XIII.—*On the Theory and Construction of a Seismometer, or Instrument for Measuring Earthquake Shocks, and other Concussions.* *By* JAMES D. FORBES, Esq. *F.R.S. Sec. R. S. Ed., Professor of Natural Philosophy in the University of Edinburgh.*

(Read 19th April 1841.)

HAVING been requested to act on a Committee of the British Association, appointed to devise and apply methods for measuring the comparative intensity of earthquake shocks, and having been shewn several ingenious contrivances by Mr DAVID MILNE (who suggested the inquiry) Lord GREENOCK, and other persons, an apparatus occurred to me which should unite the requisites of Simplicity, Compactness, Comparability, and an easy adjustment of Sensibility according to circumstances.

Mr MILNE had not failed to distinguish the ends for which instruments (which for obvious reasons were to be self-registering) ought to be devised, such as the measurement of horizontal concussions, of vertical elevation, and of heaving or angular motion of the surface. It is no part of my present object to consider the probable movements of the soil in earthquakes. I limit myself to the description of a single instrument intended to measure *lateral* shocks, such as are experienced by objects placed upon a table which is abruptly shoved forwards.

A heavy pendulum suspended from a frame in such a manner that the inertia of the bob should cause it to oscillate when its centre of suspension had been displaced by the movement of the frame with which it was connected, had already been suggested for the purpose. To obtain sufficient sensibility, a pendulum of great length would be required, nor could the sensibility be altered according to circumstances, being wholly independent of the weight of the bob. The unwieldiness of a pendulum ten or twenty feet long alone forms a strong objection to this apparatus.

The elegant inverted Pendulum or *Noddy* contrived by the late Mr HARDY,* suggested to me a different arrangement. The instrument is seen in Elevation, Section, and Plan, in Plate III. Figures 1, 2, and 3. A vertical metal-rod A B, having a ball of lead C moveable upon it, is supported upon a cylindrical steel-wire D, which is capable of being made more or less stiff by pinching it at a shorter or

* Described by Captain Kater, Phil. Trans. 1818.

SEISMOMETER.

PLATE III. Royal Soc. trans. Edin. Vol. XV. p. 219.

Fig. 1.

