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J.H. Poynting M.A.

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LVII. *Discharge of Electricity in an Imperfect Insulator.*
By J. H. POYNTING, M.A., Professor of Physics, Mason
College.*

MAXWELL has shown that the phenomena known as the Residual Discharge may be accounted for on the supposition that the dielectric is an imperfect insulator in which the conductivity varies in different parts. His theory is really quite simple and straightforward and free from any hypothesis beyond the fundamental one of electric displacement. But its very generality makes it, I believe, difficult to grasp. The idea of a yielding of displacement in the dielectric, accompanied by a conduction-current in the opposite direction, gives us no help in forming a mental picture of the process actually going on in the dielectric. A hypothesis as to the nature of electric current, which will shortly be published in the 'Philosophical Transactions,' seems to me to render the theory easier to follow; and I propose in this paper to arrange Maxwell's account of the Residual Discharge in accordance with it.

I shall first give some account of the hypothesis referred to in the special case of the discharge of a condenser. Let us suppose that we have two conductors, A and B, which we may suppose to be the two plates of a condenser charged with equal and opposite amounts of electricity, that of A being positive. Then the lines of force will run from A to B through the medium, the condition of the medium being described by saying that there is "electric displacement" from A to B. Or we may describe it without introducing the confusing term "displacement" by returning to Faraday's term "induction." We may then say that tubes of electric induction pass through the medium, each tube starting from $+1$ of electricity on A and ending in -1 on B. The total induction across any section of a tube is then always equal to 1. If we draw the level surfaces at unit differences of potential, the tubes will be divided up into cells; and if we suppose each cell to contain half a unit of energy, then the whole energy of the electrified system is accounted for. Maxwell has called these *unit cells* ('Elementary Treatise on Electricity,' p. 47). According to the views of Faraday and Maxwell, the charges on the conductors bounding the dielectric are to be regarded as the surface manifestations of the altered state of the dielectric

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corresponding to the energy put into it, somewhat as the pressure on a piston in the wall of a closed vessel of compressed water might be regarded as the surface-manifestation of the strained condition of the water.

In order to follow out the process of discharge in the medium, *i. e.*, the mode in which it is relieved from its strained condition, we will first take a simpler case in which we connect the two plates, A and B (fig. 1), of one condenser to the two plates,

Fig. 1.

C and D, of another condenser previously uncharged, and so far from A and B that there is no appreciable direct inductive action on C and D. When equilibrium is again restored, the + charge is shared between A and C, the - charge between B and D, while the difference of level has decreased. There is the same total number of tubes of induction, but each contains fewer unit cells than before, the energy corresponding to the decrease having been transferred to the wires, where it has been dissipated as heat. I shall use the term *energy-length* to indicate the line-integral of the electric intensity along its axis, this being the same as the difference of potential when there is equilibrium. We may say then that the *energy-length* of the tubes has decreased. During the change some of the electric energy was converted into magnetic energy in the medium. This might be observed if sufficiently delicate means were used.

If we confine our attention to the charges on the conductors we must say that equal quantities of + and - have moved respectively from A to C and B to D along the wires.

But taking into account the condition of induction in the medium described by the induction-tubes, we must say that the induction-tubes move sideways out from the space between A and B into the space between C and D, the motion of the charges along the wires being really the motion of the ends of

the induction-tubes. (See fig. 1, where 1-6 may be taken as successive positions of a tube.) During the motion of the tubes, some of their energy was converted into the magnetic form, the coexistence of the two forms, electric and magnetic, being a necessary condition of motion. We may illustrate this from the analogous case of a strained incompressible solid which can be sheared. If there is any mode of escape given to the strain-energy by a slipping of the surface against the constraint, then the state of strain will be propagated outwards from the interior of the solid, but some of the strain-energy will be converted into kinetic energy, and the presence of the two is a necessary condition for the propagation of the strain.

Since the energy-length of a tube diminishes as its ends move along the connecting-wires, we may represent this by supposing that parts of the tubes move into the wire. If a similar motion of electric induction took place into a dielectric, it would remain, and the dielectric would become electrically strained; but in the wire the strain breaks down rapidly, the energy being converted into heat. I think there is good reason to suppose that it is the electric energy which thus breaks down, the magnetic only being dissipated after it has been reconverted to the electric form.

