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The kingdom boasts of two batteries of machine-guns—that is, it has fourteen Gatling guns, which have never been fired off; and, as far as I could make out, the Royal Korean Artillery consists of seven men permanently absent in the provinces. As for a small arsenal of 9-inch Krupp guns, with all necessary armourers' tools, these lie rusting away, while breech-blocks and sights have all been stolen.

The "Military Adviser" to the King is an American, General Dye; but neither he nor Lieut.-Colonel Nienstead, who has the thankless post of drill-instructor to the army, are allowed any powers of discipline over the members of this very oddly constituted force.

In conclusion, I should like to draw attention to a remarkably tough paper which is made in Korea from vegetable fibre. It is in great request in China and Japan, and I am trying to introduce it into this country for military map purposes; I can strongly recommend it to any one who is in want of an almost untearable paper.

ON THE DETERMINATION OF SEA-WATER DENSITIES BY HYDROMETERS AND SPRENGEL TUBES.¹

By W. S. ANDERSON, F.C.S.

THE two principal methods of ascertaining the density of a sea-water are, first, a weighing method, preferably by weighing the water in a Sprengel tube; and, second, by using a hydrometer. The latter method is used exclusively on board ship, where weighing is generally impossible.

For some time, as chemist to the Marine Station, Granton, I have had occasion to estimate the densities of a large number of sea-waters for Dr. Murray and Mr Irvine, and have employed both methods. Some observers have supposed that there was a difference in the results obtained by these methods;² the hydrometer giving, it was stated, results lower than those of the Sprengel tube by 0.12 (water = 1000), and it was proposed that a correction of that amount should be added to the densities obtained by hydrometers in order to render the results accurate. Dr. Murray requested me to investigate the whole matter, together with any bearing the results might have on the *Challenger* work, and provided me with such hydrometers and other instruments as were required for the investigation. This paper is the result of experiments comparing the density values given by the Sprengel tube and the hydrometer; observations by the latter instrument having been taken both on land and on board ship.

Weighing the water in a Sprengel tube is admitted by all observers to be the more accurate method. The instrument is a glass tube bent into a U or V shape, with a capillary tube at each extremity, one of them having bulbs blown into it. The instrument used for the experiments

¹ Read before the Royal Society of Edinburgh, on May 21st, 1894.

² *Scot. Geog. Mag.*, Jan. 1893, Dickson on the Waters of the English Channel.

had a capacity of 82 cc., and the inner diameter of the capillaries was from 0.3 to 0.5 millimetre, so that a length of 4 to 9 millimetres weighed 0.001 grm. With a more delicate balance than was at my disposal, the Sprengel tube may be about one half the capacity, and the capillaries narrower.

The instrument was standardised in the following manner:—The dry tube was weighed in air, preferably on both sides of the balance, and the weight reduced to *vacuum*. Then distilled water (boiled and cooled) was drawn up one of the capillaries to completely fill the instrument without air-bells, and the whole was placed in a bath of the temperature at which the sea-water density was afterwards to be taken. At 0° C. a bath of melting ice was used, and into this the Sprengel tube was plunged, the capillaries being left outside. Care was taken that the line of water in the capillary containing the bulbs was not broken on the contraction of the water. On the subsequent expansion of the water, the capillary with the bulbs was emptied nearly down to a mark etched on its lower end, and after all change in the volume of the water had ceased, the excess was removed by blotting paper. As little of the capillaries as possible was allowed to project above the surface of the bath. On the removal of the Sprengel tube from the ice-bath it was placed in a bath of the temperature of the balance room, or a degree or two higher to prevent condensation of moisture on the chilled glass. The water expanded into the capillary bulbs, when the instrument was carefully dried and the weight reduced to *vacuum*.

The determination of the weights of the Sprengel tube full of pure water at the temperatures desired was the most important part of the operation, as on it depended the accuracy of the sea-water densities, and the weighings were repeated till the results were constant. The tube was also weighed filled with pure water at other temperatures, such as 4°, 10°, 15°, 15.56°, 17.5° C., as it was found better to test every sea-water at two temperatures; any error in weighing being thus easily detected.

The operation was practically the same at the higher temperatures as at 0° C., except that a bath containing 15 litres of water was used. This was heated with a flame, an accurate thermometer giving the temperature after well stirring. There was little difficulty in keeping the temperature within $\frac{1}{100}$ ° C. by careful application of the flame, especially when the temperature of the laboratory and the bath were nearly the same. The Sprengel tube was suspended between two wires over the bath, and the standardising completed for the higher temperatures as already mentioned for 0° C.

While in the water bath the sea-water, if recently collected, gave off air-bells, which accumulated into one on tapping the tube, and were drawn up into the bulbs by suction.

On subtracting the weight of the tube from its weight when full of water the standard '1000' was found, and with this value the weight of the sea-water which filled the tube, at the same temperature, was compared. The weight of the sea-water was divided by the weight of the distilled water, and the result was the density of the sea-water, represented by ρ_t . When ρ_t was required, the density at ρ_t was multiplied

by the density of pure water at $t^{\circ}\text{C.}$ ($4^{\circ}\text{C.} = 1$). Some tables at the end of the paper facilitate the reduction of ρ_{St} to ρ_{Sr} .