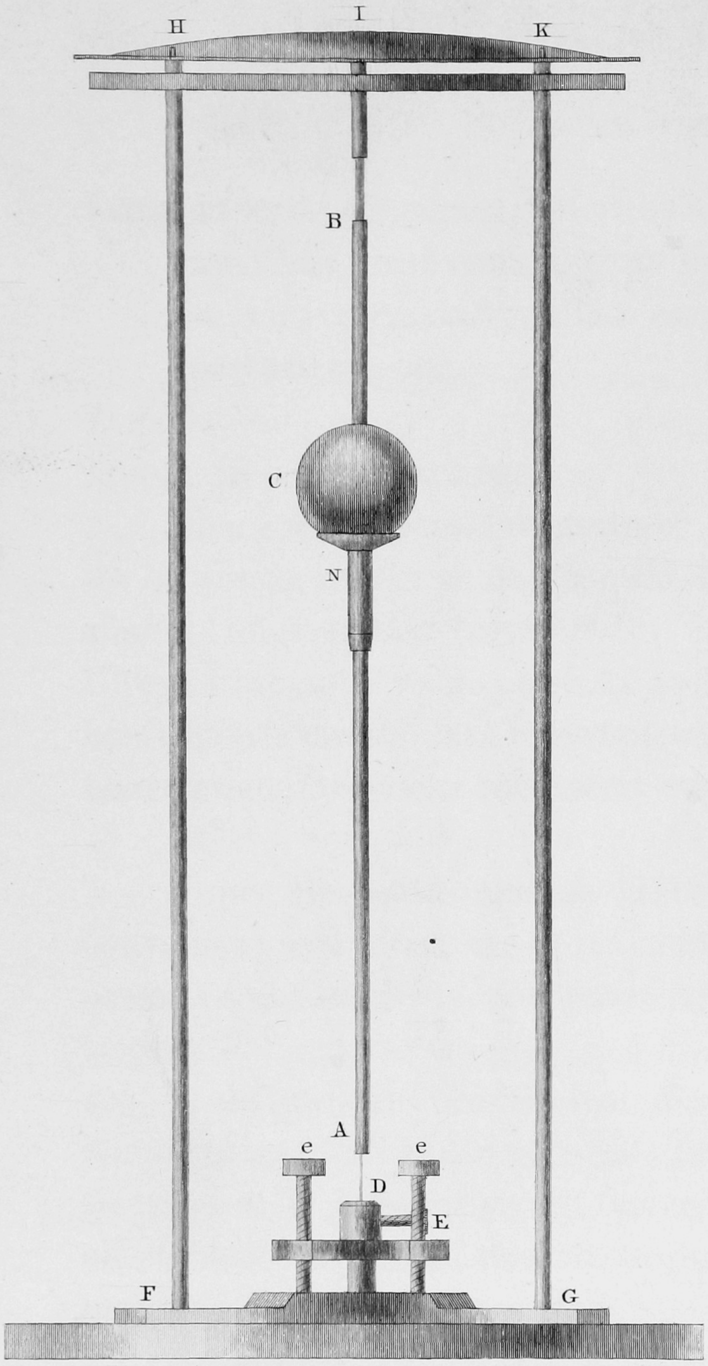


Fig. 2.

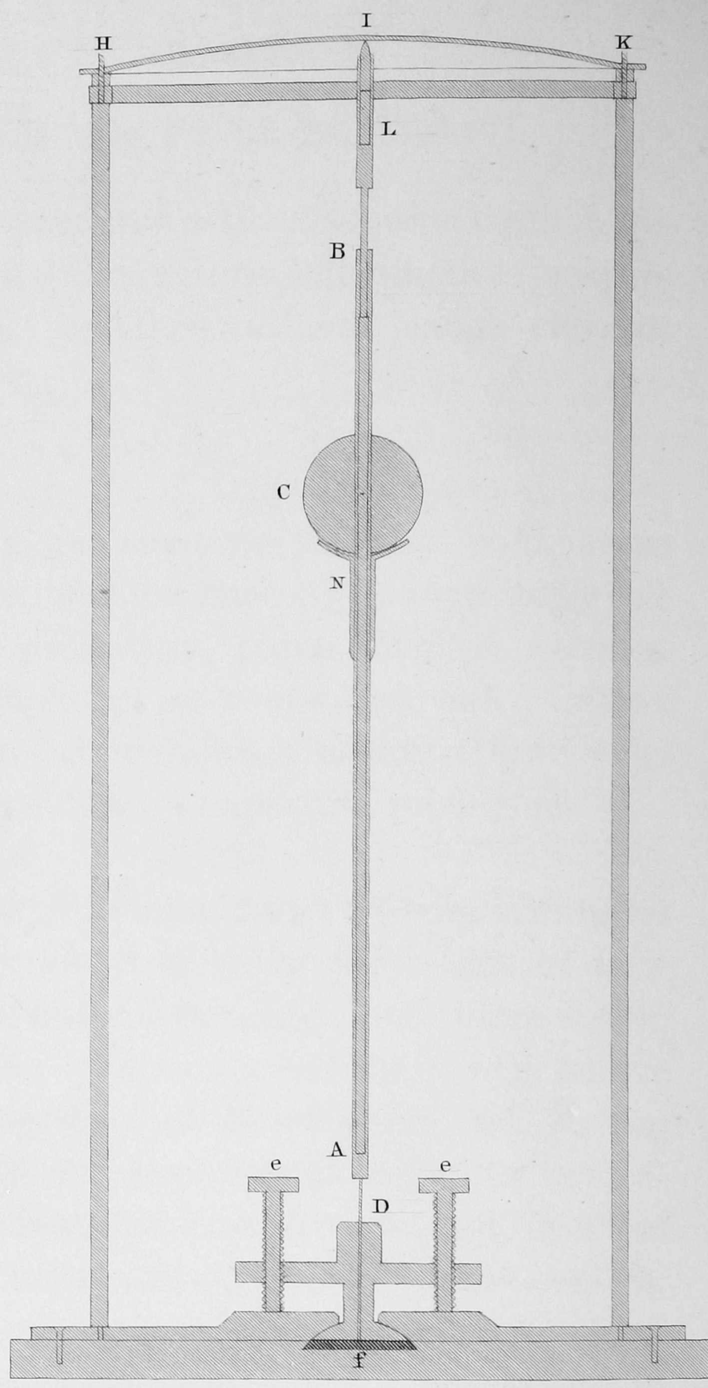


Fig. 3.