We may now consider the case in which the total discharge of a condenser takes place through a connecting wire. Considering merely the conducting plates and the wire, we say that the charges move along them towards each other and finally unite, neutralizing each other and producing heat in the wire. Regarding the medium, we must suppose the tubes of induction to move sideways towards the wire, shortening as their ends, which are represented by the charges, approach each other, and finally disappearing into the wire. Faraday describes the process by saying that "when current or discharge occurs between two bodies, previously under inductrical relations to each other, the lines of inductive force will weaken and fade away, and, as their lateral repulsion-tension diminishes, will contract and ultimately disappear in the line of discharge" (Exp. Res. vol. i. p. 529, § 1659). The so-called velocity of electricity is merely the velocity of the ends of the tubes, and this may evidently vary according to the nature of the circuit. It is quite conceivable that if the wire be in a neutral medium, *i. e.* one in which there is no surface-difference of potential, say gold in air, and if it follow the direction of a tube of induction, then a tube may move into the wire throughout its whole length at once. In this case the "velocity of electricity" would be infinite.

We know from experiment that if a galvanometer be inserted on the connecting wire then the same magnetic impulse is observed wherever in the circuit the galvanometer be placed, the impulse depending on the galvanometer-constant and on the total discharge. The same experimental result may be stated in an equivalent form, viz. that the line-integral of the magnetic intensity round a closed curve encircling the wire, if integrated for the time of discharge, is the same for all positions of the curve. On the hypothesis here described all the electric induction-tubes of the system finally pass sideways from the medium into the wire. They must, therefore, on their way pass inwards across any curve encircling the wire, so that the total number of induction-tubes cutting such a curve is the same for all positions of the curve. In the paper above referred to, I have sought to connect these two constants by supposing that the magnetic effect is due to, or, more correctly, accompanies, the motion inwards of the condition of electric induction. As soon as motion commences some of the electric energy is converted into magnetic, and the magnetic induction may be represented by ring-shaped closed tubes surrounding the wire. The two inductions, electric and magnetic, coexisting, will propagate the energy onwards till it finally arrives in the wire and is dissipated as heat, the induction there losing its directed condition.

The flowing of electric charges along the wire, which is usually considered as the essential part of the phenomena, or at least that to which attention is to be chiefly directed, becomes on this hypothesis merely the last stage of the process, which consists of a propagation from the surrounding dielectric towards the wire of electric and magnetic induction, which we may symbolize by the motion inwards of two sets of tubes, the electric tubes being, on the whole, more or less in the direction of the wire, the magnetic tubes being closed rings surrounding it. The wire plays the part of the refrigerator in a heat-engine, turning the energy it receives into heat—a necessary condition for the working of the machinery.

Let us now take the case of a condenser in which the dielectric, though homogeneous, is imperfectly insulating, so that the charge gradually disappears. According to Maxwell, in this case "induction and conduction are going on at the same time." Though Maxwell gave no precise account of the process of discharge, his theory and the mechanical illustration accompanying it are based on the supposition that two processes are going on at the same time in every part of the medium, viz. :—(1) a yielding of the electric strain or "displacement" in the dielectric equivalent to a displacement

current from the negative towards the positive plate, and (2) a conduction-current from the positive plate to the negative equal to (1) in amount. This latter is accompanied by dissipation of energy. The two equal and opposite currents, being superposed, have no external magnetic effect.

But it seems to me that we may equally well and more simply represent the facts by considering the first process alone, viz. the yielding of the electric strain, the medium being incapable of bearing it permanently. The electric energy is gradually converted into heat in the same part of the dielectric where it was previously electric, *i. e.* there is here no transfer of energy. The decrease of induction in the medium is accompanied by a corresponding decrease of charge on the plates, not by conduction of + or - electricity either way through the medium, but simply because there is a decrease of the induction in the medium of which the charges on the plates are the surface manifestations. The induction decreases equally through the whole length of a tube, so that the tube "weakens" at the same rate throughout its length.

There will be no magnetic effect in the surrounding space, for there is no movement inwards of electric induction-tubes to supply the place of those which decay.

