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[F O U R T H S E R I E S .]

ART. XXI.—*The Velocity of the Propagation of Magnetism* ;
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THE effect of self-induction in a circuit causing a retardation in the establishment of a magnetic flux is of course well known to everyone ; but an allied effect, caused not by the inductance between coil and magnetic circuit, but in the magnetic circuit itself, thus causing a very perceptible interval of time between the appearance of the magnetism at two points, has not been so thoroughly studied, although it must be present in all alternating current machinery to a certain degree. This retardation is thus due to the eddy currents in the core of the magnet in question and is a complicated function of the various physical properties of the core, the frequency and magnitude of the impressed electromotive force.

This phenomenon was first investigated in detail by Oberbech* in 1884. His apparatus consisted in a core with magnetizing coil. Beside this coil were two others whose distance apart could be varied at will. To measure the phase angle between the electromotive force generated in these coils when a sinusoidal e.m.f. was impressed on the main coil, he made use of a dynamometer, the fixed coil being in series with one of the sliding coils, and the movable one with the other. The resulting deflection he showed to be a function of various known quantities and some unknowns that were eliminated by repeating the experiment with added resistance in series with one of the two circuits, thus obtaining a different deflection. This was repeated at different distances up to 20^{cm}. His results show values of the lagging angle from 2° 16', when the distance was

* Wied. Ann., xxii, p. 73.

10^{cm} and the core a bundle of 64 small steel wires, up to 96° 12' in 10^{cm} for a steel bar of 1·2^{cm} diameter. This shows at least qualitatively what we should expect, that lamination reduces the lag. The curves of flux intensity taken along the bar appeared to be logarithmic curves whose equation was $Q = Q_0 e^{-\beta x}$. β varied somewhat, and his results were tabulated as follows:

$$\begin{aligned}\beta \text{ for soft iron} &= \begin{cases} 10\cdot27 \\ 10\cdot17 \\ 10\cdot07 \end{cases} \\ \beta \text{ for hard iron} &= \begin{cases} 14\cdot8 \end{cases} \\ \beta \text{ for steel} &= \begin{cases} 14\cdot51 \\ 16\cdot16 \\ 16\cdot37 \end{cases}\end{aligned}$$

x being measured in meters. In conclusion, the author says that the size of β varies only as the material and is independent of the diameter. This is also true of direct current phenomena; hence he argues that were it not for the opposition of eddy currents the velocity would be enormous.

When I first began investigations on this effect I was not aware of the work just cited. But inasmuch as the results were somewhat of a qualitative nature, and the method used is open to serious objection, I thought it worth while to continue the research.

The method employed was in some respects similar to that of Oberbech. A magnetizing coil was mounted on the center of a steel bar about one meter long and having a section of 2·83^{cm}². It was of the best English tool steel. This material was used in order to obtain as low a velocity as possible and thus measure it more readily. On the middle of the magnetizing coil were wound 250 turns of fine wire, and sliding on the core were two more coils, one of 500 turns and one of several thousand. As will be seen, the exact number is not an essential quantity.

In performing the experiment one of the movable coils was set at some known distance from the exciting coil at the center, and the angle of lag between it and the small coil of 250 turns was measured. For short distances, up to about 8^{cm}, one of 500 turns was used; beyond that the larger coil. The instrument by which this angle was measured was a small quadrant electrometer having double quadrants and an aluminum needle suspended by a quartz fiber and dipping into sulphuric acid through which the contact was made. The needle and one pair of quadrants constituted one terminal and the other pair of

quadrants the other. By means of two Kempke discharge keys the circuits were so arranged that the electrometer was normally on short circuit, but on pressing the keys simultaneously the terminals of the external circuit were simultaneously attached to the electrometer terminals. This method was essential, for with one terminal free and the other connected to some metal object the instrument would be most violently deflected; hence the necessity for making the contacts exactly together.

As it was purposed to measure alternating voltages, the instrument was calibrated with the use of alternating e.m.f. of the

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same frequency. These were measured by "weighing" in a Kelvin balance the current that went through a non-inductive resistance across whose terminals the electrometer was connected. The resistance was measured after each reading to allow for possible changes due to heating. It may be of interest to note that the calibration curve as obtained by this method differed considerably from one obtained with constant voltages. This was doubtless due to the capacity introduced by the cup of sulphuric acid into which the needle dipped.

Several readings were necessary in determining a single angle. Calling the small central coil wound on the exciter "A," and the more distant coil "B," we may simplify the description as follows: The electrometer was first connected to A, giving a deflection D_a which corresponded to an induced potential V_a .