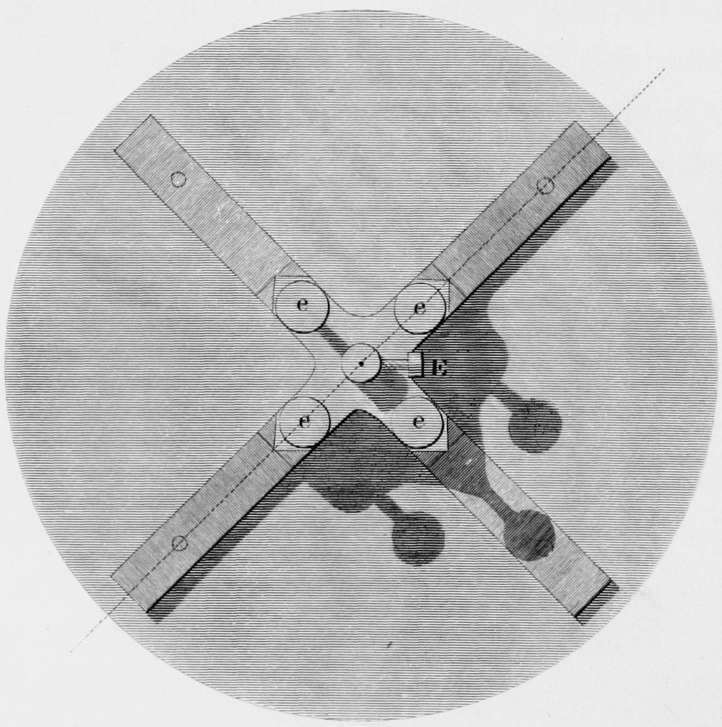
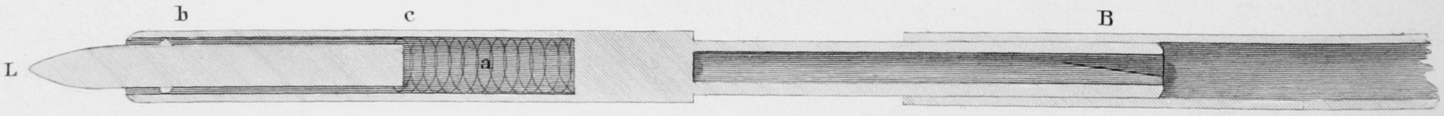
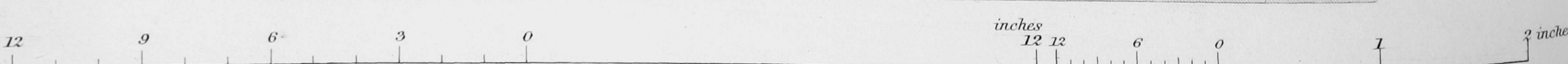


Fig. 4.





Scale to Plan Elevation & Section.

Scale to Enlarged view of Pencil piece.

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greater length by means of the screw E. It is evident that, by adjusting the stiffness of the wire, or the height of the ball C, we may alter to any extent the relation of the forces of Elasticity and of Gravity, and consequently render the equilibrium of the machine in a vertical position stable, indifferent, or unstable. Since, then, a lateral movement, which carries forward the base of the machine, can only act upon the matter in C through the medium of the elasticity of the wire, the stiffness being diminished, or the weight increased, the tendency of the rod to right itself may be diminished in any proportion, and that irrespectively of the dimensions of the instrument.

The wire D being cylindrical, the direction of the displacement occasioning the shock will at once be indicated by the plane of vibration of the pendulum, which, being once put in motion, will oscillate backwards and forwards many times before coming to rest. The pendulum is adjusted to the vertical position by four antagonist screws *e e e e*, acting on a ball and socket arrangement *f*.

The self-registering part of the apparatus, which Mr DAVID MILNE has termed a *Seismometer*, was arranged by that gentleman and by Mr JAMES MILNE, the ingenious artist who constructed it. It consists of a spherical segment H I K of copper lined with paper, against which a pencil L, inserted in the top of the pendulum-rod, is gently pressed by a spiral spring. The marks thus traced on the concave surface indicate at once the direction and maximum extent of the pendulum's vibration. The arrangement of the pencil is seen upon a larger scale in Fig. 4, where L is the pencil as before loosely fitting the cylinder *b c*, and pressed upwards by the spring *a*. The whole pencil-case moves stiffly on the extremity B of the pendulum-rod, so as to adjust the pressure against the paper.

