THE
AMERICAN JOURNAL OF SCIENCE

[THIRD SERIES.]

ART. XLIV.—*Further Study of the Solar Corona*; by
FRANK H. BIGELOW.*

IT was one of the proposed objects of the U. S. Eclipse Expedition to West Africa, in December, 1889, to make an effort to secure some photographs of the Solar Corona on a large scale, in order that certain measurements might be taken which would test fully the validity of the equation that I have assigned to the coronal curves. Inasmuch as we failed to obtain the desired pictures, in consequence of the clouds that prevailed during the minutes of totality, I have since my return to the United States made a series of measures upon photographs of the Coronas of 1878, July 29, 1889, Jan. 1, and 1889, Dec. 22, which have been generously placed at my disposal by the Superintendent of the Naval Observatory, and by the Directors of the Harvard College and the Lick Observatories respectively. I am also greatly indebted to Professor William Harkness for allowing me to use the Transit of Venus Measuring Engine, by Stackpole, and for all needful assistance. Six pictures have been measured, two La Junta negatives of July 26, 1878, taken by Professor Hall, two Creston negatives of July 26, 1878, by Professor Harkness, one positive of Jan. 1, 1889, by Professor W. H. Pickering at Willows, Cal., and one negative by Mr. Burnham at Cayenne, S. America, Dec. 22, 1889. As the computations are quite heavy the results of only one are presented with this paper.

* Read before the American Association at Indianapolis, August 22d, 1890.

AM. JOUR. SCI.—THIRD SERIES, VOL. XL, No. 239.—Nov., 1890.

The Equation of the Lines of Force.

The equation to be investigated may be derived as follows. Without raising the question of the physical constitution of masses of matter exhibiting the phenomena, we may understand by a polarized sphere, one in which magnetic or electrical potentials are to be referred to one general axis; or if not symmetrical, one in which the positive potential is distributed about one pole, and the negative potential about another pole, the poles themselves being the intersection of two nearly opposite axes with the spherical surface. The potentials at any points in space may be discussed as if the actual distribution of polarized matter were replaced (1) by a small magnet at the center of the sphere, the direction of the axes coinciding, or (2) by a surface distribution whose density varies from a maximum at the poles, with the cosine of the angle of the polar distance, to zero at the corresponding equatorial plane. The former is the

Equation of lines of force and equipotential surfaces.

result of bringing two equal masses of opposite signs very near each other at the center of the sphere, and the latter is the lamellar distribution. They each give the same equations for lines of force and equipotential surfaces respectively.

Let M_1, M_2 be any two electrified masses connected by an axis of reference; join any point P with M_1 and M_2 by straight lines, and with the axis A by any curve; across a cylindrical surface generated by AP , the flow of force from M_1 is N_1 , and from M_2 is N_2 , the constant total force being $N_1 + N_2 = N$. Let PM_1 be the curve formed by the locus of the point that will keep N the same while the position of P varies in a plane, and let the tangent to this curve at the point M_1 make the angle θ with the axis; N is called the order of a line of force. Considering a single mass, the flow of force corresponding to the circular zone whose angle at the sunmit is 2θ , is proportional to the cap whose height is $1 - \cos \theta$.

$$\frac{N}{4\pi M} = \frac{1 - \cos \theta}{2} \quad N = 2\pi M (1 - \cos \theta).$$

Introducing this into the equation and limiting the case to equal masses of opposite signs, we obtain,

$$2\pi M_1(1 - \cos \theta_1) + 2\pi M_2(1 - \cos \theta_2) = 2\pi M(1 - \cos \theta) = N;$$

$$\text{whence,} \quad M_1 \cos \theta_1 + M_2 \cos \theta_2 = M_2 + M_1 \cos \theta.$$

and by our special case,

$$\cos \theta_1 - \cos \theta_2 = 1 - \cos \theta = \frac{N}{2\pi M}.$$

Now if these masses be brought infinitely near together, the θ_1 and θ_2 will differ by an infinitely small amount, h ; we have,

$$\begin{aligned}\cos \theta_1 - \cos \theta_2 &= 2 \sin \frac{\theta_1 + \theta_2}{2} \sin \frac{\theta_1 - \theta_2}{2} \\ &= 2 \sin \theta \cdot \frac{h}{2} = \sin \theta \cdot h. \\ &= \sin \theta \cdot \frac{2a \sin \theta}{r} = \frac{2a}{r} \cdot \sin^2 \theta,\end{aligned}$$

where a is the distance apart of the particles, and r the mean distance of the point P from $M_1 M_2$.

Hence,
$$N = 2\pi (2Ma) \frac{\sin^2 \theta}{r},$$

which is the equation of a line of force.

