please make explicit use of the Principle of the Permanence of Equivalent Forms, which, after having been expounded at length and defended by Peacock (appendix to "Algebra"), has been summarily rejected as misleading and unmeaning by many recent authors. To the formal reintroduction of this principle Mr. Hayward's language exhibits a tendency to return. Outside the domain of elementary Algebra, its strict employment in the prolongation of an analytical function into a new region is indeed of common occurrence in Analysis; while its tentative application in unrestricted form, as an instrument of suggestion and discovery in the Theory of Operations, is fundamental. To the effort to widen the limits of interpretation in connexion with it, has been due most of the advances in Analysis.

It is a fundamental question in mathematical logic how far, after having carried the stream of our analysis through regions of uninterpreted symbols, and having at length arrived at a stage in which these symbols have disappeared, we are entitled to claim this procedure to be demonstrative. It is of course of the very essence of Algebra that the intermediate steps of its analysis remain uninterpreted; though in the Algebra of real quantities we have a tacit assurance that an interpretation can be supplied if necessary. Why then was there an objection to a similar procedure in the Algebra of complex quantities; and what is the source of the timidity and doubt which characterized the use of complex analysis before its geometrical interpretation was developed? Simply that complex quantities turned out to be multiple-valued, and that the selection of the proper value under given circumstances had to be settled by tracing the continuity of the quantities in a way that was to the mind practically impossible until a visual geometric representation was discovered. The Argand diagram is not essential to the logic of the matter; it rather makes Analysis possible by bringing its scope within our grasp. It simply forms a more extensive and systematic example of the method which has been in use since the time of Descartes for studying functions and approximating to their roots, by aid of their graphical representation.

The Principle of Permanence of Equivalent Forms thus lies at the very root of Algebra, but it is rendered ineffective by indeterminateness of interpretation. Its strict use, when most needed, is subject also to another hitch. It requires that the forms be expressed in exact terms; an infinite series must be expressed in the sum of n terms together with a residue R . These residues must be retained throughout the analysis until we arrive at a point where interpretation comes in; and it must then be settled how far they can be neglected in the circumstances of the actual interpretation. In the language of Mr. Hayward, it cannot be asserted about series that are not absolutely convergent, that the fundamental laws of Algebra hold without limitation.

It is perhaps a question how far the idea, thus restricted and safeguarded, is worth being expressly retained as a working principle of ordinary Algebra. In subjects like the Calculus of Operations and Finite Differences, which are still in an unsystematized stage, it cannot be dispensed with; and the extent to which its use is boldly pushed, by De Morgan and Boole, even to the discussion in an operational manner of divergent series without their

residues, contrasts with the more exact processes of recent Analysis. How far this boldness arises from the profound logical studies of these writers, and their appreciation of the imperfect character of inference at the best, may be a subject open to discussion.

In connexion with the doctrine of convergence of series, the author gives a very clear account, from Sir G. Stokes, of how it is that, on approaching certain critical points, the convergence may gradually fall off and finally disappear. The illustrations employed are algebraic series of an exceptional character; and the whole circumstances may possibly suggest to the uninitiated that it is a phenomenon of exceptional rarity. The most natural context is, of course, in connexion with the wonderful and far-reaching theory initiated by Fourier, by means of which functions arbitrarily discontinuous are expressed by seemingly continuous series. In that connexion, the necessity of explanation is so obvious that it is interesting to examine the previous attempts at elucidation. Thus De Morgan, in 1839, is able to conclude ("Diff. and Int. Calc.," pp. 233, 239) that such discontinuity cannot occur in series proceeding by powers of a real variable; that in other cases it occurs only through the series becoming divergent at the point of discontinuity. It is, however, an important question how far it would be allowable to avoid burdening an elementary exposition by complete precautions against the existence of anomalies like this, which would hardly have originally occurred to any one in that early stage.

The book ends with a wider survey, including a clear and interesting account of Cauchy's theory of the radical points of a rational function. The graphs of the cubic $x^3 + ax$, which are given as an illustration, would also form excellent and rapid examples of the Rankine-Maxwell method of graphical addition, applied to the separate graphs of x^3 and ax .

J. L.

FOSSIL PLANTS AS TESTS OF CLIMATE.

Fossil Plants as Tests of Climate, being the Sedgwick Prize Essay for the Year 1892. By A. C. Seward, M.A., F.G.S., Lecturer in Botany in the University of Cambridge. (London: C. J. Clay and Sons, Cambridge University Press, 1892.)

THIS admirable essay is really a digest of the opinions of the principal writers on fossil plants, so far as they throw light on geological climates, and a critical *résumé* of the subject up to date. It should be read by all who prefer to deduce the relative temperatures of various latitudes in the past from such solid data as assemblages of ferns, cycads, and conifers, the ancestors of living genera and species, rather than from utterly extinct belemnites, ammonites, and saurians, of whose habits little can ever be known, and which might have drifted far out of their temperature zones by warm and cold sea-currents.