HARDY's instrument was intended simply for ascertaining the stability of the support for a clock. The spring was a piece of flat watch-spring—the plane of its motion was parallel to that of the pendulum of the clock whose influence was suspected, and the time of oscillation being adjusted accurately to seconds by screwing the bob up or down, the repetition of impulses always isochronal, though individually feeble, at length urged it into considerable arcs of vibration, if the beam or wall on which it stood was not perfectly stable. The instrument under consideration, on the other hand, has a free vibration in every vertical plane, the time of its oscillation is immaterial, except in so far as the sensibility is increased as the time is greater; HARDY's instrument collects the effect of a series of isochronous impulses, this one registers the maximum effect of a single and insulated one in direction and in intensity: HARDY's was an indicator of instability, this (as we shall see) furnishes a measure of the cause of a concussion.

The admirable advantage which the balance of the gravitating and elastic forces affords will appear from the following considerations:—

I. We must first attend to the *friction* which must be overcome in order to carry the pencil across the surface receiving the trace. The moving force of the pendulum will be greater as its inertia increases, in consequence of which the bob lags behind the movement of the frame to which it is attached by the elastic wire, which frame carries along with it the concave surface over which the pencil will therefore be dragged. *To overcome the friction of the pencil, we must therefore increase the mass of the pendulum.*

II. The mass of the pendulum cannot be changed without modifying the sensibility of the apparatus; that is, the maximum vibration which a given shock will produce. But the desired sensibility is easily maintained by the pinching screw E, which must be employed to shorten the free part of the elastic wire (or a thicker wire may be introduced), until the sensibility is exactly as great as may be required.

III. Hence one and the same instrument may have any required sensibility given to it, and that *wholly irrespective of its dimensions.* The sensibility depends upon the force tending to restore the pendulum to its position of rest when the displacement = 1. The time of vibration depends on this quantity. Hence *the time of vibration is the test of sensibility.* As the condition of equilibrium approaches to indifference, the sensibility increases without limit.

IV. However weak the spring may be, and however great the sensibility, it is plain that, on the present construction (for others might easily be suggested which should give a different result), the displacement of the bob and pencil cannot by possibility exceed the forward motion which the earthquake is understood to communicate to the stand (which may be screwed to a floor). The inertia of the pendulum cannot do more than leave the extremity B as much behind A as the earthquake has shifted A forwards. If this effect be worth measuring at all, the lateral vibration of the ground must be a sensible quantity, and there is no difficulty in constructing an instrument on any scale, from an inch to 10 feet in length, in which (the time of oscillation being the same,—say one second) the maximum vibration shall have the same linear magnitude. The only consideration is, that the range may be sufficiently great to exhibit the stronger shocks without giving an inconvenient curvature to the apparatus; and for that purpose I have thought that a radius of 20 inches and a diameter of 10 inches for the spherical segment is sufficient.

V. If it be desired to *magnify* the scale of displacements, this may still be done without any increase of dimensions. Let the bob C (Plate III. Fig. 1,) be lowered upon the rod A B, so as to stand at only one-half or one-third of its height:—let the mass be increased so as to overcome the friction of the pencil as efficaciously at this diminished leverage; and let the spring be adjusted so as to give the same sensibility as before; the displacement of the bob will be the same as at first, but the displacement of the pencil will be magnified two or three times, according to their relative radii.

In practice, the pencil will not describe precisely lines upon the sphere, but very elongated ellipses. Hence it will be easy to distinguish the mark made by the first oscillation of the pendulum, which will always be contrary to the direction in which the vibration of the ground takes place.

There is one peculiarity arising from the construction of the instrument, which, at first sight, perhaps, we should scarcely expect. The maximum displacement we have seen to depend solely upon the time of one vibration, and it may be the same (for small shocks) on whatever scale the instrument is constructed. We might expect, however, that the taller instruments would oscillate longest, and be most easily set in motion; but the contrary is the fact. This arises from the circumstance, that the stiffness of the spring must increase with great rapidity as the length of the pendulum becomes greater,—that, consequently, the elastic wire bends in all its length, unlike the feeble flat spring of HARDY's instrument, which doubles over almost at a point. The elastic wire, therefore, tends to vibrate back and forwards many times before the inertia of its load has suffered a complete vibration to take place, and even the flexure of the pendulum-rod, by the powerful elastic action of the wire, will cause it to perform subordinate oscillations, which have a tendency to destroy one another, and to bring the whole to rest. This is a decided advantage when the object is (as in the present case) merely to register the first or maximum displacement, and I find that, with the size of the instrument which I have recommended (a 20-inch pendulum), the effect is sufficient and well marked.

In proceeding to investigate mathematically the action of such an instrument, and to shew how it may be most advantageously adjusted to inform us of the intensity of earthquake shocks, I must repeat, that I proceed upon the very limited hypothesis that the kind of shock desired to be measured consists in a lateral heaving of the earth's surface through a certain space with a uniform velocity, commencing and terminating abruptly. Except under such limitations, it is impossible to obtain rigorous conclusions. Experience alone can shew how far such conditions correspond with fact; but, unless theory indicate arrangements for testing their admissibility, our knowledge is likely to remain as indefinite as it is at present.