The hydrometer is an instrument the principle of which is well known; the density of water is ascertained by dividing the weight of the instrument by the volume. The volume of the glass body at the given temperature must be known, as well as the volume of the stem immersed in the water. All the results require reduction to one temperature for comparison, as it seldom occurs that the temperature of the water remains the same during a series of observations.

There was a hydrometer of the *Challenger* kind at the Marine Station. It had accompanied others on the voyage, but had never been used; all the densities having been taken with No. 0 hydrometer. This hydrometer was carefully standardised in pure water, according to the method described in full by Mr. Buchanan.¹ After the constants were obtained, the hydrometer was tested with sea-waters collected from various places in the North Sea. The results tended to show that the densities found by the hydrometer were fairly in agreement with those determined by direct weighing, but that the individual results varied to a small amount. It was hardly to be expected that a hydrometer of the ordinary size, where one division of the stem was equal to 0.05 or 0.06 in density (water = 1000) should give results up to 0.01, unless the mean of a number of readings was taken, and in practice it was found that, the greater the number of readings taken, the more correct became the mean result from the whole. One of the waters was employed for ascertaining its density at three very different temperatures. The value ρ_{S_0} , that is, the weight of sea-water at 0°C. compared with the weight of an equal volume of distilled water at 0°C. , was previously estimated by the Sprengel tube to be 1028.51. The chlorine, or total halogen, in the water was also estimated by Volhard's process, and found equal to 19.547 grammes per kilogramme. The following are the hydrometer results reduced to ρ_{S_0} :—

Temperature	1.4° to 2.7°C.	16.7°C.	26.2°C.
Density	1028.40	1028.53	1028.46
	.47	.49	.47
	.58	...	.60
	.53	...	...
	.52	...	...
	.52	...	...
	.60	...	...
	.60	...	...
	.51	...	...
	.57	...	...
Mean density	1028.53	1028.51	1028.51

Although the average density was very nearly correct, still the individual readings sometimes varied considerably—as much as three or four millimètres on the scale of the instrument. The cause of this

¹ *Challenger Report*, "Chemistry and Physics," vol. i. part 2.

was afterwards found to be the insufficient mixing of the sea-water before taking the reading. This error affected particularly the first series of readings, when the water was cooled by ice, and also the third series, when the water was warmed in a bath up to 26° C.

Two hydrometers of the *Challenger* pattern, marked 18 and 19, were kindly lent me by the Scottish Fishery Board, and were tested in like manner as the last mentioned, with the use of the constants supplied with them. The sea-water employed had a density of 1021·335, the weight of sea-water at 15·56° C. being compared with the weight of an equal volume of distilled water at maximum density. The hydrometer results were:—

Station Hydrometer.	No. 18.	No. 19.	Temperature of Readings.
1021·34	1021·36	1021·36	6·1° to 6·9° C.
·34	·30	·31	
...	·37	·36	
...	·36	·36	
...	·32	·33	
...	·34	·36	
...	·34	·37	
Mean density 1021·34	1021·34	1021·35	23° to 24° C.
...	1021·36	1021·32	
...	...	·33	
...	...	·35	
Mean density ...	...	1021·33	

Mean density from all readings = 1021·343.

This result was satisfactory, the average density being nearly the same as that obtained by the Sprengel tube, and the greatest errors $\pm 0\cdot035$, equal to $\pm$ half a millimetre of scale-reading.

Owing to uncertainty as regarded the standardising of these two instruments, it was still impossible to consider the question of hydrometer error as settled, although the preliminary experiments seemed to show that the hydrometer method was a perfect one and did not require any correction. But the personal equation in reading, and any instability of the instrument, might cause an error on the scale of one millimetre, equal to 0·06 in density; and this was half of the difference in question.

To settle this, I made a hydrometer of more than double the usual volume, leaving the stem about the same, *i.e.* 3 millimetres diameter. This had much more delicacy, and any difference could be more easily detected. The instrument was made in the manner described so fully in the *Challenger Report*. The data required for the standardising of a hydrometer are:—the weight of the instrument *in vacuo*,¹ the volume of the glass body at 0° C., the expansion of the body for each degree, and the

¹ For a hydrometer of the size mentioned, the weight may be taken to $\pm 0\cdot02$ gram. No appreciable error is introduced, as the volume is relative to the weight decided on.

volume of the stem for each millimètre of the scale. The expansion of the stem with increase of temperature is too minute to be taken into account. A set of suitable weights are necessary for taking readings at various parts of the stem. To find the volume of the instrument during a reading, the volume of the body at the observed temperature must be known, and to this volume is added the volume of the stem immersed. The total weight of the instrument, during the observation, divided by the total volume so found, gives the density of the water at the temperature. As the comparison is made with distilled water at 4° C., when each cubic centimètre weighs one gramme, the sign S will represent the density of the water, t being the temperature of the water during observation. The data for standardising may be expressed in the following formula :—

$$\frac{\text{Weight of hydrometer in vacuo (+ additional weights)}}{\text{Volume in cc. at } 0^{\circ} \text{ C. + expansion in cc. for } 1^{\circ} \text{ C.} \times t^{\circ} + \text{cc. of stem immersed}} = \text{S}_t.$$