Perhaps we may take the following as illustrating the two modes of regarding the process. Suppose that a solid is submitted to some strain and kept in the strained position, but that the energy of the strain gradually dissipates; then we may confine ourselves simply to the statement that, owing to some rearrangement of the molecules, they cease to have molecular-strain energy, the energy in each portion of the mass being transformed to heat in that portion; or we may imagine that there is a continual return from the strained towards the original position, accompanied by an equal reverse flow of the matter towards the strained position, this latter not storing up energy, but dissipating the energy given up by the yielding of the strain. The ultimate result according to each is the same, but the latter account is purely hypothetical. We may at once obtain the equation giving the value of the charge at any time in terms of the initial charge when the condenser is left insulated.

Let σ be the charge per unit area, this being equal to the electric induction across unit area in the dielectric;

a be the thickness of the dielectric;

K be the specific induction-capacity;

c be the capacity per unit area;

X be the electric intensity in the dielectric, *i. e.* force per unit electricity on a small electrified body.

We have

$$X = \frac{4\pi\sigma}{K}. \quad \dots \quad (1)$$

Now we know that the rate of decrease of charge on the ends is proportional to the charge σ , and therefore to X .

The decrease of charge or of induction in the medium is therefore

$$\frac{d\sigma}{dt} = -\frac{X}{r}, \quad \dots \quad (2)$$

where r is a constant, which we may term the specific resistance.

Hence from (1),

$$\frac{d\sigma}{dt} + \frac{4\pi\sigma}{Kr} = 0, \quad \dots \quad (3)$$

or

$$\sigma = \sigma_0 e^{-\frac{4\pi t}{Kr}}. \quad \dots \quad (4)$$

If we use p to denote the decrease of induction per second,

$$p = -\frac{d\sigma}{dt} = -\frac{K}{4\pi} \frac{dX}{dt}. \quad \dots \quad (5)$$

The energy per unit volume is $\frac{KX^2}{8\pi}$; its rate of decrease is therefore

$$-\frac{KX}{4\pi} \frac{dX}{dt}. \quad \dots \quad (6)$$

Substituting from (2) and (5), we get the expression which here corresponds to Joule's law for the heating effect, viz. rate of decrease of electric energy per unit volume $= p^2 r$.

If at any moment the two end plates be connected by a wire, transfer of induction will at once take place into the wire, and the whole system will be completely discharged. During this discharge there will be magnetic energy accompanying the motion of electric induction.

We will now investigate the more complicated case of a stratified dielectric in which the different layers have different specific resistances. Before proceeding to the mathematical account, we shall consider the process generally, taking the simple case in which K is the same throughout. Let the condenser be charged very rapidly and then insulated. At the first moment there will be equal and opposite charges on the two end plates, and the number of induction-tubes running through unit area parallel to the plates will be the same in each layer. But decay of induction and dissipation of energy

at once sets in, the rate of decay varying in different layers, so that after a time the number of induction-tubes in contiguous layers will differ and there will be charges on the separating surfaces. In those layers where the rate of decay is most rapid, there will be negative charges on the surface nearer the + plate, and + charges on the surface nearer the - plate ; but still the induction in all is in the same direction.

Now let the two end plates be connected by a wire. At once induction is propagated into the wire and transference takes place from the space between the plates until they are at the same potential, *i. e.* until the line-integral of the electric intensity, or, since K is constant, that of the induction from plate to plate is zero. The same number of tubes must have entered all parts of the wire, otherwise there would be charges at points along its length. Hence the same number of tubes running in the positive direction must have passed out from each of the layers. The result must be a reversal of the induction of some of the layers, *viz.* those in which the induction decayed most rapidly. This, of course, means that after their positive induction has all flowed out and they are quite discharged, tubes from the other layers have bent round and entered them, now charging them in the opposite direction. We may imagine the process to be somewhat as in figs. 2 and 3, representing a condenser with three layers, A, B, C, the decay having been most rapid in the middle one, so that it has become completely discharged, while there is still positive induction in A and C. 1, 2, 3, 4 (fig. 2) represent successive

Fig. 2.

positions of a tube moving out from A towards the wire : 1', 2', 3', 4' successive positions of a tube moving out from C. When they have taken up the positions 4 4' they come in contact, and where they overlap they will neutralize each other and break up into two portions, the outer part of each

forming one positive tube, as 5 (fig. 3), which will move off

Fig. 3.

to the wire, the inner parts uniting to form a negative tube 6 in B.

