PRECISE AND NUMERICAL IDENTITY.

THE following observations have been to some extent suggested by recent controversies in MIND, and are offered as a very humble contribution to the discussion of the question, What is the meaning of Identity?

It would seem *prima facie* that corresponding to the two aspects of all 'things' which can be subjects of predication (and of their names), *identity*, *identical*, may refer to intension (qualitative aspect) or extension (denotational, existential aspect).¹ If (one thing) A is identical with (another thing) B, then identical must mean exactly alike, indistinguishably similar, 'precisely identical,' as we admit that two half-crowns are, when we put a private mark upon one as a means of knowing it from the other, of discriminating it as *not* numerically identical—in order, *e.g.*, to detect theft—or as we may admit two sheets of notepaper, or two penny stamps, to be. In such a case, there is intensional or qualitative, but not denotational 'identity,' precise but not numerical 'identity'. In the contrasted denotational (or numerical) sense of identical, a thing can be identical only with itself. Thus the stamp I now take out of my purse is numerically identical with the stamp which you gave me yesterday in exchange for another. Although indeed its perceptible attributes are unchanged, and it looks just the same, this is not all, for denotationally or extensionally it is the very same stamp, it is numerically identical—it has had, we will admit, a continuous existence from yesterday to to-day.

Further, this very stamp is the stamp which the English Ambassador at Washington affixed a fortnight ago to the envelope from which you cut it—it has had a continuous existence from that time to the present—we have not here a case of "mistaken identity". It has, however, since it was posted, changed in appearance, it has not the same visible qualities—it has been defaced by the Washington postmark—it has also had a corner torn off. Thus though

¹ I use *Intension*, *Connotation*, in the senses distinguished by Dr. Keynes, *Formal Logic*, 4th ed., pp. 23, 24. "The term *intension* has been used to express in the most general way that aspect of general names under which we consider not the objects called by the names but the qualities belonging to those objects."

"We include in the *connotation* of a class-name only those attributes upon which the classification is based, and in the absence of any of which the name would not be regarded as applicable."

this stamp is in a certain sense identical, *i.e.*, numerically identical, with its original self, it is not qualitatively or intensionally 'identical,' it is not *precisely* similar. In the same non-qualitative, most usual and perhaps preferable use of *identical*, we should say that King Edward VII. is the identical person who six years ago was Prince of Wales. So the Koh-i-nor, which now belongs to him, is the identical diamond which a few years ago belonged to Queen Victoria. In these cases it is in virtue of persistence through change of time, it may be with change of intrinsic quality yet always with a certain residuum of unchangingness amid change, that we attribute identity in the sense of continued existence. Thus, if this portrait is genuine, if it is what its owner claims, it is the identical picture which Gainsborough painted of Lady Mulgrave about the year 1785. If it has not changed, so much the better; but even if damaged, it is the 'identical' picture, it is not a copy or a counterfeit.—In each sort of case, we have a kind of unity in difference: (a) qualitative unity (= similarity) in distinct subjects of attributes, (b) the unity of continued being in change of time (and perhaps also of unchangingness in circumstances and relations not arising out of time-change) with or without some change of intrinsic quality.

Can we ever assert 'identity' (exact similarity) of intrinsic quality (= 'precise identity') in anything, combined with identity of being (*continuity of being, numerical identity*), and also without any change of circumstance or relation? It seems to me clear that this could be so only if we could have something which not only exists (or is) without change of intrinsic quality but also exists or is 'timelessly,' and that we could only express our assertion in the form *A is A*. But it would be necessary to inquire more particularly, What can be meant by 'timeless'? Does *timeless* mean (i.) what is not in time, does not last through time (some time or all time), or (ii.) what is not affected by lapse of time? If what is meant is (i.), the burden of explaining what exactly is connoted by 'timeless' being, occurrence, or cognition, and of pointing out to what *things* this timelessness can be attributed, lies upon the person who asserts 'timelessness'.