The S_t so obtained may be reduced to $\text{S}_{15.56}$, for comparison with other results, by means of Table I., page 588. In finding the data for the large hydrometer the weight *in vacuo* was first obtained, and then the volume of one millimètre of the stem; found by loading the hydrometer with weights and sinking the stem in distilled water. On dividing the volume of water so displaced, at the temperature chosen, by the number of millimètres immersed, the result obtained was the value of one millimètre of the scale, and was equal to 0.0098 cc. The stem was also tested in a sea-water of known density by using it as a hydrometer. The weight used in sinking the stem was divided by the stem immersed, and the result was an approximation to the density of the water. The volume of the body of the instrument at any temperature (t) was next obtained by floating it in distilled water and taking readings, generally three or four, at various parts of the stem. It was thought better to take readings at every degree up to 29° C., a wrong reading or an irregularity in glass-expansion being thus easily detected. It was found to be of the greatest importance that all the water should be of the same temperature, and this was only secured by thorough mixing.¹ At first it was erroneously supposed that the water could be well mixed by plunging the hydrometer many times through it. But the bottom layer was often found to differ in temperature from the surface by nearly half a degree. This difficulty was overcome by attaching a stiff rubber washer or ring to a glass rod, and, before taking a reading, the water was well mixed up by moving this up and down encircling the hydrometer, which was held in the middle of the water. No water remained on the stem more than ten millimètres above

¹ On 1st October 1893, Professor Pettersson, Stockholm, wrote the following in a letter to Mr. Irvine: "The most dangerous error of the hydrometer—because the experimenter has no means of correcting it, at least when he works on board a ship—is this: you obtain very sensible indications by using a hydrometer with a large body. But, remember, there must then be equilibrium of temperature through the whole system—water, glass, etc. That takes time, and meanwhile the water in which swims the instrument is heated or cooled by the wind, the warm water rises, the cooler sinks down; you do not know if your instrument does not swim in a fluid of which the uppermost layer has a different temperature from the lower: you may not stir, for then a sharp reading is rendered impossible for a minute or so."

the surface, so that there was free vertical motion, and every reading was checked twice or thrice by disturbing the equilibrium of the hydrometer. The temperature of the water was taken before and after every reading, of which about 150 were noted. The volume of the body of the hydrometer, at any temperature, was calculated by multiplying the total weight of the instrument by the volume of pure water at that temperature ($4^{\circ}\text{C.} = 1$), and subtracting the volume of the stem immersed. The last datum was obtained by multiplying the reading in millimètres by 0.0098 cc. (volume of one millimètre of the scale). The experiments gave the following data for the new hydrometer:—Weight *in vacuo* = 353.36 grammes; volume of body at $1^{\circ}\text{C.} = 354.504$ cc.; expansion of body for $1^{\circ}\text{C.} = 0.0095$ cc.; volume of each millimètre-division of the stem = 0.0098 cc. These values having been proved to be nearly accurate for pure water, they were applied to the determination of sea-water densities. An Atlantic water collected in $51^{\circ} 20' \text{ N. lat.}$, and $30^{\circ} 0' \text{ W. long.}$ was chosen, and was used for all the subsequent experiments, as it was expected to follow Dittmar's tables of sea-water expansion more closely than any other. The chlorine was estimated by Volhard's process to be equal to 19.696 grammes per kilogramme. The density was taken carefully by the Sprengel tube at 0°C. , 15°C. , and 17.5°C.

$${}_0S_0 = 1028.685, {}_{15}S_{15} = 1027.338, \text{ and } {}_{17.5}S_{17.5} = 1027.185.$$

Reducing these to the standard hydrometer temperature, the resulting density was ${}_4S_{15.56} = 1026.353$, and was probably correct to ± 0.005 . The density at 0°C. is ${}_0S_0 = 1028.685$, and the D value, or the ratio between the chlorine and 1028.685 (-1000), is 1.4565, a value generally admitted to be correct for the whole of the North Sea, and which may also hold good for the whole ocean, with the exception of small enclosed areas, water confined in sea-muds, estuaries, and comparatively fresh water like that of the Baltic.

The densities of the Atlantic water, obtained by the large hydrometer, were as follows:—

${}_4S_{15.56}$		
1026.35	9	
.35	5	
.35	8	
.33	7	
.35	9	
.34	1	
Mean	1026.35	1
1026.36	7	
.36	6	
.36	2	
.36	5	
Mean	1026.36	5

Temperature between 8.4° and 8.5°C.

Temperature between 18.6° and 18.8°C.

1026·34	1	Temperature between 28·7° and 29·2° C.
·34	7	
·35	0	
·34	1	
<hr/>		
Mean	1026·34	5

Mean of all readings = 1026·353.

These results were very satisfactory, the mean densities obtained by the Sprengel tube and hydrometer being identical. The difference between the lowest reading and the highest was 0·03, equal to an error in reading the scale of $\pm$ half a millimetre.

It was thus shown decisively that a delicate hydrometer, if properly standardised, and all precautions taken to secure correct readings, will give good results, not requiring any correction whatever to make them agree with the Sprengel tube results.¹

Any errors of reading, during the experiments, were mainly due to insufficient mixing of the water. When the water was colder than the air of the laboratory, the temperature of the surface rose on standing, and a layer of cold water remained at the bottom of the hydrometer jar. When standardising, readings should be taken in distilled water warmed up gradually and mixed well before reading. During the cooling of the water a second series of readings should be taken, and these ought to agree with the first series.