Term $2a \cdot M$ is the moment of the system, and if we choose to deal only with the outside of a large sphere, it is equivalent to the layers of gliding on a polarized sphere, and these again to $u\sigma_o$, where u is the volume of the sphere $\frac{4}{3}\pi R^3$, and σ_o of the surface density at the poles. Therefore,

$$N = \frac{8\pi}{3} \cdot (\pi R^3 \sigma_o) \frac{\sin^2 \theta}{r}.$$

From this equation we may draw typical lines of force by assuming $\pi R^3 \sigma_o$ equal to unity, and this is the best form for use until we have the means of determining the value of σ_o ; thus we finally get,

$$N = \frac{8\pi}{3} \cdot \frac{\sin^2 \theta}{r}.$$

The Equation for Equipotential Surfaces.

The potential of a mass M_1 at a distance r_1 is, $V_1 = \frac{M_1}{r_1}$, and of a Mass M_2 , at a distance r_2 is, $V_2 = \frac{M_2}{r_2}$. If these masses are equal and of opposite signs, at the point of intersection of r_1 and r_2 ,

$$V = V_1 + V_2 = \frac{M_1}{r_1} - \frac{M_2}{r_2} = M \left(\frac{1}{r_1} - \frac{1}{r_2} \right).$$

These surfaces are of an ovoidal form, with a single sheet, tending to merge into spheres in proportion as they approach the centers of action.

If the two points are located infinitely near together,

$$V = M \cdot \frac{r_2 - r_1}{r_1 r_2} \text{ becomes } V = M \cdot \frac{r_2 - r_1}{r^2} = M \cdot \frac{2a \cdot \cos \theta}{r^2}.$$

An equivalent of the moment of the system $2aM$ is as before, $2aM = \frac{4}{3}\pi R^3 \sigma_o$, and the value of the potential at any point outside the sphere (r, θ) is $V = \frac{4}{3}\pi R^3 \sigma_o \cdot \frac{\cos \theta}{r^2}$.

Typical forms of the curves may be found by employing simply the expression, $V = \frac{\cos \theta}{r^2}$.

Gauss' Theorem of Polar intercepts.

2.

For Gauss' Theorem.

On the sphere whose radius is R , construct a line of force one of the points being (r, θ). Through (r, θ) draw a circle with radius r , a tangent to this circle making an angle m with the polar axis, and also a tangent to the line of force making the angle n with the axis. The triangle PTS will have the interior angles l, m, n , at corresponding vertices. Now,

$$PS \cdot \cos l = OS \cdot \sin \theta$$

$$PS \cdot \sin l = TS \cdot \cos \theta$$

$$\text{Hence, } OS \cdot \tan l = TS \cdot \cot \theta = TS \cdot \frac{\tan l}{2},$$

as can be proved by the discussion of the resolved forces. Therefore,

$$ST = 2OS, \text{ and } OT = 3OS.$$

The intercept from the center cut off by the force tangent is one-third the intercept cut off by the circle tangent.

This formula is also convenient for computing the inclination of the line of force to the normal as it leaves the surface of the sphere. If the lines are seen on the section of the meridian plane perpendicular to the line of vision, it applies directly, $\tan l = 2 \cot \theta = 2 \tan m$. If the angle actually seen is on any other meridian section, the projected value of the angle must be converted from its oblique to the perpendicular plane, by turning it through the angle α . Since l is the angle

of the line of force with the tangent, $90-l$ is its angle with the normal.

If l' is the apparent measured angle of inclination of a line with the radius extended, then $\tan l = \tan l' \sec \alpha \sec \theta$.

In a preceding paper I applied this theorem to some photographs of coronal lines, finding a strong tendency to represent the intercept ratio in the four quadrants, and yet with such marked exceptions as might well lead to the interpretation that the agreement was fortuitous between the graphical representations of the mathematical formulæ and the natural phenomenon. It is now my object to carry out the comparison with considerable accuracy, paying regard to the distortions of the coronal lines as produced by the perspective effects. It may be possible sometime to include the corrections due to differential refraction, non-coincidence of the centers of the disks of the Sun and the Moon with the axis of vision, and such others as exist, but at present they must be omitted because the photographs used are too small to permit settings for measures within these limits of precision.

The Negative here discussed is

Marked No. 16, July 29, 1878, La Junta, Col., by Professor Asaph Hall. It was taken with a Dallmeyer's patent portrait and group lens, size No. 8 D; effective focal length 37.89 inches, clear aperture 6.0 inches; the plates were the dry washed emulsion; the size of the image of the Moon's disk is 0.362 inch, but the limb is not circular because of the Moon's motion in right ascension. The measurements were made by centering the image so that in revolving the table the edge of the disk remained tangent to the micrometer thread. Having selected the polar line of the corona by inspection, the readings of the N. and S. poles respectively were taken; then as many settings on individual rays as was practicable were made, the polar angle and the distance in micrometer revolutions read. The corresponding values of r and θ were made the basis of the computation.