If (ii.) is meant, then it would appear that *timeless* can at any rate be asserted of any and every truth. In fact if truths were not all unchangeable, any assertion of change would become unmeaning. And in this sense we may too by a convenient and indeed necessary logical device attribute timelessness to Concepts (*i.e.*, to Abstract and to General Ideas), the *raison d'être* of which is to fix and symbolise exact qualitative one-ness. Absolute unvaryingness (precise 'identity') and continuous existence (continuous being, 'numerical' identity) of concepts may however be allowed to be a fiction, an ideal limit to which they approach more nearly, the more nearly they fulfil their indispensable function in knowledge.

(1) If we assume that both Subject and Predicate in a categor-

ical assertion can be taken *in intension only*, difference of quality or content would force us to say of A that it is not-A.

(2) Difference of being (extension or denotation) only, would force us to say that *A is-not A*. (1) and (2) are both impossible forms of assertion, and it seems that we cannot assert difference of intension only or difference of extension only. In the admissible and significant *A is not B*, difference of intension makes possible the assertion of difference (=denial of identity) of denotation.

(3) If there is complete absence of all difference of any sort, we can only say that *A is A* (Greenness is Greenness, Humanity is Humanity, etc.), the whole of A and nothing but A. Thus it appears that significant affirmation of *identity with itself* of any A, the identity of which is exclusive of *all* difference of every sort, is barred, and *A is A* seems a mere form unless we *interpret* it to mean *A is AB*, or *A is B*, both of which are embraced under *S is P*, the ordinary formula of categorical affirmation. In this formula the identity asserted is denotational, the copula has a denotational import, while the difference of symbols has an intensional import. If not, *S is P* would be an impossible form of assertion. For intensionally *S is not-P*, and unless denotationally *S is P*, we should have to say *S is not P*. This account is confirmed by analysis of any categorical affirmation (*cf.* MIND, 1893, p. 441, etc.).

(4) There may be many cases of 'identity' of both denotation and intrinsic quality, in difference of time, circumstance and relation (though we have to remember that so-called 'identity' of quality means only exact resemblance and is quite distinct from identity of denotation, extension, or being). Take such things as a postage stamp, a coin, a stone, a book, a picture. This book (*e.g.*) is the one which Colin gave me for a Christmas present; that sovereign is the one which you paid me yesterday in discharge of your debt; that picture is the one that was bought by a Mr. Campbell at the Price sale for ten thousand guineas. All these assertions are expressed in the form *A is AB*, and we see that while (as before observed) the affirmative copula imports identity of denotation, the difference of symbol (*A, AB*) imports here *not* alteration of the intrinsic quality of the subject (*A*) spoken of, but attributions (*B*) of time, circumstance and relation, all of which (as above remarked) are included in *intension* as differentiated from *connotation* (see note above, p. 384).

Assertions of change or otherness of intrinsic quality in a (denotationally) identical subject are conveyed by propositions of the form *A is B*. *E.g.*, the amethysts which Betty picked up at Aberystwyth have been cut and made into a necklace; the last pair of shoes which I had from Box is nearly worn out. And any assertions in which we affirm of any subject (*A*) a Predicate having an altogether different content (*B*) (whether *B* connotes intrinsic quality, or circumstance, or relation), fall also into the form *A is B*.

We have then the *primâ facie* possible forms:—

- (1) A is not-A,
- (2) A is-not A,
- (3) A is not B,
- (4) A is A,
- (5) A is AB,
- (6) A is B.

From the above it appears that in significant categorical affirmation there *cannot* be complete 'identity' of intension as between Subject and Predicate, while there is complete identity of denotation, and in categorical negation there cannot be difference of denotation only or of intension only. In *S is P*, I affirm *identity of denotation in difference of intension*, in *S is not P*, I deny *identity of denotation in difference of intension*, and as already pointed out the identity necessary to affirmative assertion must be denotational (numerical, etc.) identity.¹

There is one kind of thing which taken strictly cannot enter into relations of thought or reality, and any attempts at affirmation about which fall into form (4), since nothing but itself could be affirmed of it as Predicate—*viz.*, Abstract Ideas in the sense in which Locke rejects them. We could not, *e.g.*, say of Humanity that it is anything but Humanity, taking it honestly in connotation only, (and as it is abstracted from Extension) its whole Intension is comprised in its Connotation. