the value of M can be approximately found. By this process, which is rather laborious, the following approximate values were found, when $A=20$, $a=10$, $b=5$ cm and $n_1 n_2=100,000$:—

q cm.	M millihenries.
0	9.3
5	9.7
10	9.9
15	6.7

It is interesting to notice that the maximum value of M

does *not* occur when the axes of the coils are coincident ($q=0$). To avoid the labour of the calculation by quadratures, some rough experiments were made with coils of finite section (about 0.6×0.6 sq. cm.). From one set of these the series of curves shown in fig. 13 was obtained. The coils were of mean radii $A=10$, $a=5$ cm. respectively, $n_1 n_2$ being 100,000, and q and b were given various values. It will be seen that for $b=2.6$ the maximum M does not occur

Fig. 13.

at the symmetrical position (just as in the case calculated, above). For equal coils, however, the coaxial position gives the maximum M . The curves also show that the value of q for which M vanishes and changes sign becomes greater and greater as the mean planes of the coils are set at greater distance from one another. The ratio of A/a has been made 2 in this example, as that is a suitable proportion for use in a practical instrument. Having now the curves connecting M with q , the distance between the axes of the two coils, it is easy to investigate the variety of angular scales obtainable

by turning one of the coils round an axis parallel to its own axis.

Let a definite value of b be taken, say $b=5$, which is convenient in practice, and in fig. 14 let the smaller coil be

Fig. 14.

movable about an axis perpendicular to the plane of the paper at Z, and let the radius $ZQ=\rho$. Let the distance ZO' of the axis Z from O' (the point where the axis of the other coil cuts the plane of the paper) be h . Let the angular scale reading $YZQ=\phi$.

Then by choosing various values for ρ and h we can get a wide variety of movement of the centre Q (of the movable coil) relatively to O' , and hence a variety of scale calibrations. Since $\cos \phi = \frac{q^2 - h^2 - \rho^2}{2h\rho}$, the angle ϕ corresponding to a reading M can be got from q , h , and ρ . The scale can be most easily set off by the following geometrical construction. Draw a circle with centre Z and radius ρ , and make ZO' equal to h . Then find on the ($b=5$) curve in fig. 13 the q corresponding to a particular value of M. With this value of q as radius, a circle with centre O' will cut the circle X at the point of the scale required. By taking successive values of M the direct-reading scale can be set off round the circumference of the circle X.

As an example, in fig. 15 are given three scales for $h=5.5$, $\rho=5.5$; $h=7$, $\rho=7.2$; and $h=10$, $\rho=4.2$ respectively, when $A=10$, $a=5$, $b=5$, and $n_1 n_2 = 33,000$.

Although these scales give very different ranges, the

general characteristic of all three is an openness at both ends; no proportions that I have tried tend much to make the scale uniform throughout. The openness at the lower values is a distinct advantage, however, when dealing with

Fig. 15.

small inductances; but care must be taken, when marking that part of the scale, that the subdivisions shall be accurately marked and not put in merely by interpolation.

Scale (2) in fig. 15 is the best of the three and is nearly that which I have used in practice.

In conclusion I desire to express my best thanks to Dr. Glazebrook, for his kind encouragement and valued advice.

Sept 21, 1907.

DISCUSSION.

Dr. FLEMING asked the author how his method compared in accuracy with the Carey Foster method, using a commutator to add up the impulses. He had frequently measured mutual inductance by determining the self-inductances of the primary and secondary joined in series, with the windings first helping each other and then opposing each other. He asked if the author's methods were applicable when dealing with the high frequencies used in wireless telegraphy.

Mr. A. RUSSELL agreed with the author in thinking that mutual inductance was sometimes easier to calculate and easier to measure than self-inductance. The variable mutual inductance standard shown was very compact, and would be a great acquisition to a physical laboratory. In calculating self-inductance the variable current density over the cross-section of the wire sometimes brought in serious difficulties. The measurement of self-inductance in these cases might possibly be made by Mr. Campbell's method, but the limitations introduced into his formulæ by the assumption that the back E.M.F. was proportional to the rate at which the total current was changing would have to be considered.

Mr. CAMPBELL, in reply to Dr. Fleming, said the methods of measuring mutual inductance described in the paper were more convenient and direct reading than that of Carey Foster, although they were not so absolute. They were of higher accuracy than the difference method, in which the primary and secondary were put in series and their combined self-inductance measured both with the windings helping and opposing. In reply to Dr. Fleming and Mr. Russell, the methods of measurement described were not intended, in general, for use with high frequencies of the order of those used in wave telegraphy, where resonance methods appear to be the only ones available. Mr. Campbell mentioned that the correction for the small inductance of the rheostat in method (B) could be got rid of by making the rheostat of constant inductance, this being attained by running a manganin and a copper wire side by side with a travelling contact across them, the current entering by the copper at one end and leaving by the manganin at the other.

CAMPBELL'S VARIABLE MUTUAL INDUCTANCE.