When the difference of potential between the end plates is zero, suppose the wire to be removed. The induction still remaining decays. If it decayed in the same proportion throughout, the difference of potential would always remain zero. But it decays in greater proportion in the negative layers, since in these the dissipation is by hypothesis most rapid. Hence in the line-integral of the induction from plate to plate the negative terms decrease more rapidly than the positive, and so the total value becomes positive. Then on again connecting with a wire, another positive discharge occurs. The process may evidently be repeated, the discharge always being positive, until finally it becomes insensible.

The analogy between the residual discharge and the phenomenon of elastic recovery in strained solids, pointed out by Kohlrausch, suggests a simple illustration.

Suppose that we build up a cube with successive layers of substances with the same instantaneous rigidity but with different viscosities. Let this be placed between two plates, A, B (fig. 4), the lower plate being fixed. Let rigid trans-

Fig. 4.

verse partitions, r, r , be passed through the layers and attached

by hinges to the two plates, and then let the upper plate be acted on by a force in a direction perpendicular to the partitions, so that a shearing-strain is given to the whole cube. The partitions r, r are merely put so that the distortion from the original position shall always be the same throughout. When a given strain has been produced, let the upper plate be also fixed. Now if the rate of dissipation of strain-energy were the same throughout the layers the stress would also be the same throughout, though gradually decreasing, and on removing the constraint the upper plate would return by a certain amount and then remain in its new position. But the dissipation is not uniform, and after a time the stress in some of the layers is greater than in others. Hence, on removing the constraint from A and allowing it to return, when those in which dissipation has been most rapid have become entirely free from strain-energy, there is still some remaining in the other layers. These latter will therefore strain the former, and we shall have a reverse stress in some of the layers. Thus A will come to a new position of equilibrium; not so far, however, as its first position. Suppose that it is now again fixed. At first no force is necessary to keep it in position; but the stress exerted by the negative layers decays more rapidly than that exerted by the positive, and soon, on releasing A, it will return still further towards its original position. The process may be repeated, the successive discharges of momentum imparted to A being always in the same direction.

[*Added April 15, 1886.*—The supposition of stratification made by Maxwell is, no doubt, very artificial, and was made for the sake of simplicity in the mathematical treatment. He states that “an investigation of the cases in which materials are arranged otherwise than in strata would lead to similar results, though the calculations would be more complicated; so that we may conclude that the phenomena of electric absorption may be expected in the case of substances composed of parts of different kinds, even though these individual parts should be microscopically small.

“It by no means follows that every substance which exhibits this phenomenon is so composed.” (*Electricity and Magnetism*, 2nd ed. vol. i. p. 419.)

Probably in the case of blown glass or any dielectric made up of heterogeneous parts, which has been flattened by rolling, there is more or less approach to the stratified condition; but in other cases, such as shellac or paraffin, we might fairly expect the dielectric to be similarly constituted in all directions. We can only therefore take Maxwell's investigation

as showing in a general way that heterogeneity would introduce absorption phenomena ; and we cannot expect the results obtained on the supposition of such a special arrangement to agree with those of experiment. We may regard the stratified arrangement as giving a superior limit, as it were, this being the constitution most favourable to the production of the phenomena in the way supposed. The inferior limit would be given by an arrangement in which each portion of the substance of the same kind stretched from plate to plate with the same cross section throughout. In this case there would be no residual discharge produced. Using Maxwell's notation (see below), the resistance per unit cross section may be shown to be

$$R' = \frac{(a_1 + a_2 + \dots)^2}{\frac{a_1}{r_1} + \frac{a_2}{r_2} \dots}$$

instead of

$$R = a_1 r_1 + a_2 r_2 + \dots,$$

and R' is always less than R . In any intermediate composition, in which portions of more conducting matter are insulated from each other by less conducting matter, we shall have residual discharge.