North Polar Setting $251^{\circ} 40'$.

South Polar Setting $244^{\circ} 30'$.

The angle from the N. Pole through the east to the S. Pole is $172^{\circ} 50'$.

The North Pole lies $1^{\circ} 10'$ east of Sun's Axis.

The South Pole lies $6^{\circ} 0'$ east of Sun's Axis.

The reading for the North Pole is 7.55 divisions.

The reading for the South Pole is 7.45 divisions.

Correction for micrometer readings is for N. P. readings -1.00 ; for S. P. reading -0.90 . The diameter of the disk is 13.10 divisions. The factor for reduction to the unit radius is 6.55.

A complete record of the readings is given in table No. 1.

TABLE I.

S. W. Quadrant.			S. E. Quadrant.		
Ray.	Radius.	Circle.	Ray.	Radius.	Circle.
1	<i>d</i>		1	<i>d</i>	
	7.55	254° 19'		7.55	250° 51'
	9.32	254 36		9.20	249 45
	10.50	255 0		10.70	249 20
2	7.50	256 1	2	7.55	250 33
	9.45	258 31		8.90	248 8
	10.90	260 12		10.20	244 0
3	7.55	258 30	3	7.58	242 33
	9.45	261 17		8.92	237 0
	10.87	262 56		10.30	233 10
4	7.55	260 40	4	7.56	237 18
	9.03	263 17		9.40	230 52
	10.60	265 38		11.25	223 8
5	7.55	264 41	5	7.57	233 26
	9.00	272 23		9.55	225 49
	10.12	276 5		10.80	220 36
6	7.60	276 52	6	7.56	230 48
	9.40	285 17		10.00	220 8
	10.52	290 41		11.80	213 50
7	-----	--- --	7	7.58	220 26
	9.10	290 42		9.60	213 43
	10.28	299 8		11.18	208 12
8	-----	--- --	8	7.66	217 2
	9.60	299 10		9.62	208 6
	11.10	307 38		11.16	201 23
	12.35	314 32		12.68	194 44
			9	7.57	210 0
				9.85	201 36
				11.85	192 37
N. E. Quadrant.			N. W. Quadrant.		
1	<i>d</i>		1	<i>d</i>	
	7.45	70° 50'		7.50	63° 15'
	9.00	73 38		9.10	62 30
	10.70	75 0		10.45	60 0
2	7.45	74 32	2	7.45	61 13
	9.30	77 52		9.20	59 0
	11.00	81 0		10.52	57 46
3	7.45	78 8	3	7.45	59 10
	9.33	81 36		9.20	55 29
	10.50	83 26		10.95	51 29
4	7.45	81 20	4	7.45	55 32
	9.33	87 51		9.44	50 25
	10.72	91 12		10.94	45 6
5	7.45	86 5	5	7.45	51 48
	9.20	90 57		9.30	44 16
	10.80	95 58		11.10	35 8
6	7.45	93 47	6	12.24	31 4
	9.60	100 30		13.52	26 56
	10.90	105 1			
7	7.45	103 1		-----	--- --
	9.45	111 18		9.20	26 56
	10.91	115 48		11.56	13 16
8	-----	--- --		13.10	5 2
	9.10	122 50			
	10.71	130 39			

TABLE II.

S. W. Quadrant.				S. E. Quadrant.			
Ray.	log r.	θ .	N.	Ray.	log r.	θ .	N.
1	0·00000	2° 39'	0·018	1	0·00000	0° 49'	0·002
	0·10388	2 56	0·017		0·09757	1 55	0·007
	0·16148	3 20	0·019		0·17053	2 20	0·009
2	9·99667	4 21	0·049	2	0·00066	1 7	0·003
	0·11062	6 51	0·092		0·08139	3 32	0·026
	0·17940	8 32	0·122		0·14755	7 40	0·106
3	0·00000	6 50	0·119	3	0·00208	9 7	0·209
	0·11062	8 37	0·146		0·08249	14 40	0·444
	0·17808	11 16	0·212		0·15224	18 30	0·594
4	0·00000	9 0	0·205	4	0·00066	14 22	0·515
	0·08848	11 37	0·277		0·10804	20 48	0·824
	0·16603	13 58	0·333		0·19448	28 32	1·229
5	0·00000	13 1	0·425	5	0·00133	18 14	0·817
	0·08685	20 43	0·858		0·11573	25 51	1·220
	0·14375	24 25	1·026		0·17053	31 4	1·506
6	0·00330	25 12	1·507	6	0·00066	20 42	1·200
	0·10804	33 37	2·001		0·13800	31 32	1·670
	0·16240	39 1	2·284		0·21718	37 50	1·912
7	-----	-----	-----	7	0·00199	31 14	2·242
	0·09225	39 2	2·688		0·11826	37 57	2·413
8	0·15131	47 28	3·211		0·19150	43 28	2·557
	-----	-----	-----	8	9·00723	34 38	2·655
8	0·11826	47 30	3·469		0·11927	43 34	3·024
	0·18808	55 58	3·731		0·19065	50 17	3·195
	0·23876	62 52	3·829		0·25120	57 26	3·261