I. The pendulum (Plate III. Fig. 1.) being displaced, to find the force tending to redress it.

Let F = the force in grains, which, when applied to the rod AB at distance = 1 from the centre of motion (a point in the spring D), will balance the force of the spring when the lateral displacement is also = 1.

k = radius of gyration of the pendulum, which will be nearly equal to the distance of the centre of the ball C from the middle of the wire D.

M = weight of pendulum in grains.

θ = angular displacement.

The elastic force for unity of displacement at radius k is equal to $\frac{F}{k^2}$.

For an angular displacement θ or distance $k\,\theta$, it is $\frac{F}{k}\theta$. This tends to *redress* C. The effect of gravity in *displacing* C is

$$Mk \sin \theta,$$

or $Mk\,\theta$ nearly, when the displacement is not large.

Hence the pressure on C tending to stability is

$$\left(\frac{F}{k} - Mk\right)\theta.$$

The accelerating force on C is

$$\left(\frac{F}{Mk} - k\right)\theta g; \quad \ldots\ldots \quad \text{Eq. } (a).$$

g being the accelerating effect of the force of gravity. Hence equilibrium is

$$\left.\begin{array}{l}\text{Stable}\\ \text{Indifferent}\\ \text{Unstable}\end{array}\right\} \text{as } \frac{F}{M} \begin{array}{c}>\\=\\<\end{array} k^2.$$

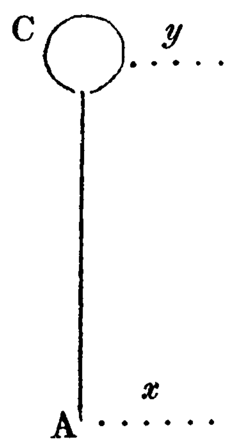
When the pendulum-rod forms an angle θ with the vertical, the displacement s of the ball, which moves with a radius k, is $k\,\theta$. Hence $\theta = \frac{s}{k}$, and the redressing force is (by Eq. (a))

$$\left(\frac{F}{Mk} - k\right)\frac{s}{k} \cdot g = \left(\frac{F}{Mk^2} - 1\right)s \cdot g.$$

And if ϕ represent the Accelerating Force for unity of displacement of the ball

$$\phi = \left(\frac{F}{Mk^2} - 1\right)g \quad \ldots\ldots\ldots\ldots \quad (b).$$

II. To determine the motion of the pendulum when a uniform velocity is suddenly communicated to the base.



x = movement of stand,

y = movement of ball,

$s = y - x$ = relative movement of ball.

Let the motion of A be uniform for a certain time t with velocity V, then $x = Vt$,

$$s = y - Vt \quad \ldots\ldots\ldots\ldots\ldots \quad (1).$$

$$\frac{ds}{dt} = \frac{dy}{dt} - V \quad \ldots\ldots\ldots\ldots \quad (2).$$

$$\frac{d^2 s}{dt^2} = \frac{d^2 y}{dt^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (3).$$

Force causing C to follow A (having always an opposite sign from s), $= \phi s$ (ϕ being the elastic action of the wire on C for $s = 1$) as above; Eq. (b).

Therefore $\frac{d^2 y}{d t^2} = -\phi s.$

$$\frac{d^2 s}{d t^2} = -\phi s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

$$\left(\frac{d s}{d t}\right)^2 = -\phi s^2 + c.$$

When $s = 0$ and $y = 0$, $\frac{d s}{d t} = -\text{V}$ (for the point A begins to move with a velocity V),

$$\frac{d s}{d t} = \sqrt{\text{V}^2 - \phi s^2}$$

$$d t = \frac{d s}{\sqrt{\text{V}^2 - \phi s^2}} \quad \text{and} \quad t = \frac{1}{\sqrt{\phi}} \sin^{-1} \frac{s}{-\text{V}} \sqrt{\phi} + c'.$$

When $s = 0$, $t = 0$ $\therefore$ $c' = 0$,

$$s = -\frac{\text{V}}{\sqrt{\phi}} \sin(\sqrt{\phi} \,.\, t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

The greatest value of $s = \pm \frac{\text{V}}{\sqrt{\phi}}$, ... (5*).

occurs when $\sqrt{\phi} \,.\, t = 90°, 270°$, &c.

or when $t = \frac{\pi}{2\sqrt{\phi}}, \frac{3\pi}{2\sqrt{\phi}}$, &c. . . . . (6).