It has been stated that one source of errors in hydrometer work has connection with the size or diameter of jar used, and this is no doubt true, but only indirectly. Nearly all the experiments with the large hydrometer were made in a glass jar of 13 cc. diameter, but a few readings taken in a jar of only six centimetres (two centimetres more than the diameter of the hydrometer) showed that the results were less reliable. The instrument tended to cling to the sides of the jar, and came more quickly to rest than in a wide jar, but the readings were the same. The difficulty in mixing the water sufficiently showed that for such delicate work readings ought never to be taken in a jar of a diameter less than two and a half times the diameter of the hydrometer.

Occasionally there are observational errors, such as reading 43 or 33

¹ *Capillarity*.—The influence of capillarity on a hydrometer was expected, during the experiments stated above, to manifest itself in a direct, instead of an indirect manner. This effect is caused by the difference in weight of the hillocks of pure water and sea-water drawn up round the stem, and was estimated by Dittmar to be equal to + 0·04 in density. In the hydrometers examined by me this influence was not manifest, not even in the large hydrometer where the error would have been equal to + '02. In a larger and more delicate hydrometer, the capillarity error, if it exists, would tend to disappear. But experiments with capillarity tubes of various diameters showed that sea-water will rise in them to the same height as distilled water, and this would tend to show that the influence is in the other direction, though hardly perceptible. I am inclined to think that it may be neglected altogether, and, so far as sea-water hydrometers are concerned, it is of no great importance, as such instruments ought always to be compared by some weighing method before they are extensively employed.

for 37, and there is greater liability to read wrong when the figures run down the scale instead of up.

Although hydrometers give accurate results on land, it is known that results may be different at sea, no doubt owing to the rolling and lurching of the ship and the consequent instability of the instrument and difficulty in reading. Under such conditions the temperature of the water ought to be within a few tenths of a degree the same as that of the surrounding air, and well mixed before every reading, of which three should be taken at different places on the stem.

The following differences in density between the hydrometer readings at sea and the subsequent determinations by weighing on land may be placed in a table, each of the columns representing a water obtained during a cruise in the North Sea.

Waters with difference of	Waters with difference of	Waters with difference of	Waters with difference of
7 . $\pm$ '03	1 . + '06	1 . + '01	1 . + '03
5 . $\pm$ '05	1 . + '15	1 . + '03	2 . + '05
3 . $\pm$ '08	2 . + '17	1 . + '08	1 . + '08
3 . $\pm$ '11	1 . + '18	4 . + '10	6 . + '10
3 . $\pm$ '16	1 . + '19	2 . + '12	2 . + '12
3 . $\pm$ '21	1 . + '20	1 . + '14	1 . + '14
3 . $\pm$ '80	2 . + '21	1 . + '16	3 . + '17
1 . -1'11	1 . + '23	1 . + '17	

The large errors in the first column are no doubt due to reading the scale of the hydrometer the wrong way. The other results are difficult to explain, as a smooth sea lasted during the cruise. In the second column the results are given of a cruise in bad weather, and in the third and fourth column results recorded when the weather was fairly good. Why the errors are all plus may be difficult to explain. If the readings were all taken when the vertical motion of the ship was downwards, the tendency would be to give plus results; and minus results would occur when the ship was rising during a reading. The observer, however, is inclined to consider the errors as due to temperature. There seems to be no doubt that an error of 0'16 in density, or 3 millimètres of the scale, is not uncommon, even on a comparatively smooth sea. The waters of one cruise were also examined for density by a hydrometer of the same size on land, and gave the following differences when compared with the Sprengel tube results:—

At Sea.	On Land.
+ '01	- '02
+ '03	'00
+ '09	- '07
+ '10	- '06
+ '14	+ '08
+ '10	+ '04
+ '17	- '02

At Sea.	On Land.
+·12	+·06
+·08	·00
+·16	+·03
+·12	—·04
+·09	—·04

The errors at sea are all plus quantities, and the greatest is equal to 3 millimètres of reading. On land the plus and minus quantities nearly balance, and the greatest error is equal to 1·5 millimètres. Experiments at sea might yet be tried with a hydrometer of a capacity of 600 cc., where an error of 3 millimètres would only equal ± 04 .

If the samples of water collected are to be afterwards tested in the laboratory, they must be filtered and well stoppered to prevent evaporation, which sometimes takes place notwithstanding every precaution. Whether glass should be used at all for storing waters is questionable.¹

From Dr. Murray and Dr. Buchanan I obtained the hydrometer used on board the *Challenger* for all the waters collected. It was examined for the purpose of seeing what corrections, if any, would be necessary for the interpretation of the results as detailed in the *Report*,² in the hope that the variableness or constancy of the composition of ocean water might be detected. Professor Dittmar had estimated the error of this hydrometer to be —·08 in density, and the error of the scale to be +·0002 cc. per millimètre. That is to say, the data he found for the hydrometer differed by the above quantities from the data given by Dr. Buchanan. My examination of the hydrometer gave the following general results. Like Dittmar, I found it to be too low generally, but not so low at the higher temperatures. At 10° C. the error in density was —·07, at 24° C. only —·01, and the scale error was +·0001 cc. per millimètre. With reference to the *Challenger* hydrometer, and to any inferences drawn from its present condition, it is assumed in the following

