FURTHER DISCUSSION ON "ON THE USE OF CONSTANT POTENTIAL GENERATORS FOR CHARGING RADIOTELEGRAPHIC CONDENSERS AND THE NEW RADIOTELEGRAPHIC INSTALLATIONS OF THE POSTAL AND TELEGRAPH DEPARTMENT OF FRANCE"
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In the issue of June, 1917, of the "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," page 199 and following, Mr. L. Bouthillon had published an interesting paper on the use of constant electromotive force for charging the radiotelegraphic condensers. I desire to mention that I have already indicated in British Patent 27,247 (1910), page 2, lines 14-30, the enormous advantages of the "periodic charging" versus the "aperiodic charging."

Moreover, the system described by Mr. Bouthillon is practically the same as the one experimented with by Major Fracque in 1910 at the Eiffel Tower station.

A very complete theory of this charging method has also been published by Major Fracque (see for instance "La Lumière Electrique," January 16, 1915, pages 45-46), so that I cannot agree with Mr. Bouthillon when he says that the above system is new.

In concluding, let me say that the use of constant electromotive force for charging does not give the *maximum maximorum efficiency* (greatest of the maximal efficiencies).

Let us suppose that a condenser is connected across the terminals of a direct-current supply by means of a self-inductance L and an ohmic resistance R (this resistance taking account of the dielectric losses in the condenser). We propose to find the wave of the charging current which will give the maximum maximorum efficiency, the time, and the increase of the charge having given values τ and ΔQ . It seems admissible to reduce to such a problem that of the most economical charging

of a radiotelegraphic condenser. If the current is i at the time t , this problem consists in fact to determine the minimum value of the integral

$$W = R \int_0^\tau i^2 dt,$$

the integral

$$\Delta Q = \int_0^\tau i dt$$

having a fixed value.

In other words, we ought to try to find the shape of wave which gives the maximum square root of the mean square, or effective value, the average value being fixed.

It is possible to find the answer by means of the general method which is applicable to such a problem; but we prefer a very elementary solution, which has been already applied by M. Marius Latour for a problem concerning the electric transmission of energy.¹

Put:

$$i = \frac{\Delta Q}{\tau} + x.$$

We can immediately write from (2):

$$\int_0^\tau x dt = 0.$$

Therefore, when substituted in (1):

$$W = R \frac{\overline{\Delta Q^2}}{\tau} + 2R \frac{\Delta Q}{\tau} \int_0^\tau x dt + R \int_0^\tau x^2 dt,$$

the first integral of the second member is zero, and the minimum value of W manifestly occurs when $x = \text{constant} = 0$, since the second integral contains only essentially positive elements.

Consequently it may be said that the most economical charging for a radiotelegraphic condenser is obtained when the current has a constant value during the whole charging. This value is naturally:

$$I = \frac{\Delta Q}{\tau}.$$

If we call Q the initial charge of the condenser, the efficiency can be defined by the ratio: $\frac{\text{stored energy}}{\text{stored energy} + \text{wasted energy}}$ or:

¹Cf. Marius Latour, "L'Eclairage Electrique," February 23, 1901, page 279, number 8, volume XXVI.

$$\begin{aligned}\gamma &= \frac{\frac{1}{2} \times \frac{(Q+\Delta Q)^2}{C} - \frac{1}{2} \times \frac{Q^2}{C}}{\frac{1}{2} \times \frac{(Q+\Delta Q)^2}{C} - \frac{1}{2} \times \frac{Q^2}{C} + R \frac{\Delta Q^2}{\tau}} \\ &= \frac{\tau \left(1 + 2 \frac{Q}{\Delta Q}\right)}{\tau \left(1 + 2 \frac{Q}{\Delta Q}\right) + 2 C R}\end{aligned}$$

or finally

$$\gamma = \frac{\tau \left(1 + 2 \frac{Q}{\Delta Q}\right)}{\tau \left(1 + 2 \frac{Q}{\Delta Q}\right) + \frac{\delta T}{\pi^2}},$$

δ being the logarithmic decrement, and T the natural period of the circuit.

It will be noted that with the Poulsen arc working with oscillations of the 2nd type of M. Blondel, one realizes in practice the charging with a current of constant value by means of the extremely high inductance inserted in the charging circuit.²

It is theoretically possible to determine the shape of wave for applied electromotive force which allows us to realize exactly such a method of charging. But it will not be of practical interest, because, for one thing, the initial conditions are always uncertain.

²H. Barkhausen, "Das Problem der Schwingungserzeugung," Leipzig, 1907.