CHICAGO GEOMETRY SYLLABUS.

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PREFACE.

This syllabus lays no claim to completeness, nor does it intend to suggest any order in which theorems or groups of theorems may be presented. The syllabus has, however, two distinct aims, namely: to show the possibility of emphasis in presentation and of applications with a concrete setting.

Geometry should not be taught as one "dead level of monotony." A clear and lasting impression of a vast field can be obtained only through distinctions in emphasis. This is a fundamental characteristic of the human mind. It is hoped that by some such classification of theorems as is here indicated, the dependence of the minor theorems upon the more important ones can be clearly shown and that by careful choice and grouping of exercises the important theorems can be emphasized.

The outlines for concrete work and applications are intended to be suggestive only. It is hoped that each teacher will find therein abundant material from which to choose. While every teacher realizes the necessity of problems with concrete setting, it has until very recently been extremely difficult to obtain any variety of such problems. The references here inserted may prove a suggestive and helpful source of supplementary problems.

GENERAL SUGGESTIONS.

A. The following theorems may be assumed or proved informally:

1. Only one perpendicular can be drawn to a line from a given point, whether the point be within the line or without the line.
2. All straight angles are equal.
3. All right angles are equal.
4. Only one line can be drawn bisecting a given angle.
5. The sum of two or more angles on one side of a straight line having a common vertex is two right angles.
6. The total angular magnitude about a point is four right angles.
7. Complements and supplements of equal angles are equal.
8. Vertical angles are equal.

¹This Syllabus was presented to the mathematics section of the high school teachers of Chicago sometime ago and by them presented to the superintendent of schools. Since issuing the report, some books have appeared which might have been added to the list of references, such as Betz and Webb Geometry, by Ginn & Co., and Source Book of Problems for Geometry by Mable Sykes, Allyn & Bacon,

9. Any side of a triangle is less than the sum of the other two.
10. If the sum of two angles is two right angles, their exterior sides form a straight line.
11. Only one parallel can be drawn to a line through a given point.
12. A perpendicular from a point to a line is the shortest distance from the point to the line, and the converse.
13. Circles with equal radii are equal, and the converse.
14. A diameter bisects the circle.
15. A straight line can intersect a circle at most in two points.
16. Angles at the center of a circle are measured by the subtended arc.
17. In the same circle or in equal circles, equal angles at the center subtend equal arcs, and the converse.

B. Constructions.

I. The ordinary simple constructions should be introduced from the beginning as needed.

II. Figures for theorems should be constructed according to the hypothesis and (except occasionally in blackboard work) should be constructed exactly. It is often desirable for the pupil to discover by measurements made on an accurate figure, facts and relations that serve as a basis for future theorems.

III. Original problems in construction should be attacked by special methods. Two such methods deserve special attention:

a. Method by analysis, such problems as:

1. Construction of triangles and quadrilaterals from given data. Those requiring the construction of auxiliary triangles may be introduced wherever possible, *e. g.*, construct a triangle ABC given the altitude and the median upon a and the side b . This may be given with the construction for a right triangle given the hypotenuse and a side.

2. Construction of lines, *e. g.*, construct a line that shall pass through a given point and make a given angle with a given line.

3. Construction of equivalent figures, where an algebraic analysis is used.

b. Method by intersection of loci. Such problems as the construction of triangles and circles, and the determination of points.

IV. References.

1. Wright, Exercises in Concrete Geometry (chapter on loci).

2. J. Petersen, *Methods and Theories for solution of Problems in Geometrical Construction*. Copenhagen. Translation may be obtained through G. E. Stechert & Co., New York.

3. Mahler, *Ebene Geometrie*; Part III, Leipzig.

4. Morris and Husband, *Practical Plane and Solid Geometry*.

5. Wentworth-Smith (*Constructions classified*).

6. Durrell, *Plane and Solid Geometry*.

7. Wentworth and Hill, *Geometry Exercise Manual* (*Constructions classified*).

8. Nichols, E. A., *Elementary and Constructional Geometry*.

9. Wells, *New Plane and Solid Geometry*. Gives directions for construction of figures according to the hypothesis.

FIRST SEMESTER.