¹ The following examples of evaporation and change in sea-water are interesting. Two or more samples of the same *Challenger* water, No. 1687, were collected, and one of them was examined twelve years ago by Dittmar. Two of the samples were also examined last year:—

	Chlorine per Kilogramme.	Alkalinity (CO ₂) per Kilogramme.	Year of Analysis.
Sample A,	{ 20·64	53·05	1878-82
	{ 20·63	52·61	1893
Sample B,	20·83	66·71	1893

So that while one sample had remained the same for about a dozen years, the other had evaporated sufficiently to increase the density by 0·30 (water=1000), and the alkalinity was greater than this evaporation could account for. Whether this increase of alkalinity is caused by solution of the alkali of the glass, or by the decomposition of sulphates by organic matter, is doubtful. Not only may the action of the glass tend to increase the alkalinity, but, as the following fact shows, it may tend to *decrease* the alkalinity of the water, a tendency which has, I believe, been overlooked. A bottle of sea-water, with 0·1 gramme mussel flesh added, after standing for nine months, was examined for alkalinity, and found to have lost 14 milligrammes of combined carbonic acid. A white deposit on the bottom of the bottle was supposed to be carbonate of lime, but on examination turned out to be mainly silicate of magnesia along with silicates of alumina, iron, and a trace of lime, the action of the alkaline sea-water on the glass being thus very distinct. What reaction had taken place, and what was the influence of the organic matter, is not yet clearly understood.

² *Challenger Report*, "Chemistry and Physics," vol. i., part 2.

part of the paper that the volume of the glass and its power of expansion have not altered since it was first made, though it is not impossible that changes may have occurred during twenty years which would alter the readings. In examining the *Challenger* hydrometer I checked the brass weights and the table for holding them (see page 10), and compared them with the values of the weights as given in the *Challenger Report*. The two weights principally used during the cruise had become lighter, Nos. IV. and V. having lost 0·0015 and 0·003 grammes respectively, and the table for laying them on had lost 0·003 grammes. No doubt these weights had become lighter by use, the other unused weights having remained constant. Two readings were taken with the hydrometer at 10° C., the table and weights and all the data as given by Buchanan being employed. The Atlantic water used for the other hydrometer experiments was also used in this case.

$$\begin{array}{r} {}^4S_{15.56} \\ 1026.335 \\ 26.355 \\ \hline \end{array}$$

Mean 1026.345

The correct density, as ascertained by the Sprengel tube, was 1026.353, and therefore the error of the instrument at 10° C. was only about —·01 in density. It is plain that towards the end of the cruise the given data would tend to render the observed densities more correct than they would be at the commencement, at which time the real mass of the brass weights would agree with the values given in Buchanan's data. To show this more fully we should subtract 0·04 of density from the mean result of the two readings given above, 0·02 for the loss in weight of the table, and 0·02 for the loss in weight of No. V. brass weight, and the resulting density is 1026.305, too low by —·05, which would give the error of the hydrometer when the cruise began. Further experiments were carried out, using another set of weights altogether, showing that the error was more probably —·07 at 9° C.

	${}^4S_{15.56}$		
	1026.34	6	
	.26	9	
	.25	5	
	.25	7	
	.31	2	
	.25	6	
	<hr/>		
Mean	1026.28	3	
	1026.37	2	
	.36	5	
	.33	4	
	.30	1	
	.33	4	
	.33	9	
	<hr/>		
Mean	1026.34	1	

Temperature from 9.2° to 9.4° C.

Temperature from 23.4° to 24° C.

The mean error at 9° C. was equal to $-.07$, and at 24° C. only $-.01$. There was also a distinct tendency to read higher than the mean at the lower part of the stem of the hydrometer, and correspondingly lower at the upper part of the stem. The corrections necessary for this tendency I found to be $-.02$ and $+.02$ respectively.

From the above experiments the following data were found for the hydrometer as it is at present, and beside these values I have placed Buchanan's, as found by him before the cruise.

	Buchanan.	Anderson.		Buchanan.	Anderson.
Weight of Hydrometer,	160.2128	—	Volume of body at 0° C.,	160.277	160.262
Weight of Table,	0.836	0.833	Expansion for 1° C.,	0.00456 cc.	0.00527 cc.
„ No. I.	0.856	—	Volume of one millimetre of stem,	0.00865 cc.	0.00856 cc.
„ No. II.	1.601	—			
„ No. III.	2.4225	—			
„ No. IV.	3.2145	3.213			
„ No. V.	4.071	4.068			
„ No. VI.	4.8245	—			

I have assumed, then, that the hydrometer itself has not altered since it was made, but that the weights have become lighter by use, and in the following table all the corrections have been taken into account for the period of the voyage, and for the temperature of the water.

Temperature of water ° C.	1st Period, from Feb. 1873 to Mar. 1874.	2nd Period, from April 1874 to April 1875.		3rd Period, from May 1875 to May 1876.	
	Correction for any weight.	Correction for weights No. I., II., III., VI.	Correction for weights No. IV. and V.	Correction for weights I., II., III., VI.	Correction for weights No. IV., V.
0°	+ .10	+ .09	+ .08	+ .08	+ .06
10	+ .07	+ .06	+ .05	+ .05	+ .03
17	+ .04	+ .03	+ .02	+ .02	.00
24	+ .01	.00	-.01	-.01	-.03

For the stem correction .02 must be subtracted when a high reading is noted, and .02 added when a low reading is noted.