Perhaps if any criticism can be made, it is that too little has been said by the author as to what is known of the Mesozoic floras, which, if scanty, are extremely interesting. In fact only the widely-separated Palæozoic and Cenozoic floras are fully dealt with. Owing to the magnitude, difficulty, and freshness

of the problems presented by the former, they have received the larger share of attention and have ever attracted some of the most acute and philosophic of scientific workers. But while the researches of such investigators as Williamson and Renault into the actual structures and affinities of the carboniferous plants have been rewarded with the most brilliant successes, attempts to speculate and theorize have only been productive of barren controversy. All inferences as to the temperatures in which they flourished have merely been inductions from unknown data: their affinities with existing plants are so remote that they can tell us of little beyond moist climates and spongy, marshy soils, liable to inundation, with possibly an atmosphere more highly charged with carbonic acid than at present. But that neither the flora nor identical conditions were uniformly present over the whole land during the deposition of the carboniferous, becomes every year more apparent; and perhaps few would now maintain that fossil floras met with in widely-different latitudes must necessarily be contemporary because similar, or reject as impossible the correlation of the *Glossopteris* floras of the southern hemisphere simply because they are so dissimilar.

Tertiary floras, however, have to be approached from almost totally different standpoints, for here minute investigations into vegetable structure can only exceptionally lead to important results. On the other hand it may be possible to predicate the climate that any group among them would have required, with almost perfect accuracy. Allowing that even most closely-allied species may have had different habits, enough remain that are practically identical with living species. These not only prove to us that in every land in our hemisphere the temperatures remained warmer throughout the Tertiary period than at present, but also that the temperatures were far from equable during Eocene time. Thus it is impossible to hesitate as to the evidence of the flora in the lower stages of our Eocene, which exhibits an abundance of planes, poplars, and alders and an absence of all approach to sub-tropical essences; nor as to that of the London Clay, with its tropical *nipas*, *sabals*, and a host of others almost indistinguishable from species existing at the present day. There is scarcely need of the corroborative evidence of the *Mollusca* as to cooler seas in the Thanets, nor of tropical conditions in the large turtles, crocodiles, snakes, and nautili of Sheppey. In fact, the temperature of the spots occupied by Reading, Bournemouth, or Mull at a particular stage of the Eocene could be predicated from the fossil floras almost as accurately as from living plants. If the same cannot be said of the Arctic regions it is simply that the specimens brought home are, perhaps from the exigences of travel and inexperience of the collectors, for the most part so imperfectly preserved and fragmentary, that few of the determinations can carry the smallest weight. It may suit quidnuncs to accept indeterminate fragments as evidence of the growth of palms and cycads in the Greenland Eocene—it is time the Miocene age of these beds was relegated to the limbo of Coal-measure palms and yuccas—and to become excited over the presence of a sub-tropical flora within the Arctic circle; but as a fact it is doubtful whether anything has been discovered there which might not have grown in our own temperature, if

slightly modified, a state of things which it is conceivable the damming back of the Arctic seas by the land connection which then existed between Europe and America, aided by an active Gulf Stream, might have brought about. When we come to the Miocene, worked out as that of Switzerland was by Heer, or still more the Quaternary, with such data as those laboriously amassed by Clement Reid, the inferences as to climate are still more irresistible.

As to evidence of the age of rocks, plants are less trustworthy, because they have neither been so perfectly studied nor are their zones as yet at all properly defined. All we can say is that certain assemblages are found in association at the beginning of the Tertiary, and that changing temperatures have since compelled them not to disperse, but to migrate far and wide. Fewer probably of the species are extinct than is generally supposed, and the primitive associations have held together perhaps to the present day, with many gaps from extinction and desertion and a large infusion of recruits through the ordinary causes of evolution, stimulated by the increase in browsing mammalia. Whether, on the other hand, the marine deposit zones are really entitled to the weight attached to them as evidences of age, except locally, is not so clear. They are usually the littoral deposits of a limited area, where some changes of level or current have apparently suddenly driven out the fauna and introduced new colonies more adapted to the changed conditions. If we could follow the subsequent wanderings of these assemblages under the sea our faith in their sudden extinction and consequently in their chronological value might be greatly modified. At all events, many of the less conspicuous groups of mollusca, when critically examined, prove to have surprisingly near relatives in distant seas at much later periods, and even at the present day.

J. STARKIE GARDNER.

OUR BOOK SHELF.

Pioneers of Science. By Oliver Lodge, F.R.S. (London and New York: Macmillan and Co., 1893.)

THIS book consists of eighteen lectures on the history and progress of astronomy, which were delivered by Dr. Lodge in 1887. "The lectures having been found interesting," he thought it "*natural* to write them out in full and publish," and, although this can scarcely be considered a sufficient excuse, the intrinsic merits of the work are abundant justification for its existence. In Part I., "From Dusk to Daylight," the progress of astronomy from Copernicus to Newton is traced in a series of vivid pictures of Copernicus, Tycho Brahe, Kepler, Galileo, Descartes, and Newton; while Part II., "A Couple of Centuries' Progress," brings the history of gravitational astronomy from Newton down to the present time. In these latter lectures Roemer and Bradley are associated with the velocity of light and aberration; Leverage and Laplace with the solar system and the nebular hypothesis; Herschel with the motion of "fixed" stars; Bessel with the distances of stars; Adams and Leverrier with the discovery of Neptune; and Lord Kelvin and George H. Darwin with tides. Dr. Lodge has been able, by judiciously combining clear statements of scientific facts and laws with interesting personal details, to give his lectures all the charm of a romance. The book is an admirable introduction to the study of astronomy, and no better gift for a beginner could well be chosen; while to those to whom many of