It appears probable, from experiments of Dr. Schulze-Berge ('Nature,' March 4, 1886, p. 432), that the resistance of certain dielectrics is not proportional to the thickness, but is much less for thin layers than might be expected. May this not possibly arise from the size of the heterogeneous portions being comparable with the thickness of the dielectric, so that the more easily conducting portions may stretch in some parts from plate to plate? If so, we approximate more nearly to the inferior limit.]

The mathematical account of the residual discharge on this hypothesis is practically the same as Maxwell's ; but it may perhaps be worth while to give it with the necessary alterations, as these seem to make it somewhat more straightforward and evident.

We shall suppose with Maxwell, "for the sake of simplicity, that the dielectric consists of a number of plane strata of different materials and of area unity," and that the induction is in the direction of the normal to the strata.

Let $a_1, a_2, \&c.$, be the thickness of the different strata ;

$X_1, X_2, \&c.$, be the electric intensity within each stratum ;

$p_1, p_2, \&c.$, be the amount of decay of induction per second in each stratum ;

Let $f_1, f_2, \&c.$, be the induction in each stratum ;

$u_1, u_2, \&c.$, be the total number of tubes of electric induction entering each layer sideways, *i. e.* crossing in through its boundary per second ;

$r_1, r_2, \&c.$, be the specific resistance referred to unit of volume ;

$K_1, K_2, \&c.$, be the specific inductive capacity ;

$k_1, k_2, \&c.$, be the reciprocal of the specific inductive capacity ;

E be the electromotive force due to a voltaic battery placed in the part of the circuit leading from the last stratum towards the first, which we shall suppose good conductors ;

Q be the total number of induction-tubes which have left the battery and entered the wires and dielectric up to the time t .

Then, since the same number of tubes enter all parts of the circuit in a given time,

$$u_1 = u_2 = u_3 = \dots = u \text{ say.} \quad . \quad . \quad . \quad . \quad (1)$$

These tubes tend to increase the induction in the layers. But at the same time decay is going on, so that we have

$$u_1 = p_1 + \frac{df_1}{dt}, \quad u_2 = p_2 + \frac{df_2}{dt}, \quad \&c., \quad . \quad . \quad . \quad (2)$$

whence

$$p_1 + \frac{df_1}{dt} = p_2 + \frac{df_2}{dt} = \&c. \quad . \quad . \quad . \quad . \quad (3)$$

We also have by Ohm's law,

$$p_1 = \frac{X_1}{r_1}, \quad \&c.; \quad . \quad . \quad . \quad . \quad (4)$$

and by the relation between induction and intensity,

$$X_1 = 4\pi k_1 f_1, \quad . \quad . \quad . \quad . \quad (5)$$

whence

$$u = \frac{X_1}{r_1} + \frac{1}{4\pi k_1} \frac{dX_1}{dt}. \quad . \quad . \quad . \quad . \quad (6)$$

Let us suppose that at first there is no charge, and that suddenly the E.M.F., E is made to act. Then, if at once Q tubes enter the dielectric,

$$X_1 = 4\pi k_1 Q, \quad \&c.; \quad . \quad . \quad . \quad . \quad (7)$$

and since

$$E = a_1 X_1 + a_2 X_2 + \dots, \quad . \quad . \quad . \quad . \quad (8)$$

$$E = 4\pi(k_1 a_1 + k_2 a_2 + \dots)Q.$$

The instantaneous capacity C , which is equal to $\frac{Q}{E}$, is given by

$$C = \frac{1}{4\pi(k_1a_1 + k_2a_2 + \dots)} \quad \dots \quad (9)$$

But dissipation at once sets in, and if the electromotive force E be continued uniform, a steady state will ultimately be reached in which the dissipation in each layer is equal to the number of fresh tubes reaching that layer. The number of tubes entering being the same throughout, the dissipation p is also the same throughout.