N. E. Quadrant.				N. W. Quadrant.			
1	log r.	θ .	N.	1	log r.	θ .	N.
1	0·00000	3 40	0·034	1	0·00330	11 15	0·199
	0·09225	0 52	0·001		0·09757	12 0	0·289
	0·17499	1 30	0·004		0·16376	14 30	0·360
2	0·00000	0 2	0·000	2	0·00000	13 17	0·442
	0·10804	3 22	0·023		0·10284	15 30	0·472
	0·18808	6 30	0·069		0·16694	16 44	0·473
3	0·00000	3 38	0·037	3	0·00000	15 20	0·589
	0·10959	7 6	0·099		0·10284	19 1	0·702
	0·16603	8 56	0·138		0·18593	23 1	0·835
4	0·00000	6 50	0·119	4	0·00000	18 58	0·885
	0·10959	13 21	0·346		0·11522	24 5	1·056
	0·17587	17 42	0·517		0·18549	29 24	1·317
5	0·00000	11 35	0·338	5	0·00000	22 42	1·248
	0·10284	16 27	0·530		0·10804	30 14	1·657
	0·17940	21 28	0·742		0·19236	39 22	2·164
6	0·00000	19 17	0·914	6	0·23837	43 26	2·287
	0·12328	26 00	1·212		0·28825	47 34	2·347
	0·13800	30 31	1·572	6	-----	-----	-----
7	0·00000	28 31	1·909		0·10284	47 35	3·604
	0·11573	36 48	2·302		0·21152	61 14	3·956
	0·18419	41 18	2·388		0·27012	69 28	3·940
8	-----	-----	-----	8	-----	-----	-----
	0·09757	48 20	3·736		-----	-----	-----
	0·17543	56 9	3·849		-----	-----	-----
	0·25638	61 40	3·597		-----	-----	-----

The first process was to substitute the resulting values of $r \cdot \theta$ in the formula for the lines of force, $N = \frac{8\pi}{3} \cdot \frac{\sin^2 \theta}{r}$. The computed values are collected in table No. II.

It is observed that the values of N derived from measures on the same ray are not equal. If all the circumstances are fully accounted for, they would be alike and the angle θ would show the same polar distance for the beginning of the line N . But it is seen that there is a marked progression in these values, which indicates that some systematic error adheres to the work. An inspection of the case suggests that the projection of the ray from its position in space upon the apparent plane of the disk must be taken fully into account, for it is not probable that any large number of the visible rays start from the body of the sun in the very plane of the disk. To such rays the formula should have applied. The problem is therefore to discover at what point of the sphere each ray originates, and to assign angular coördinates to the same.

The figure illustrates how a ray springing from the surface of the sun is seen projected so as to lie across a series of true

Ray projected across a series of N -lines.

N -lines represented as proceeding from the disk. What was computed in the last operation is in fact the particular N -line that passed through the point as measured in its projected situation. We must now discover a means of determining through what angle α the plane of the ray N must be rotated in order to be seen on the plane of the disk. Also we must carefully distinguish between the pole of the corona and

the selected pole on the disk, for although these two poles lie in the same plane whose position angle with the sun's axis at the center of the disk has already been given, yet the angular distance of the coronal pole from the plane of the disk, being at this time unknown, has an immediate effect upon the angle α that is being sought. All the planes containing the rays intersect in the coronal axis, and if this was in the plane of the disk the rotation angle α would be the same for all measured points on the ray; otherwise α measured at the pole on the

disk will exhibit a progression of values as we pass along the ray. As each quadrant has been considered independently, no effort was made to generalize the angle α , but it is counted from the plane of the disk to the plane passing through the point on the ray, at the axis of intersection of the planes through the center of the sun.

$$N = \frac{8\pi}{3} \cdot \frac{\sin^2\theta}{r}.$$

Let $x_1 = r_1 \sin \theta_1$ and $y_1 = r_1 \cos \theta_1$, the accents indicating the first point that is measured on a ray.

$$r = \sqrt{x^2 + y^2} \quad \sin^2\theta = \frac{x^2}{r^2} = \frac{x^2}{x^2 + y^2}.$$

$$N = \frac{8\pi}{3} \cdot \frac{x^2}{(x^2 + y^2)^{\frac{3}{2}}}.$$

As a first approximation, assume that the axis of the planes of rotation lies in the plane of the apparent disk; hence by rotating a ray from the edge of the disk to its actual position, the values of y are unchanged, while those of x are reduced. The radius of revolution is $x_1 = r_1 \sin \theta_1$, therefore $x = x_1 \sec \alpha$.