An attempt has been made to apply the corrections of the above table to about 70 of the *Challenger* densities, those, namely, which were partially checked by Professor Dittmar,¹ by an estimate of the chlorine and other constituents of the water.

¹ *Challenger Report*, "Chemistry and Physics," vol. i. p. 23.

It is interesting to note that the mean D value was 1·4571, which is very near the value for the North Sea and the Atlantic waters. Considering the doubt attached to hydrometer estimations at sea it is very questionable whether the differences of D value, found by dividing Buchanan's densities reduced to 0° C. by Dittmar's chlorines, can be accepted as evidence of difference in the composition of sea-water at various places in the ocean. There are only two or three of the *Challenger* waters still in existence, but the following comparison of the estimate of density by the hydrometer during the voyage, and a later estimate by a Sprengel tube is interesting. That the chlorine had remained the same since the first analysis is a sign that the density had undergone no alteration in the interval, and had probably remained unchanged since collection.

CHLORINE.			DENSITY.		
Water No.	(Dittmar) 1878-82.	1893.	Hydrometer value reduced to 0°S ₀ .	Sprengel tube value 0°S ₀	Difference.
1700	20·57	20·58	1029·79	1029·96	—·17
1687	20·64	20·63	30·00	30·03	—·03

This result would tend to show that errors of 0·17 in density, equal to three millimètres on the scale of the hydrometer, may have occurred in the determinations during the cruise.

If it be assumed that the differences of the value of D are due to the hydrometer error, and not to any real difference in the ratio of chlorine to the other salts, it may be interesting to place in a short table the errors of density required for the seventy waters, to reduce the values to one uniform value of D. Besides these D values derived from Buchanan's results, I have attempted to derive a ratio, or D value, from Dittmar's estimate of the total salts, as well as chlorine.¹ A difference in the ratio of chlorine to the total salts in sea-water is really synonymous with a change of the D value, and ought to compare with the ratio of chlorine to density.

The chlorine is given in the *Report* in parts per 100 of the total salts.¹ A mean D value of 1·457, and a hypothetical density of 1080·747, have been used by me as standards, and the probable D value has been found for each analysis. The variations from the mean values were found in terms of the density, and these variations are collected and compared with the variations from the mean obtained from the hydrometer densities. The variations are also given in millimètre divisions of the scale.

¹ *Challenger Report*, p. 23.

Variation.	By the Hydrometer.		From Dittmar's Analysis.	
	+ Results.	— Results.	+ Results.	— Results.
Within '03 of density	9	6	19	18
" '06 "	9	7	11	12
" '12 "	7	6	4	3
" '17 "	5	4		
" '22 "	5	4		
" '28 "	3	1		
" '35 "	...	2		
Within 1 millimètre of scale	19	12	32	28
" 2 " "	7	8	3	4
" 3 " "	5	6	...	...
" 4 " "	9	2	...	...
" 5 " "	...	2	...	...
" 6 " "	...	1	...	...

There is not one of Dittmar's results which shows a variation greater than 0·12 in density, equal to two millimètres on the scale of the hydrometer, while twenty-five of Buchanan's results give greater variations. It would thus seem that the probable error at sea, owing to the various causes before mentioned, is very much greater than any errors attaching to the hydrometer constants. Only in 36 out of the 70 waters do the D values, from both methods, agree in direction from the mean value, and in the majority of these 36 waters they disagree to a considerable extent.

If a difference, between the results derived from both methods, of 0·04 in density be considered of comparatively small importance, only 13 out of the 70 waters are in satisfactory agreement, and it is to be observed that they are all practically normal waters.

No.	D value.	No.	D value.
22	1·4587	434	1·4590
226	74	437	88
283	70	865	91
312	73	875	81
386	84	922	92
392	61	962	67
393	79		

But the table of the full analysis referred to¹ cannot be used for such a purpose as the determination of the D value within certain limits, as the analysis of such a complex substance as sea-water is difficult. The irregularity in the precipitation of lime,² and the fact that the soda must be estimated by difference, might give rise to errors, and the varying amounts of surplus base shows that such must have crept in. On combining together the bases and acids found by analysis, an excess of base

¹ *Challenger Report*, "Chemistry and Physics," vol. i. p. 23. ² *Op. cit.* p. 9.

is generally left over. The amount of the surplus base, from the analysis of each water, is given in the column of alkalinity,¹ in units in the fifth place of decimals, and on page 125 (vol. i., "Chemistry and Physics"), is given the correct quantity of surplus base, in terms of combined carbonic acid, as found on titration with acid. Only in very exceptional circumstances does the alkalinity really fall below 110 or rise above 160 units of alkalinity per kilogramme, but from the full analysis there are the following values:—

3	waters with	400 or over.
9	"	" 300-400
33	"	" 200-300
14	"	" 160-200
10	"	" 110-160
6	"	" 70-110
2	"	below 70

The mean of all is 220, or 100 too high, and the conclusion is evident that the total bases have as a rule been slightly over-estimated, and the results cannot be used for calculation of the D value within that probable error.