A. Preliminary Concrete Work.

Note. These exercises are grouped here together for convenience. They may be scattered through the early part of the work as desired.

I. Test for straight edge (sighting).

II. Test for right angle; mechanics' test for square.

III. Measurements of lengths; use of dividers.

IV. Measurement of angles; use of protractor.

a. In constructing an angle of given size.

b. In measuring a given angle.

c. Approximate construction of regular polygons of any given number of sides by means of angles at the center of the circumscribed circle.

V. Problems requiring the solution of an algebraic equation and making use of the facts concerning complementary angles, supplementary angles, the angular magnitude about a point on one side of a straight line, and the total angular magnitude about a point. See Myers, *Geometric Exercises*; Wright, *Exercises in Concrete Geometry*; Wentworth-Smith, pages 19 and 24; Stone-Millis, pages 14 and 15.

VI. Construction of an angle at a given point in a given line, equal to a given angle (without protractor).

VII. Construction of a perpendicular bisector to a given line. Tests for accuracy. The perpendicular bisectors of the

sides of a triangle should be concurrent and the medians should be concurrent. Construct accurate figures.

VIII. Construction of a perpendicular to a given line from a given point in the line. Test for accuracy. If 10, 12 and 6 are the sides of triangle, the solution of the equations $x+y=12$, $y+z=6$, and $x+z=10$, will give the points of tangency of the inscribed circle. Locate these points and erect perpendiculars to the sides at these points. Inscribe the circle.

IX. Construction of perpendicular to a line from a given point without the line. Test for accuracy. The three altitudes of a triangle should be concurrent.

X. Bisecting a given angle. Test for accuracy. The bisectors of the angles of a triangle should be concurrent.

XI. Construction with concrete setting.

a. Tile patterns to scale. See any tile catalogue. All tiles are derived in some way from the six-inch square, and all tile patterns drawn to scale.

b. Ornamental figures based on square net works. See Diefenbach: *Geometrische Ornamentik*, Max Spielmeier, Berlin.

c. Ornamental figures based on circles. See Diefenbach, or Wentworth-Smith, p. 11-12.

d. Construction of mechanical devices. Stone-Millis, p. 17 and 30-31.

XII. Construction of figures from simple problems. Wentworth-Smith, p. 13-14.

XIII. Preliminary work on construction of loci: *e. g.*, a chord of a circle moves so as always to pass through a given point. Find by experiment the path of its middle point.

XIV. Verification of Pythagorean proposition. The use of this theorem increases the amount of concrete work possible, especially in connection with the theorems on tangents.

Note. Eggar, *Manual of Geometry*, Macmillan, is suggestive on this whole topic.

B. Congruent Triangles.

I. Preliminary work.

Construction of various triangles from given data, followed by the tracing or cutting out and the super-position of such triangles.

II. Fundamental theorems.

a. Two triangles having two sides and the included angle, etc.

b. Two triangles having two angles and the included side, etc.

III. Important dependent theorems.

a. Two triangles having three sides of one, etc.

b. Theorems concerning isosceles triangles.

c. In the same circle or in equal circles equal arcs subtend equal chords and the converse.

d. Two right triangles having the hypotenuse and a side, etc.

e. Such of the theorems given below under arcs, chords, and tangents, as may be proved here if preferred.

IV. Applications.

Theorems and constructions depending on the above theorems. Simple problems in measuring inaccessible distances. Explanation of the use of triangular supports. For suggestions, see Stone-Millis, pp. 47, 48, Godfrey and Siddons, pp. 87, 88, 91, 115, 119. Slaughter and Lennes, pp. 15-17. Experimental construction of loci.

C. *Parallels.*

I. Preliminary work.

Construction with draftsman's triangle and ruler, and with dividers and ruler, of two parallel lines, of a line equally distant from two parallel lines. Measurement with protractor of angles of parallels cut by transversals, to find all equal and all supplementary angles.