Substituting,
$$N = \frac{8\pi}{3} \cdot \frac{r_1^2 \sin^2\theta_1 \sec^2\alpha}{(r_1^2 \sin^2\theta_1 \sec^2\alpha + r_1^2 \cos^2\theta_1)^{\frac{3}{2}}}.$$

Since the points are on the same ray, we take

$$\begin{aligned} \frac{r_1^2 \sin^2\theta_1}{(r_1^2 \sin^2\theta_1 \sec^2\alpha + r_1^2 \cos^2\theta_1)^{\frac{3}{2}}} &= \frac{r_2^2 \sin^2\theta_2}{(r_2^2 \sin^2\theta_2 \sec^2\alpha + r_2^2 \cos^2\theta_2)^{\frac{3}{2}}}. \\ \frac{(x_2^2 \sec^2\alpha + y_2^2)^{\frac{3}{2}}}{(x_1^2 \sec^2\alpha + y_1^2)^{\frac{3}{2}}} &= \frac{x_2^2}{x_1^2}. \\ \sec^2\alpha &= \frac{x_2^{\frac{4}{3}} y_1^2 - y_2^2 x_1^{\frac{4}{3}}}{x_1^{\frac{4}{3}} x_2^2 - x_2^{\frac{4}{3}} x_1^2}. \end{aligned}$$

The application of this formula gives the values of α corresponding properly to the mid-point between the two points from which it was derived. They also have a progressive value, indicating that the pole of the corona is not on the plane of the disk. It will not be far wrong to assume that value of α which is nearer the first α than the second, as would be seen by inspection from the points on a sphere.

Now substituting these values of α in

$$N = \frac{8\pi}{3} \cdot \frac{x^2 \sec^2\alpha}{(x^2 \sec^2\alpha + y^2)^{\frac{3}{2}}}.$$

we find that the ranging nature of the NS has ceased, and that there remain only such irregularities as are occasioned by the inaccuracy of the measurements themselves.

TABLE III.

S. W. Quadrant.					S. E. Quadrant.				
Ray.	α .	Adopted α .	θ .	Mean θ .	Ray.	α .	Adopted α .	θ .	Mean θ .
1	87° 16' 87 48	87°	35° 0' 31 41 30 21	32° 41'	1	88° 34' 87 18	88° 50'	31° 41' 33 25 29 28	31° 31'
2	84 16 82 0	84	29 22 31 23 30 44	30 30	2	88 27 85 49	88 40	30 28 32 34 (22 23)	31 31
3	82 25 79 36	82	33 59 32 26 30 50	32 25	3	79 26 71 53	79	34 25 34 59 31 48	33 44
4	77 40 72 50	78	30 17 31 25 30 14	30 38	4	71 21 65 10	71	33 45 34 1 31 38	33 8
5	74 55 64 15	75	33 51 35 51 33 20	34 21	5	65 43 60 40	66	34 1 34 19 33 25	33 55
6	57 4 51 1	59	36 33 36 35 35 41	36 16	6	62 8 48 6	62	34 52 34 54 32 38	34 8
7	----- 48 51	48	38 59 38 57	38 58	7	41 54 34 33	42	35 8 35 13 35 11	35 11
8	33 15 23 12	33	41 6 41 7 40 30	40 54	8	40 20 31 39 28 16	40	39 7 39 11 38 39 37 26	38 18

N. E. Quadrant.					N. W. Quadrant.				
Ray.	α .	Adopted α .	θ .	Mean θ .	Ray.	α .	Adopted α .	θ .	Mean θ .
1	----- 89° 22'	89°	30° 48' 28 35	29° 42'	1	----- 74° 14'	75°	33° 5' 30 17 30 2	31° 8'
2	89 48 88 53	88 50'	28 32 (19 27)	28 32	2	58 50 16 3	65	28 2 27 41 26 11	27 18
3	85 21 81 45	85	28 43 32 55 30 59	30 52	3	73 12 64 40	73	36 30 34 5 32 5	34 40
4	81 20 74 38	81	31 8 33 39 30 32	31 45	4	67 55 45 6	68	34 52 33 30 32 26	33 36
5	74 50 69 57	75	32 41 33 17 31 32	32 50	5	59 17 54 58 39 30 28 10	59	34 24 35 0 34 23 33 5 31 29	33 37
6	61 37 68 32	64	33 25 33 18 34 34	33 46	6	30 16	30	41 55 41 58 40 49	41 34
7	48 59 30 58	49	36 0 36 23 35 16	35 53					
8	23 20	24	42 25 42 23 40 16	41 41					

Take the typical formula for N and compute $\sin^2 \theta = \frac{3}{8\pi} \cdot N$.