Next by connecting to B, D_b and thus V_b was obtained. Then by connecting A and B in series in such a way that they were in accord, V_{a+b} , and finally when opposed $V_{a'-b}$, were observed. The accent above "a" in the last quantity refers to the necessity of reversing the connections of A (or B), and, strangely enough, the deflection given by any coil was slightly altered on reversal. The reason for this I was unable to determine, unless, indeed, it is due to inequalities between the positive and negative loops of the commercial current which was used. This seems unlikely, and I purpose to investigate the matter further. This effect was not harmful if properly observed and allowed for.

With the four readings just referred to it was possible to calculate the angle of lag in two ways that can best be seen in graphic form.

If the two vectors V_a and V_b were in phase, then V_{a+b} must be equal to their arithmetical sum, otherwise it will be less, and

this was always found to be the case. The construction can be seen from a glance at the diagram, which gives us ϕ , or the angle of lag. Similarly with the two coils opposed, $V_{a'-b}$ will be greater than the arithmetical difference of $V_{a'}$ and V_b , as shown above. In the results recorded here the latter method was used as being less liable to error, for it involved no large deflection like D_{a+b} and for small values of ϕ it is also true from trigonometrical reasons.

Determinations of ϕ were made at intervals up to nine inches from the central coil, and from these values the velocities for the parts in question were calculated. From inspection of the curve it is clear that $\frac{d\phi}{dx}$ is not a constant, hence the velocity cannot be constant. The velocity at any point in the bar was found from the angle curve by observing the change in lag for a short distance on either side of the point in question. Calling

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this distance “ d ” and the angular difference “ δ ,” we have $v = \frac{360 \cdot d \cdot n}{\delta}$, where n = the frequency, and in this case was 60 cycles per second.

In comparing the curves of magnetic leakage and velocity it is evident that as the flux in the bar diminishes the velocity increases with a consequent lessening of the lag per unit length. This is what would be expected, for with a decrease of flux the eddy currents must grow smaller, and there is less opposition to the advance of the magnetism. Beyond nine inches the velocity

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increases with great rapidity and presumably becomes equal to the velocity of light when the flux density is infinitely small, or when the core is a non-conductor. To test this latter case I made an observation with an air core, and with coils A and B about 6 inches apart. In this case the induced e.m.f. was too small to appreciably affect the electrometer, so a very sensitive D'Arsonval ballistic galvanometer was used. Instead of an alternating current a constant voltage was impressed on the exciting coil and the throw of the galvanometer observed at both “make” and “break.” Now if B is so adjusted that it gives exactly the same throw as A, then when they are in series and opposed there should be no throw provided the resistance and self-induction of both are made equal. This condition was fulfilled by adding an external coil in series with A, thus mak-

ing that circuit similar to that of B. The result, as expected, was no deflection, although so sensitive was the apparatus that a change of one millimeter in the position of B overthrew the equilibrium. This at least showed that the velocity must be exceedingly great as compared to that in the steel bar.

It only remains to show that the mathematical theory predicts results similar to that observed. In an article by J. Zenneck,* the author develops the theory of the propagation of magnetism, which, though it takes no account of hysteresis, should give at least approximate values when properly used. The fundamental equation is similar to that for variable currents:

$$(1) \quad Q \dot{w}_m = - \frac{\delta V_m}{\delta x} - p_m \frac{\delta Q}{\delta t},$$

where Q = flux,
 w_m = reluctance,
 V_m = magnetic potential,
 p_m = coefficient of self-induction of the core.

By applying the law that the rate of leakage is proportional to V_m , i. e.

$$(2) \quad C_m = - \frac{\delta Q}{\delta x} / V_m,$$

where C_m is the magnetic analogue of capacity, and combining (1) and (2), the equation (3) is obtained:

$$(3) \quad C_m p_m \frac{\delta Q}{\delta t} + C_m w_m Q = \frac{\delta^2 Q}{\delta x^2},$$

whose solution must be of the form:

$$(4) \quad Q = Q_0 e^{-\beta x} \cdot e^{i(\pi n t - \gamma x)},$$

where β is the damping factor, n = twice the frequency and $\gamma = \frac{\pi n}{v}$, v being the desired velocity. This solution satisfies the equation when

$$(5) \quad \begin{cases} \beta^2 - \gamma^2 = C_m \cdot w_m \\ \text{and } 2\beta\gamma = C_m \cdot \pi n p_m. \end{cases}$$

To find v it is necessary to know three of the quantities, β , C_m , w_m and p_m . As p_m presents the greatest difficulty, the first three were selected by the author of this article. β was deter-

* Drude's Ann., 1903, No. 4, p. 845.

mined directly from the leakage curve, assuming it to be fairly well represented by the equation $\beta = \log \frac{Q_o}{Q_x} / x$. The average value was .16, which gives a curve fitting the original fairly well near its middle point. This agrees quite closely with Oberbeck's value for steel; the difference in the position of the decimal point is due to his choice of the meter as the unit of length instead of the centimeter.