The fact remains, however, that from theoretical considerations small changes in the relation of chlorine to density in ocean water must occur. Bottom waters were found to be more alkaline than surface waters, owing to the solution of carbonate of lime shells falling from the surface, and to the action of mud.⁴ The extremes of alkalinity found by Dittmar for all the waters he examined, would raise the D value in some cases from 1.457 to 1.458, and in other cases lower it to 1.456. But, as already shown, the alkalinity of waters may alter during storage in glass bottles. Other causes of small differences in the D value of ocean waters are the abstraction of bromine by sea-weeds, and also the freezing of sea-water at the Poles.

But after a consideration of all these facts, it must be concluded that the data as yet obtained are not sufficient to throw any certain light on the non-uniformity or otherwise of the composition of ocean water, any differences that occur being probably hidden by the analytical errors, and the errors entailed by storage of water.

TABLE I.

An extended form of this table was found useful in hydrometer work for reducing the densities ${}_4S_t$ to ${}_4S_{15.56}$. It is a reconstruction of Dittmar's table, *Report*, page 70, giving the result of all the calculations of Dittmar's table simply as a quantity to be added or subtracted. For example, given by hydrometer ${}_4S = 1023$, what is ${}_4S_{15.56}$? The temperature is found on the horizontal line, and opposite in the column headed 1023 stands 1.65; $1023 - 1.65 = 1021.35$, which is the required ${}_4S_{15.56}$. Intermediate results are found by interpolation. When t is below 15.56 the figures are subtracted from ${}_4S_t$, when t is higher they are added to ${}_4S_t$.

¹ *Challenger Report*, "Chemistry and Physics," vol. i. p. 23.

² Murray and Irvine on Blue Muds, *Transactions of the Royal Society of Edinburgh*, vol. xxxvii., part ii., No. 23.

Minus.	t° C.	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030
	0·	1·83	1·88	1·93	1·97	2·02	2·07	2·12	2·17	2·22	2·27
	1·0	1·81	1·86	1·90	1·94	1·99	2·03	2·08	2·12	2·17	2·21
	2·0	1·78	1·82	1·86	1·90	1·95	1·99	2·03	2·07	2·11	2·15
	3·0	1·73	1·77	1·81	1·84	1·88	1·92	1·95	1·99	2·03	2·06
	4·0	1·67	1·70	1·74	1·77	1·80	1·83	1·87	1·90	1·94	1·97
	5·0	1·59	1·62	1·65	1·68	1·71	1·74	1·77	1·80	1·83	1·86
	6·0	1·50	1·53	1·56	1·58	1·61	1·64	1·66	1·69	1·71	1·74
	7·0	1·40	1·42	1·44	1·47	1·49	1·51	1·54	1·56	1·58	1·61
	t° C.	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029
Plus.	8·0	1·26	1·28	1·30	1·32	1·34	1·36	1·38	1·40	1·42	1·44
	9·0	1·14	1·15	1·17	1·19	1·21	1·22	1·24	1·26	1·27	1·29
	10·0	1·00	1·01	1·03	1·04	1·06	1·07	1·08	1·10	1·11	1·13
	11·0	·84	·85	·86	·87	·89	·90	·91	·92	·93	·94
	12·0	·68	·69	·70	·71	·72	·72	·73	·74	·75	·76
	13·0	·50	·51	·51	·52	·52	·53	·54	·54	·55	·56
	t° C.	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028
	14·0	·31	·32	·32	·32	·33	·33	·34	·34	·34	·35
	15·0	·12	·12	·12	·12	·12	·13	·13	·13	·13	·13
	16·0	·09	·09	·09	·09	·10	·10	·10	·10	·10	·10
Plus.	17·0	·31	·31	·31	·32	·32	·32	·33	·33	·33	·34
	18·0	·54	·55	·55	·56	·57	·57	·58	·58	·59	·59
	t° C.	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027
	19·0	·78	·79	·80	·81	·81	·82	·83	·84	·85	·85
	20·0	1·03	1·04	1·05	1·06	1·07	1·08	1·09	1·10	1·11	1·12
	21·0	1·29	1·30	1·32	1·33	1·34	1·35	1·36	1·37	1·39	1·40
	22·0	1·56	1·57	1·58	1·60	1·61	1·62	1·63	1·65	1·66	1·67
	23·0	1·83	1·84	1·86	1·87	1·89	1·90	1·92	1·93	1·95	1·96
	t° C.	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026
	24·0	2·09	2·10	2·12	2·14	2·15	2·17	2·19	2·20	2·22	2·24
Plus.	25·0	2·37	2·39	2·40	2·42	2·44	2·46	2·48	2·50	2·52	2·54
	26·0	2·66	2·68	2·70	2·72	2·74	2·76	2·78	2·80	2·83	2·85
	t° C.	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024
	27·0	2·93	2·95	2·97	2·99	3·01	3·04	3·06	3·08	3·10	3·12
	28·0	3·23	3·25	3·28	3·30	3·32	3·35	3·37	3·40	3·42	3·44
	29·0	3·54	3·57	3·59	3·62	3·65	3·67	3·70	3·72	3·75	3·77
	30·0	3·85	3·88	3·91	3·94	3·96	3·99	4·02	4·04	4·07	4·10
	31·0	4·17	4·20	4·23	4·26	4·29	4·32	4·35	4·38	4·41	4·44

If ${}_4S_{15}$ be preferred as the standard, reduce the sea-water as in above table and then *add* the following quantity, which depends on the density at ${}_4S_{15.56}$.