The angle θ derived from this is the distance from the coronal pole at which the observed rays have sprung up from the surface of the sun. The values may be slightly in error by the small inaccuracy of the chosen α , but the changed projection resulting from this imperfection is not great.

This table shows that the coronal rays have their bases in a zone about 34 degrees from the coronal pole, the belt being about ten degrees in width, but its maximum density at the parallel of 34 degrees. The conclusion is drawn that there are no visible rays in the neighborhood of the poles, and hence the appearance of the corona is similar to that of the terrestrial aurora.

Position of the Coronal Poles.

We find the location of the north and south poles of the corona in the following way.

Let r, θ_1 represent the measured coördinates of each point, with the numerical suffix.

α_1 = angle at pole of disk to the point on the ray.

p_1 = angular distance from pole of disk to point.

θ' = angular distance from coronal pole to point.

r' = radius of spherical surface through point.

B = angle at coronal pole from pole of disk to point.

c = angular distance of coronal pole from pole of disk.

Then $x_1 = r_1 \sin \theta_1$.

$y_1 = r_1 \cos \theta_1$.

$z_1 = x_1 \tan \alpha_1$. $D_1 = x_1 \sec \alpha_1$.

$$\tan p_1 = \frac{D_1}{y_1} = \tan \theta_1 \sec \alpha_1.$$

$$r^2 = x_1^2 \sec^2 \alpha_1 + y_1^2.$$

$N = \frac{8\pi}{3} \cdot \frac{\sin^2 \theta_2}{r}$, where $r = 1$ for typical N, and θ_2 is the mean angle as computed.

$$\sin^2 \theta' = \frac{3r}{8\pi} \cdot N.$$

In the Spherical Triangle ABC we have given, $a = \theta'$, $b = p_1$, $A = 90^\circ - \alpha_1$.

Hence,

$$\sin B = \sin p_1 \cos \alpha_1 \operatorname{cosec} \theta'.$$

$$\tan \frac{1}{2}c = \frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} \cdot \tan \frac{1}{2}(p_1 + \theta').$$

P = projection of coronal pole c on a plane through the poles of the ecliptic.

TABLE IV.
S. W. Quadrant.

Ray.	$\log r'$.	θ' .	p .	A.	B.	c .	Mean c .
1	0.12501	38° 35'	41° 29'	3° 0'	176° 49'	2° 51' 18"	
2	0.08771	34 9	36 3	6 0	173 42	1 58 4	
3	0.11735	37 51	40 44	8 0	171 29	2 56 34	
4	0.09399	34 36	37 18	12 0	167 11	2 45 20	
5	0.11606	40 10	41 46	15 0	164 30	1 12 16	
6	0.09166	41 6	42 25	31 0	148 6	1 8 56	
7	0.17873	50 45	50 28	42 0	138 13	-0 23 42	
8	0.16308	52 11	52 27	57 0	122 41	0 30 34	1° 44' 58"

S. E. Quadrant.

1	0.08655	37 17	35 0	1 10	178 47	1 37 56	
2	0.08100	35 6	39 57	1 20	165 8	0 0 0	
3	0.11272	39 13	40 40	11 0	168 48	0 51 38	
4	0.09148	37 24	38 11	19 0	160 39	0 49 34	
5	0.08849	38 10	39 0	24 0	155 32	0 54 26	
6	0.08014	37 59	38 50	28 0	151 25	0 58 48	
7	0.04482	37 21	39 13	48 0	129 15	2 51 32	
8	0.05173	41 8	42 2	50 0	128 47	1 23 50	1 10 58

Mean value of c for the south pole, 1° 27' 58".

N. E. Quadrant.

Ray.	$\log r'$.	θ' .	p .	A.	B.	c .	Mean c .
1	0.21388	39° 20'	40° 55'	1° 0'	178° 58'	1° 35' 0"	
2	0.09159	34 45	36 4	5 0	174 50	1 20 14	
3	0.09717	36 3	37 27	9 0	170 42	1 24 30	
4	0.09678	37 19	38 23	15 0	164 38	1 5 16	
5	0.08194	38 41	38 36	26 0	154 3	-0 5 28	
6	0.05725	38 46	39 38	41 0	138 4	1 8 58	
7	0.12036	49 48	50 53	66 0	111 53	2 46 18	1° 9' 21°

N. W. Quadrant.

1	0.09564	35 15	37 33	15 0	164 8	2 24 2	
2	0.05720	29 20	29 11	25 0	155 7	-0 6 38	
3	0.12128	40 52	43 10	17 0	162 12	2 24 30	
4	0.10837	38 49	42 32	22 0	156 10	4 2 34	
5	0.07499	37 8	39 5	31 0	147 28	2 17 1	
6	0.13909	51 9	51 39	60 0	119 18	1 0 32	2 0 20

Mean value of c for the north pole, 1° 34' 51".