We have then

$$p = \frac{X_1}{r_1} = \frac{X_2}{r_2} = \&c. \quad \dots \quad (10)$$

and substituting in (8)

$$E = (r_1a_1 + r_2a_2 + \dots) p.$$

Hence, if $R = r_1a_1 + \dots$

$$p = \frac{E}{R} \quad \dots \quad (11)$$

In this state we have the induction given by

$$f_1 = \frac{X_1}{4\pi k_1} = \frac{r_1}{4\pi k_1} p = \frac{Er_1}{4\pi k_1 R} \quad \dots \quad (12)$$

If we now suddenly connect the extreme strata by means of a conductor of small resistance, E will be suddenly changed from the value E_0 to zero, and Q' tubes will pass out from each layer of the dielectric into the wire. If then X_1^1 be the new value of the intensity,

$$Q' = \frac{X_1}{4\pi k_1} - \frac{X_1^1}{4\pi k_1};$$

whence

$$X_1^1 = X_1 - 4\pi k_1 Q' \quad \dots \quad (13)$$

Since, then, the difference of potential is zero,

$$a_1 X_1^1 - a_2 X_2^1 + \dots = 0;$$

substituting from (13) we get

$$a_1 X_1 + a_2 X_2 + \dots = 4\pi(a_1 k_1 + a_2 k_2 + \dots) Q';$$

or

$$Q' = \frac{a_1 X_1 + a_2 X_2 + \dots}{4\pi(a_1 k_1 + a_2 k_2 + \dots)} = CE = Q, \text{ from (8) and (9).} \quad (14)$$

Hence the instantaneous discharge is equal to the instantaneous charge.

By (10) and (11) we may put (13) in the form

$$\begin{aligned} X_1^1 &= pr_1 - 4\pi k_1 Q, \\ &= \frac{Er_1}{R} - 4\pi k_1 Q. \quad \dots \quad (15) \end{aligned}$$

Let us next suppose the connection broken immediately after the discharge. No fresh tubes enter any layer; so that putting $u=0$, we have from (6)

$$u = \frac{X_1}{r_1} + \frac{1}{4\pi k_1} \frac{dX_1}{dt},$$

or
$$X_1 = X_1^1 e^{-\frac{4\pi k_1 t}{r_1}},$$

where X_1 is now the value of the electric intensity at any time t after the connection is broken.

Substituting from (15) and putting E_0 for the initial value of E ,

$$X_1 = E_0 \left(\frac{r_1}{R} - 4\pi k_1 C \right) e^{-\frac{4\pi k_1 t}{r_1}} \dots \dots (16)$$

The value of E at any time is

$$\begin{aligned} E &= a_1 X_1 + a_2 X_2 + \dots \\ &= E_0 \left\{ \left(\frac{a_1 r_1}{R} - 4\pi a_1 k_1 C \right) e^{-\frac{4\pi k_1 t}{r_1}} + \left(\frac{a_2 r_2}{R} - 4\pi a_2 k_2 C \right) e^{-\frac{4\pi k_2 t}{r_2}} + \&c. \right. \\ &= E_0 \left\{ \left(\frac{r_1}{R} - 4\pi k_1 C \right) a_1 e^{-\frac{4\pi k_1 t}{r_1}} + \left(\frac{r_2}{R} - 4\pi k_2 C \right) a_2 e^{-\frac{4\pi k_2 t}{r_2}} + \dots \right\}. (17) \\ &= E_0 \left\{ \left(\frac{r_1}{k_1} - 4\pi C R \right) \frac{a_1 k_1}{R} e^{-\frac{4\pi k_1 t}{r_1}} \dots \right. \end{aligned}$$

The instantaneous discharge obtained at any time t will be, as before, CE.

If the terms be arranged in descending order of magnitude of $\frac{r_1}{k_1}$, then the exponentials are also in descending order of magnitude, or the negative terms decrease more rapidly than the positive, and E is positive.

LVIII. *Note on a "Relation between the Critical Temperatures of Bodies and their Thermal Expansions as Liquids."* By T. E. THORPE, Ph.D., F.R.S., and A. W. RÜCKER, M.A., F.R.S.*

A PAPER bearing the above title was published by us in the Journal of the Chemical Society of London for April 1884, and has recently been discussed by MM. A. Bartoli and E. Stracciati†. As these gentlemen have done us the honour to make use of a formula deduced by us from the

* Communicated by the Physical Society: read April 10, 1886.

† *Ann. Chim. Phys.* Mars 1886, p. 384.