${}_4S_{15.56}$	
1000	+·095
1010	+·105
1026	+·125
1034	+·135

After the density ${}_4S_{15.56}$ is found, comparison with a determination by weight may be necessary. The result may be reduced to ${}_0S_0$ in the following manner:—

Find the difference between 1026 and the observed density, and multiply the difference by 1·0514. If the ${}_4S_{15.56}$ be greater than 1026, add the product to 1028·18, and the result is the density at ${}_4S_0$. If ${}_4S_{15.56}$ be less than 1026, subtract the product from 1028·18.

After ${}_4S_0$ is found add 0·132, and the result is the density of the water at ${}_0S_0$.

To reverse the process begin by subtracting 0·132, and multiply the difference between the remainder and 1028·18 by 0·9511, and add the result to, or subtract the result from, 1026.

If the density by weighing be taken at 15° C. or 17·5°, two commonly occurring temperatures, it may be reduced to ${}_4S_{15.56}$ by subtracting the following values:—

${}_{15}S_{15}$	To find ${}_4S_{15.56}$	$S_{17.5}$	To find ${}_4S_{15.56}$
1020	—0·972	1020	—0·856
24	—0·981	23	—0·846
28	—0·990	26	—0·835
32	—0·999	29	—0·824
		32	—0·813

But the density by weighing may be taken at any temperature, as ${}_tS_t$, and reduced to ${}_4S_t$ by multiplying the density (${}_tS_t$) by the density of distilled water at t° C. (4° C. = 1.) The reverse, ${}_4S_t$ to ${}_tS_t$ may be found by dividing ${}_4S_t$ by distilled water at t° C.

To convert the densities of ordinary sea-water obtained by Sprengel tubes to comparable temperatures, the following table will be found useful. Column *b* gives the D values of sea-water at temperatures from 0° C. to 30° C. They are calculated from Dittmar's Tables. (See *Report*, pages 70 to 81.) The D value is the density $S_t - 1000$, divided by the chlorine per kilo. It may be used in the following manner for reducing ${}_tS_t$ to ${}_rS_r$, as in Sprengel tube determinations, by multiplying the density minus 1000 by the D at T° , and dividing by the D at t° . A more convenient way of doing this is by using the logarithms, Column *c*, which, added to the logarithm of the density (minus 1000), give the logarithm of the density at ${}_0S_0$. To reduce from t to T subtract as before 1000, find the logarithm of what is left, add the logarithm opposite t° . This gives the logarithm of ${}_0S_0 - 1000$, which is next calculated to ${}_rS_r - 1000$ by subtracting the logarithm in a line with T° . When a water, such as one diluted with river-water, does not expand like ordinary sea-water, it is better to take the density at 0° C. by the Sprengel tube.

TABLE II.

a. Temperature °C.	b. D value.	c. Logarithm to reduce to $_{0}S_0$	a. Temperature °C.	b. D value.	c. Logarithm to reduce to $_{0}S_0$
0	1·4584		16	1·3869	·0218360
1	1·4528	·0016709	17	1·3836	·0228857
2	1·4475	·0032545	17·5	1·3821	·0233225
3	1·4422	·0048492	18	1·3805	·0238245
4	1·4371	·0063897	19	1·3774	·0248141
5	1·4320	·0079337	20	1·3748	·0256372
6	1·4274	·0093395	21	1·3719	·0265390
7	1·4226	·0107939	22	1·3696	·0272830
8	1·4179	·0122328	23	1·3672	·0280417
9	1·4135	·0135918	24	1·3652	·0286967
10	1·4093	·0148732	25	1·3632	·0293171
11	1·4049	·0162429	26	1·3612	·0299438
12	1·4012	·0173766	27	1·3592	·0306017
13	1·3972	·0186220	28	1·3576	·0311122
14	1·3939	·0196451	29	1·3559	·0316490
15	1·3902	·0207994	30	1·3545	·0320977
15·56	1·3883	·0213934			

The D given above is equal to $\frac{S^t - 1000}{\chi}$ and is not the D of Dittmar (on page 57 of the *Report*), which is the density of sea-water, $_{4}S_t$, minus the density of pure water, $_{4}W_t$, divided by the amount of chlorine, that is, $\frac{_{4}S_t - _{4}W_t}{\chi}$. The value of D in the table may be reduced to that of Dittmar's D by multiplying by $_{4}W_t$ and subtracting ·0016 from the product.

THE CAMPAIGNS OF ALEXANDER THE GREAT IN
TURKESTAN.¹

By J. W. McCRINDLE, M.A., M.R.A.S.

THE geographers who, in days not far remote, set themselves to determine the routes followed by Alexander the Great in his Asiatic Expedition, and to identify the districts, cities, and strongholds which in succession submitted to his arms, met with comparatively few difficulties until they came to deal with the regions in which the great conqueror found himself after he had passed through the Caspian Gates in pursuit of Darius. The countries lying between these Gates and the far-distant banks of the Jaxartes and Indus, which had been subjected to Persian rule by Cyrus and his successors, presented a striking contrast, both in

¹ *Alexander des Grossen Feldzüge in Turkestan.* Von Franz v. Schwarz. München : Dr. E. Wolf, 1893. Pp. 103.