If θ' is smaller than p , c is positive, and the coronal pole is on the earth side of the plane of the disk.

This table is computed by taking the first point measured on each ray, and selecting the values of r_1 , θ_1 from Table II, and α , θ_0 from Table III. As shown by the formula, $r' \theta'$ is the computed point whose coördinates of projection were meas-

ured; p is the angular distance from the pole on the disk; A and B the interior angles of the triangle at the respective poles, and c the required angular distance of the coronal pole from the disk plane.

Now investigate the heliographic latitude over which these coronal streamers appear to terminate as definite lines. By many computations it is found that the r for the extreme points measured is equivalent to the $\log r' = 0.39800$, of lines which leave the sun at polar distance $\theta' = 34^\circ$.

$$N = \frac{8\pi}{3} \cdot \frac{\sin^2 \theta}{r}, \text{ where } r = 1 \text{ at the surface; hence } N = 2.620.$$

$$\text{Then, } \sin^2 \theta = \frac{N \cdot 3r'}{8\pi}; \text{ and } \theta = 62^\circ 10'.$$

The corresponding equatorial distance is $90^\circ - 62^\circ 10' = 27^\circ 50'$, which is in the midst of the zone of the sun spots at the minimum of the period.

The heliographic latitude and longitude of the north and the south coronal poles, as seen on July 29, 1878, are obtained as follows.

Location of the coronal poles for July 29, 1878.

- K Poles of the ecliptic.
- S Poles of the sun's axis.
- E Poles of the plane of the equator.
- C Poles of the corona.
- D The center of the disk.
- KP The plane through the poles of the ecliptic perpendicular to the radius vector to the earth.
- M The projection of poles of the corona on the disk.
- N " " " sun " "
- P " " " equator " "
- O The direction of the ascending node at 74° longitude.

The North Pole.

KN Position angle KDS = + $4^{\circ} 27' \cdot 6$ counted from N to W.
 KP " EDK = $-14^{\circ} 22' \cdot 1$ " N " E.
 PN " EDS = $-9^{\circ} 54' \cdot 5$ " N " E.
 NM Position angle of coronal pole = $-1^{\circ} 6'$ counted from N to E.

MC Angular distance of coronal pole from plane of disk, on the side towards the earth = $1^{\circ} 34' \cdot 8$.

PM Position angle of coronal pole = $-11^{\circ} 0' \cdot 5$ counted N to E.

KS The inclination of the sun's equator = $7^{\circ} 15'$.

NKS = $(270^{\circ} + \odot) - (270^{\circ} + 74^{\circ}) = 126^{\circ} 10' \cdot 97 - 74^{\circ} = 52^{\circ} 10' \cdot 9$.

$\tan NK = \cos NKS \tan KS = 8 \cdot 89212$. $NK = 4^{\circ} 27' \cdot 6$.

$\sin SN = \sin NKS \sin KS = 8 \cdot 99866$. $SN = 5^{\circ} 41' \cdot 6$.

$ST = SN - CM = 4^{\circ} 6' \cdot 8$. $CT = MN \cos CM = 1^{\circ} 5' \cdot 97$.

$\cos CS = \cos CT \cos ST = 9 \cdot 99880$. $CS = 4^{\circ} 15' \cdot 4$.

$\cot CST = \sin ST \cot CT = 0 \cdot 57257$. $CST = 14^{\circ} 58' \cdot 8$.

$NKS = 52^{\circ} 10' \cdot 9$. $NSK = 38^{\circ} 2' \cdot 6$. $NSC = 14^{\circ} 58' \cdot 8$.

$CSK = NSK - NSC = 23^{\circ} 3' \cdot 8$. $OSK = 90^{\circ}$.

$OSC = OSK - CSK = 66^{\circ} 56' \cdot 2$. Heliographic longitude.

$90 - SC = 85^{\circ} 44' \cdot 6$. " latitude.

Coördinates of the north coronal pole:

Long. $66^{\circ} 56' \cdot 2$

Lat. $+85^{\circ} 44' \cdot 6$

The South Pole.

KN = KDS = $-4^{\circ} 27' \cdot 6$ counted E from S.

KP = EDK = $+14^{\circ} 22' \cdot 1$ " W " S.

PN = EDS = $+9^{\circ} 54' \cdot 5$ " W " S.

PM = EDC = $3^{\circ} 54' \cdot 5$ " W " S.

PKS = $52^{\circ} 10' \cdot 9$.

KS = $7^{\circ} 15'$.