The calculation of C_m is more difficult, particularly as Dr. Zenneck is not quite explicit in explaining V_x ; but I venture to suggest the following method as applied to a point four inches from the center of the exciting coil. The difference of potential of the two faces of a coil of n turns carrying a current I is $\frac{4\pi n I}{10}$ as determined by the amount of work done in carrying a unit pole from one face around outside the coil to the other. This work would be increased if the medium were of greater permeability than air; moreover, the work done would be half the total if the unit pole were carried only from one face to the far end of the bar, where the flux is almost zero. The fall of potential would then vary as Q along the bar. From these considerations I calculated V_x by the following equation:

$$V_x = \frac{4\pi n I}{10} \cdot \mu \cdot \frac{Q_x}{2Q_o} = 27,$$

where $n = 4077$ (No. turns in coil),

$I = .015$ ampères,

$\mu = 2.95$,

and $\frac{Q_x}{Q_o} = \frac{3.1}{12.9}$, as may be seen from the leakage curve.* The

value of μ was found by carefully measuring the resistance of the exciting coil and the currents flowing in it under an alternating pressure of 120 volts, when the steel core and an air core were successively used. The impedance in one case was 7998 ohms, in the other 2727. Knowing the resistance and the frequency, it was easy to calculate the coefficient of self-induction for iron (L_i) and for air (L_a), and $\mu = \frac{L_i}{L_a}$.

Referring to equation (2) we still have $\frac{\delta Q}{\delta x}$ to determine, which can be done readily by differentiating $Q = Q_o e^{-\beta x}$ and substituting the values of Q_o , β and x . β has been determined, $x =$

* These and other values were taken from the original curves as plotted in the laboratory. Those prepared for photographic reproduction give the general form correctly but are not strictly accurate copies.

$10 \cdot 16^{\text{cm}}$, and Q_o is found from the equation $Q_o = \frac{10 \cdot \sqrt{2} E \cdot 2}{\pi n w}$, where $E = 12 \cdot 9$, $n = 500$, $w = 377$, and Q_o is the average value of the flux at the center of the exciting coil. Hence $\frac{\delta Q}{\delta x} = -Q_o \beta e^{-\beta x} = -\frac{6146 \times \cdot 16}{e^{1 \cdot 63}} = 193$. Finally $C_m = -\frac{\delta Q}{\delta x} / V_x = \frac{193}{27}$, the negative sign vanishing because $\frac{\delta Q}{\delta x}$ is negative.

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The average flux density at a point four inches from the center is 522 lines per square centimeter, obtained similarly to Q_o above, and the permeability at this density was found to be 110, as shown in the permeability curve of the bar used.* This gives the reluctance per unit length as $w_m = \frac{1}{110 \times 2 \cdot 83}$; 2.83 being the cross section of the bar.

We are now in a position to make the final calculation, employing equation (5) and solving it for γ , which is equal to

$$\sqrt{\beta^2 - C_m w_m}. \quad \text{We have } \gamma = \cdot 0513 \text{ and } v = \frac{\pi n}{\gamma} = 7336^{\text{cm}}/\text{sec or}$$

* The author owes sincere thanks to Messrs. E. E. Moran and D. H. Miller, students in Yale University, for the determination of the BH curve of the steel bar.

245^{ft}/_{sec} as against 257^{ft}/_{sec} actually observed. It must be admitted that values of v calculated for other points along the bar did not agree so closely with observation, those nearer the center being too small and those farther out too large; but considering the fact that equation (5) makes γ exceedingly sensitive to very small errors in the values β , C and w , a closer agreement could hardly be expected.

A more approximate method suggested by Zennech gives $\beta/\gamma = 2.4$ for cases of moderate permeability and frequencies of about fifty cycles. Using $\beta = .16$ this gives $v = 5712^{\text{cm}}$ /_{sec}, which is as close as could be expected from the approximate nature of the assumptions.

In conclusion we may, I think, accept as proved that the velocity of magnetism in metallic paths, and especially those of high permeability, is small compared to that in non-conducting bodies. That this effect is due to eddy currents and therefore variable, diminishing as the flux diminishes along the bar. That this velocity is not a simple function of x , but depends on the physical properties of the bar, such as ohmic resistance and permeability, as well as the magnetic density at the point in question. And that this retardation is great enough to cause a very perceptible lag where the density is high and the lamination poor; great enough indeed, it would seem, to call for recognition in the design of many forms of electromagnetic machinery.

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