II. Important theorems.

a. Two lines parallel to a third line are parallel to each other.

b. Two lines perpendicular to the same line are parallel.

c. If a line is perpendicular to one of two parallel, etc.

d. Theorems regarding alternate-interior angles and exterior-interior angles, and the converse theorems.

III. Applications.

Theorems and construction depending on the above theorems. Arithmetical and algebraic problems based on facts established in the above. See Wentworth-Smith, p. 52, and similar exercises in all the newer books.

D. *The Sum of the Angles of a Triangle.*

I. Preliminary work.

Construct a triangle, and with a protector find the sum of the angles; extend each side in succession, measure each exterior

angle and find what two interior angles are together equal to it.

II. Fundamental theorem.

The sum of the angles of a triangle is two right angles.

III. Important dependent theorems.

a. If two triangles have two angles of one equal respectively to two angles of the other, the third angles are equal.

b. An exterior angle of a triangle equals, etc.

c. Theorems for the sum of the interior and exterior angles of polygons.

IV. Applications.

Theorems and constructions depending upon the above theorems. Accurate construction of figures for the above theorems and verification by measurement with protractor. Arithmetical and algebraic problems, based on the above theorems. See Wentworth-Smith, p. 52, 53. Wright: Exercises in Concrete Geometry, r. 5-9. Regular polygons and tiled floors. See Slaught and Lennes, p. 77-79.

E. Quadrilaterals.

I. Preliminary work.

Construct a parallelogram given two sides and the included angle; construction to be based on the definition of a parallelogram.

II. Important theorems.

a. A quadrilateral is a parallelogram if two opposite sides are equal and parallel, or if both pairs of opposite sides are equal.

b. The opposite sides and angles of a parallelogram are equal.

c. The diagonals of a parallelogram bisect each other.

III. Applications.

Theorems and constructions based on the above theorems. Exercises from parquet floor patterns. See Slaught and Lennes, p. 76-82. Experimental construction of loci; *e. g.*, a parallelogram ABCD has vertex A, and also the direction of sides AB and AD, fixed; find the locus of vertex C if the sum of the sides AB and AD is constant.

F. Arcs, Chords, and Tangents.

The theorems in this section may be given in connection with those under B, C, and D, or they may be grouped here.

I. Important theorems.

a. Radius perpendicular to a chord, etc., and the converse.

- b. Equal chords are equally distant from the center.
- c. Radius to the point of contact is perpendicular to the tangent and the converse.

II. Applications.

Theorems and constructions based on the above theorems. Construction of ornamental drawings based on circles; also architectural figures as easement cornices, moldings, various arches, etc. See Stone-Millis, Chapter on Circles; Godfrey and Siddons, first part of Book III; Slaught and Lennes, p. 92-94. Experimental construction of loci.

G. Measurement of Angles.

I. Preliminary work.

Draw various inscribed angles and a central angle that intercept the same arc; measure each with the protractor. Draw and measure several inscribed angles intercepting the same minor arc, several intercepting the same major arc, and several inscribed in a semi-circle.

II. Fundamental theorem.

An angle at the center of a circle is measured by its subtended arc.

III. Important dependent theorems.

- a. An inscribed angle is measured by one-half its intercepted arc.
- b. An angle formed by a tangent and a chord, etc.
- c. An angle formed by two secants, a secant and a tangent, etc.
- d. An angle formed by two chords, etc.
- e. Parallel lines intercept equal arcs, etc.

IV. Applications.

Theorems and constructions depending upon the above theorems. Proofs by means of circles of various theorems previously proved otherwise; e. g., the sum of the angles of a triangle. Construction for tangents from a point without a circle and for tangents to two circles. On a given straight line to construct a segment to contain a given angle. Problems from carpentry, pattern making, marine surveying, and railroad surveying. See Stone-Millis, Chapter on circles; Slaught and Lennes, pp. 107-111; Godfrey and Siddons, Book III, Angle properties; Breckinridge, Mersereau, and Moore, Shop Mathematics, pp. 68-71. Arithmetic and Algebraic exercises. See Myers Second Year Mathematics, pp. 137-139.