SN = $5^{\circ} 41' \cdot 6$. MN = $6^{\circ} 0'$.

KN = $4^{\circ} 27' \cdot 6$. MC = $1^{\circ} 28' \cdot 0$.

ST = $7^{\circ} 9' \cdot 7$.

$\cos SC = \cos ST \cos CT = 9 \cdot 99420$. $CT = 5^{\circ} 59' \cdot 88$.

$SC = 9^{\circ} 20' \cdot 5$.

$\cot CST = \sin ST \cot CT = 0 \cdot 07400$.

$CST = 40^{\circ} 8' \cdot 5$.

$SKN = 52^{\circ} 10' \cdot 9$.

$NSK = 38^{\circ} 2' \cdot 6$.

$CSN = 40^{\circ} 8' \cdot 5$.

$CSK = 78^{\circ} 11' \cdot 1$. $OSK = 90$.

$OSC = 168^{\circ} 11' \cdot 1$. Heliographic longitude.

$90 - SC = 80^{\circ} 39' \cdot 5$. " latitude.

Coördinates of the south coronal pole:

Long. $168^{\circ} 11' \cdot 1$

Lat. $-80^{\circ} 39' \cdot 5$

Difference of longitude.

$101^{\circ} 14' \cdot 9$

It is interesting to compare these coördinates of the polarization of the sun with the similar coördinates of the magnetism of the earth, as given by Erman and Petersen for 1829, from Gauss' Theory.

North Pole, Long.	266° 3'·8
Lat.	73° 21'·
South Pole, Lon.	150° 44'·9
Lat.	— 72° 40'·4
Difference of Longitude,	115° 18'·9

This fact of general agreement of the difference of longitude may be purely accidental, or it may point to some fundamental law of polarized rotating spheres.

This analysis of the solar corona suggests certain conclusions.

1. The force seems to be repulsive, the law of its action being expressed by a transcendental equation of the second degree. This agent is sufficient for the transportation of finely subdivided matter, and harmonizes with the lack of density of the sun's atmosphere, as indicated by its failure to influence the motion of comets passing within its limits.

2. The individual streamers are grouped in a zone about ten degrees wide, whose maximum density is at 34 degrees from the coronal poles. The number of such rays is not great, but their actual dimensions are enormous. The average linear *visible* extension is about one solar radius, and regarding the residual propulsion and curve of the trajectory their extremities are located normally above the sun spot belts. At this place the incandescence of the material particles apparently ceases, and if condensation sets in, there would exist the conditions required for the precipitation of cool masses, whose fall upon the surface of the sun is generally supposed to produce the spots. It is at this zone of maximum for the coronal rays that the deficiency of the prominences has been observed, and there may well be a physical connection between these two classes of phenomena. The re-entering form of the curves is also consistent with the existence of atmospheric currents flowing from the polar regions towards the equator, and a study of the angles of inclination of the prominences relatively to normals may develop some evidence on this point. The condensed bodies of light, seen on two axes at 40 degrees from the poles, are doubtless due to the perspective effects of the maximum zone as it passes around the sides of the sun; and the structureless equatorial wing is no doubt a floating mass of matter, cooling in the process of preparation for precipitation. This return of the ejected matter to the sun is considered necessary to account for the relation of the total flow of

energy outwards and the rate of change in its volumetric dimensions, as derived from theory.

3. The location of the coronal poles at successive eclipses will afford a means of determining the period of the rotation of the sun on its axis, in consequence of the large number of revolutions occurring between such epochs. It would be necessary to assume that the axis of polarization of the sphere remains the same, as is probably the case.

4. The accelerated angular velocity of the equatorial belts as compared with the polar regions, would result from the descent of cool matter from high altitudes above the surface of the sun, each particle being considered as a satellite approaching its center of gravity. The motion of the spot belts in latitude synchronously with the display of energy as recorded by the maxima and minima, may be due to a corresponding motion of the maximum zone towards and from the poles, with the accompanying elevation of the ends of the coronal stream lines; or to changes in the relative energies of propulsion, as a function of the time. It is possible that a picture of the corona may be sometime taken which will show these streamers distributed in two parallel belts a few degrees apart. It is a most interesting question in physics as to the reason of the location of this coronal zone at the computed distance from the poles, since it suggests also a problem similar to that of the terrestrial auroras. At this place the linear distance across the greatest number of equipotential surfaces, is apparently the shortest, hence it may be the path of least resistance. There should also be a connection between the distribution of the actinic light of the corona and the equipotential surfaces, and it is not unlikely that the light is simply proportional to the potential.

This discussion suggests the importance of securing photographs of the corona sufficiently large to admit of accurate measurements; the necessity of studying the relation between the position of the streamers and the spots at the moment of the eclipse; and the possibility of deriving the period of the sun's axial rotation from the coronas of successive eclipses.