In order that the movement should not be oscillatory, $\sqrt{\phi} \,.\, t$ must be always $\angle 90°$ for all values of t;—or $\sqrt{\phi} \angle \frac{\pi}{2t}$.

When t is infinite $\sqrt{\phi} \angle 0$, which is impossible, if ϕ be a redressing force at all.

III. To determine the motion of the pendulum after the motion of the base suddenly stops.

Whilst x varies uniformly (with velocity $=$ V)

$$s = -\frac{\text{V}}{\sqrt{\phi}} \sin \sqrt{(\phi \,.\, t)}.$$

Let x abruptly cease to increase when $t = \text{T}$ (and then let $s = \text{S}$)

$$\text{S} = \frac{-\text{V}}{\sqrt{\phi}} \sin(\sqrt{\phi} \,.\, \text{T}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

By (1) the displacement S is then $(y - \text{VT})$ . . . . (8),

$$y = \text{VT} + \text{S} = \text{VT} - \frac{\text{V}}{\sqrt{\phi}} \sin(\sqrt{\phi} \,.\, \text{T}).$$

The absolute velocity of C is then $\frac{d y}{d \text{T}} = \text{V} - \frac{\text{V}}{\sqrt{\phi}} \cos(\sqrt{\phi} \,.\, \text{T}) \sqrt{\phi}$

$$= \text{V}\{1 - \cos(\sqrt{\phi} \,.\, \text{T})\} \quad . \quad . \quad . \quad (9);$$

C is proceeding in space with this velocity, and under the action of a force $\phi\, s$, always tending to the point A (now stationary).

In this second stage

$$\frac{d^2 s}{dt^2} = -\phi\, s \text{ as before;}$$

$$\left(\frac{ds}{dt}\right)^2 = -\phi s^2 + k.$$

At the moment that $s = \mathrm{S}$, $\frac{ds}{dt}$ became $= \frac{dy}{d\mathrm{T}}$, because x ceasing to increase, y henceforth $= s$.

Hence, by (9) when $s = \mathrm{S}$, $\frac{ds}{dt} = \mathrm{V}\,\{1 - \cos(\surd\phi\,.\,\mathrm{T})\} = \mathrm{U}$ . . . . (10).

But $\mathrm{U}^2 = -\phi\,\mathrm{S}^2 + k$, or $k = \mathrm{U}^2 + \phi\,\mathrm{S}^2$;

$$\frac{ds}{dt} = \sqrt{\mathrm{U}^2 + \phi\,\mathrm{S}^2 - \phi\, s^2} \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (11)$$

$$dt = \frac{ds}{\sqrt{\mathrm{U}^2 + \phi\,\mathrm{S}^2 - \phi\, s^2}}$$

$$t = \frac{1}{\surd\phi}\,.\,\sin^{-1}\frac{\surd\phi}{\sqrt{\mathrm{U}^2 + \phi\,\mathrm{S}^2}}\, s + k'.$$

When $t = \mathrm{T}$, $s = \mathrm{S}$

$$\mathrm{T} = \frac{1}{\surd\phi}\sin^{-1}\sqrt{\frac{\phi}{\mathrm{U}^2 + \phi\,\mathrm{S}^2}}\,.\,\mathrm{S} + k'$$

$$t - \mathrm{T} = \frac{1}{\surd\phi}\left\{\sin^{-1}\sqrt{\frac{\phi}{\mathrm{U}^2 + \phi\,\mathrm{S}^2}}\,.\,s - \sin^{-1}\sqrt{\frac{\phi}{\mathrm{U}^2 + \phi\,\mathrm{S}^2}}\,.\,\mathrm{S}\right\} \quad . \qquad (12).$$

Let $\sin^{-1}\sqrt{\frac{\phi}{\mathrm{U}^2 + \phi\,\mathrm{S}^2}}\,\mathrm{S} = \Theta$

$$\surd\phi\,.\,(t - \mathrm{T}) + \Theta = \sin^{-1}\sqrt{\frac{\phi}{\mathrm{U}^2 + \phi\,\mathrm{S}^2}}\,.\,s$$

$$\mathrm{Sin}\,\{\surd\phi\,(t - \mathrm{T}) + \Theta\} = \sqrt{\frac{\phi}{\mathrm{U}^2 + \phi\,\mathrm{S}^2}}\,.\,s \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (13).$$