H. Loci.

I. Preliminary work.

The use of the word path instead of locus is recommended. Constructions of paths of moving points; this should be done by locating by construction a sufficient number of points to determine the nature of the path. A sufficient number of such constructions should be given to familiarize the class with the idea of motion. These exercises may be introduced from the first and scattered along as they apply.

II. Proofs for very simple theorems may be omitted or given informally, *e. g.*, Locus of all points at a given distance from a given point, or at a given distance from a given line.

III. Formal proofs for important theorems.

In all such proofs, the two-fold idea involved should be clearly understood.

a. Locus of all points equally distant from two given points.

b. Locus of all points equally distant from the sides of an angle.

IV. Applications.

a. Construction of inscribed, circumscribed and escribed circles.

b. Construction of circles and triangles and determination of points.

c. Theorems concerning concurrent lines in triangles.

d. Problems with concrete setting, *e. g.*, reversed curves, easement curves, etc. See Stone-Millis, p. 160; Trefoils and Gothic windows; Slaught and Lennes, p. 93-94, 101 and 108-110.

I. Inequalities (optional).

I. Preliminary work is hardly necessary. It might be worth while to have pupils measure the sides of some triangles and compare so as to discern the relation of any side to the sum of the other two.

II. Fundamental theorems.

Any side of a triangle is less than the sum of the other two.

III. Dependent theorems.

a. If two sides of a triangle are unequal the angles opposite them are unequal, etc., and the converse. The perpendicular is the shortest distance from a point outside a line to a given line.

Of two unequal oblique lines from a point in a perpendicular the more remote from the foot of the perpendicular is the greater, and the converse.

b. If a perpendicular be erected at the middle of a straight line, the distance from any point not in the perpendicular, etc.

c. Two lines from a point in a triangle to the extremities of one side are together less than, etc.

d. If two triangles have two sides equal and the included angles unequal, etc., and the converse.

Chords subtending unequal arcs are unequal, etc., and the converse.

IV. Concrete applications.

A few arithmetical and algebraic problems may be given.

Second Semester.

A. Equal Ratios and Parallels.

I. Preliminary work.

a. Measurement of lines; possibility of incommensurable lines; approximate ratios; H. C. D. of two lines. Draw two lines, measure in inches and sixteenths of an inch, find the ratio of the lines; measure the lines in millimeters and find the ratio; find the ratio by H. C. D. These ratios should agree to the third decimal place.

b. Construct a line parallel to the base of a triangle. Find approximate ratios of segments and sides.

II. Fundamental theorem.

If a series of parallels cut off equal segments on one transversal, etc.

III. Important dependent theorems.

a. A line parallel to the base of a triangle divides the sides into segments that have the same ratio, and the converse.

b. Parallel lines cut off proportional segments on any two transversals.

IV. Applications.

a. Constructions dependent on the above: constructions for third and fourth proportionals and for dividing a line into parts proportional to any number of given parts.

b. Numerical and algebraic problems. See Wentworth-Smith, p. 159; Short and Elson, pp. 174, 180.

B. Equal Ratios and Similar Figures.

I. Preliminary work.

Construct two mutually equiangular triangles and find the approximate ratios of corresponding sides; these ratios should agree to the third decimal place.

II. Fundamental theorem.

If two triangles are mutually equiangular, corresponding sides have the same ratio.

III. Important dependent theorems.

a. Two triangles are similar if corresponding sides have the same ratio, or if two pairs of corresponding sides have equal ratios and the included angles are equal.

b. The ratios of corresponding lines of similar polygons are equal; *i. e.*, corresponding altitudes of similar triangles.

c. Important cases of equal ratios: In a right triangle the altitude upon the hypotenuse is a mean proportional, etc.

IV. Applications.

a. Theorems dependent upon the above, especially theorems in which it is required to prove two ratios equal. The usual methods of proving two ratios equal should be emphasized.

b. Constructions dependent upon the above. Fourth proportionals, mean proportionals, and similar figures.

c. Definitions of sine, cosine, and tangent; problems in heights and distances. See Stone-Millis, pp. 82-99; Slaught and Lennes, p. 137; Short and Elson, pp. 193-203.

d. Numerical and algebraic problems. See Stone-Millis, p. 179; Wentworth-Smith, pp. 176 and 187; Short and Elson, p. 188; Slaught and Lennes, pp. 140-144; Myers II, pp. 28-34, p. 52; Wright, Concrete Exercises in Geometry, pp. 14-20. Algebraic work should be emphasized here. Review of quadratics and radicals is advised.

e. Drawings to scale, diagonal scales, etc. Stone-Millis, p. 80.

C. Areas.

I. Preliminary work.

By using squared paper and also by drawing and cutting out the figures the equivalence of various parallelograms having equal bases and equal altitudes may be shown. This may be done with triangles. In various ways the truth of the Pythagorean proposition may be illustrated, *e. g.*, Penrigals dissection.

Squared paper may be used to advantage in work on areas. Estimating is also valuable.

II. Fundamental theorem.

The area of a rectangle equals, etc.

III. Important dependent theorems.

a. Measurement of areas.

1. The area of a parallelogram equals, etc.
2. The area of a triangle equals, etc.
3. The area of a trapezoid equals, etc.

b. Comparison of areas.

1. Pythagorean proposition.
2. The areas of similar polygons have the same ratio as the squares of homologous sides.
3. Parallelograms and triangles having equal altitudes, equal bases, or equal altitudes and bases, etc., are to each other, etc.

IV. Applications.

a. Constructions depending on:

1. Parallelograms and triangles having equal bases and equal altitudes.
2. Pythagorean proposition.
3. Algebraic analysis.

b. Theorems depending on the above, *e. g.*, proving areas equal by equal bases and altitudes, or by the sum of congruent or equivalent parts.

c. Surveyor's method of finding areas of plats irregular in outline.

d. Numerical and algebraic problems involving Pythagorean and other propositions including areas of various polygons. The algebraic work is of special importance. Review of radicals and quadratics is strongly advised. For suggestive exercises see recent texts in algebra and geometry.

e. Graphic representation of square roots.

f. Geometrical illustration of algebraic identities, and numerical illustration of identities on squared paper.

Note. Much suggestive material may be found in the chapter on areas in Godfrey and Siddons, *Elementary Geometry*. Also Myers, *Second Year Mathematics*, Chap. IV; Stone-Millis, Chap. X. and XI; Wentworth-Smith, Chap. IV; Slaughter and Lennes, pp. 170-175.

D. Inscribed and Circumscribed Polygons.

Note. The object of this series of theorems is the measurement of the circle. Theorems involving theory of limits may be proved informally without theory of limits. Stress should be laid on the fact that the numerical results are only approximate.

I. Fundamental theorems.

1. An equilateral polygon in a circle is regular.
2. A circle can be circumscribed about or inscribed in a regular polygon.
3. If a circumference is divided into equal parts, tangents through the points of division form a regular circumscribed polygon.
4. Regular polygons of the same number of sides are similar.
5. The area of a regular polygon equals one-half the product of its perimeter and apothem.

II. Dependent theorems.

- a. The ratio of perimeters of two regular polygons.
- b. The apothem of a regular inscribed polygon as a variable.
- c. The perimeter and area of such polygon as variables.
- d. The ratios of the circumferences and areas of two circles.
- e. The ratio of the areas of two similar segments.
- f. The area of a circle equals one-half the product, etc.
- g. A side of a regular hexagon equals the radius, etc.
- h. To find the side of a regular inscribed polygon of double the number of sides, etc.
- i. To compute the value of π .
- j. Problems for constructing regular polygons of three, four, five and six sides, and of two, four, eight, etc., times as many sides.

III. Applications.

Probably those most worth while are arithmetical and algebraic problems involving the above theorems. As many of these as are desired can be selected from recent textbooks, in which they are numerous.