The greatest ($\pm$) value of s is when $\surd\phi\,(t - \mathrm{T}) + \Theta = 90^\circ, 270^\circ$ &c.; and it is then

$$s_{\prime} = \sqrt{\frac{\mathrm{U}^2 + \phi\,\mathrm{S}^2}{\phi}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (14).$$

It recurs regularly ($\pm$) as $\surd\phi\,.\,t$ increases by π, or as

$$t \text{ increases by } \frac{\pi}{\surd\phi} \quad . \; . \; . \; . \; . \; . \; . \qquad (15).$$

By (14) and (10)

$$s_{\prime} = \sqrt{\frac{\mathrm{V}^2\left(1 - \cos(\surd\phi\,.\,\mathrm{T})\right)^2 + \phi\,\mathrm{S}^2}{\phi}} = \sqrt{\frac{\mathrm{V}^2\{1 - 2\cos(\surd\phi\,.\,\mathrm{T}) + \cos^2(\surd\phi\,.\,\mathrm{T})\} + \phi\,\mathrm{S}^2}{\phi}}$$

By (7) $$S = \frac{-V}{\sqrt{\phi}} \cdot \sin(\sqrt{\phi} \cdot T), \quad \text{and} \quad \phi S^2 = V^2 \sin^2(\sqrt{\phi} \cdot T)$$

$$s_{,} = \sqrt{\frac{2V^2\left(1 - \cos(\sqrt{\phi} \cdot T)\right)}{\phi}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16),$$

$$\text{or } s_{,} = V \cdot \sqrt{\frac{2 \operatorname{versin}(\sqrt{\phi} \cdot T)}{\phi}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17).$$

Hence;

[1.] The total displacement $s_{,}$ is greater as V (the velocity of the shock) is greater.

[2.] $s_{,}$ is a maximum when

$$\cos(\sqrt{\phi} \cdot T) = -1$$

$$\text{or } \sqrt{\phi} \cdot T = 180°, \text{ \&c.}$$

$$\text{or } T = \frac{\pi}{\sqrt{\phi}}, \; \frac{3\pi}{\sqrt{\phi}}, \text{ \&c.} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18).$$

That is, by (Eq. (5)) when C is in its mean position with respect to A, or $S = 0$, and $\frac{ds}{dt}$ *positive*, i. e. C moving to the right hand of A.

[3.] The value of $s_{,}$ is then (16) $$\frac{\sqrt{4V^2}}{\sqrt{\phi}} = \frac{2V}{\sqrt{\phi}} \quad . \; . \quad . \; . \; . \quad (19)$$

or *twice the greatest value* of s (Eq. (5*)). [The reason is evident, for C is then moving positively with its maximum velocity relatively to A, or V;—the stopping suddenly of A doubles the relative velocity.]

[4.] $s_{,}$ is equal to *nothing*, or the motion is destroyed if

$$\cos(\sqrt{\phi} \cdot T) = +1$$

$$\text{or } \sqrt{\phi} \cdot T = 0°, 360°, \text{ \&c.}$$

$$\text{or } T = 0°, \frac{2\pi}{\sqrt{\phi}}, \text{ \&c.}$$

[5.] Suppose the shock to be violent, or V very great in comparison with the velocity which the elasticity of the wire is capable of generating in a unit of time, that is, V to be large compared to ϕ, the point A must be moved forwards before C has sensibly changed its position. This follows from Eq. (16) which is equivalent to

$$s_{,}^2 = \frac{2V^2 - 2V^2\cos(\sqrt{\phi} \cdot T)}{\phi} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20).$$

To find the value of $s_{,}^2$ when $\phi = 0$ (compared to V).
Differentiating numerator and denominator,

$$\frac{2V^2 T \sin(\sqrt{\phi} \cdot T) \cdot d \cdot \sqrt{\phi} \cdot}{d\phi} = \frac{V^2 T \sin(\sqrt{\phi} \cdot T)}{\sqrt{\phi}}$$

which is still $\frac{0}{0}$ when $\phi = 0$.

Differentiating again, $\frac{V^2 T^2 (\cos \sqrt{\phi} \,.\, T) \, d \sqrt{\phi}}{d \sqrt{\phi}} = V^2 T^2 \cos (\sqrt{\phi} \,.\, T)$ . . (21).

When $\phi = 0$, $s_{,}^2 = V^2 T^2$ or $S_{,} = V\,T$, ... (22)

or exactly the displacement of A.

[6.] Hence (where ϕ is small, and T not very great, so that $\cos (\sqrt{\phi} \cdot T)$ is nearly 1), $s_{,}$ is greater as ϕ is less, and its greatest value is V T.

[7.] Since, by the action of a short sudden blow, $s_{,}$ can never be greater than V T, there is no advantage obtained by using a tall instrument, since, evidently, the smallest and largest alike can only exhibit a deviation due to the whole lateral displacement of the foot of the pendulum.

IV. To deduce the duration and measure of the lateral shock of an earthquake from observation.

For a given velocity V, and given stiffness of wire ($\sqrt{\phi} =$ const.), the final deviation will increase from $T = 0$ to $T = \frac{\pi}{\sqrt{\phi}}$ $\left(\text{by } (18)\right)$.

Therefore, by having instruments for which $\sqrt{\phi}$ varies, we may make sure that $T < \frac{\pi}{\sqrt{\phi}}$, and between these limits the displacement will *measure the duration* of the shock for a given velocity V.

To eliminate the velocity; Let different instruments be provided for which $\sqrt{\phi}$ varies. This is inversely as the time of one vibration backwards or forwards, determined by the difference of two values of t in (6), viz. $\frac{\pi}{\sqrt{\phi}}$.

Then the maximum vibration of each instrument (consistent with the limitation of T) being *observed,* may be called $s_{,}$ and $\sigma_{,}$, the corresponding forces being ϕ and ϕ'.

By (16)

$$\left.\begin{aligned} \phi\, s_{,}^2 &= 2\,V^2 (1 - \cos (\sqrt{\phi} \,.\, T) \\ \phi'\, \sigma_{,}^2 &= 2\,V^2 (1 - \cos (\sqrt{\phi'} \,.\, T) \end{aligned}\right\} \quad . \;.\; .\; .\; .\; .\; . \quad (23).$$

Dividing the second by the first,

$$\frac{\text{versin} (\sqrt{\phi'} \,.\, T)}{\text{versin} (\sqrt{\phi} \cdot T)} = \frac{\phi'\, \sigma_{,}^2}{\phi\, s_{,}^2} \quad . \;.\; .\; .\; .\; .\; .\; .\; .\; .\; .\; .\; . \quad (24),$$

from which T, the duration of the lateral shock, may be deduced. For this purpose let the pendulums be so arranged that the time of vibration of one shall be double that of the other (but for the longest let $T < \frac{\pi}{\sqrt{\phi}}$, as above) ; then, since the times of vibration are as $\frac{1}{\sqrt{\phi}}$, let $\sqrt{\phi'} \,.\, T = 2 \sqrt{\phi} \,.\, T$.

The following table may give a sufficient approximation :—

$\sqrt{\phi}\,.\,T$	$\sqrt{\phi'}\,.\,T$	Log. $\frac{\phi' \sigma_{,}^{2}}{\phi\, s_{,}^{2}} = 4\,\frac{\sigma_{,}^{2}}{s_{,}^{2}}$	$\sqrt{\phi}\,.\,T$	$\sqrt{\phi'}\,.\,T$	Log. $\frac{\phi' \sigma_{,}^{2}}{\phi\, s_{,}^{2}} = 4\,\frac{\sigma_{,}^{2}}{s_{,}^{2}}$
0°	0°	0.6020600	50°	100°	0.5166115
5	10	0.6012329	55	110	0.4979178
10	20	0.5987485	60	120	0.4771213
15	30	0.5945972	65	130	0.4541184
20	40	0.5887629	70	140	0.4298366
25	50	0.5812230	75	150	0.4009933
30	60	0.5719475	80	160	0.3705679
35	70	0.5608990	85	170	0.3373218
40	80	0.5480316	90	180	0.3010300
45	90	0.5332907			

From the preceding table the values of $\sqrt{\phi}\,.\,T$ and $\sqrt{\phi'}\,.\,T$ will be found by looking in the third column for the logarithm of $\frac{\phi'\,\sigma_{,}^{2}}{\phi\, s_{,}^{2}}$, or of four times the ratio of the squares of the extreme oscillations of two pendulums, derived from observation.

Since $\sqrt{\phi}$ and $\sqrt{\phi'}$ are known by the time of oscillation of the pendulums, the value of T may be found, or the duration of the shock.

The velocity of its motion will then be obtained from that of $s_{,}$ by Eq. (17).

I am not aware of the methods which have been employed for estimating the lateral shocks on railways (see Mr NICHOLAS WOOD'S Report on the Great Western Railway), but I apprehend that some modification of the preceding instrument might be found useful for such experiments.

EDINBURGH, 27*th February* 1841.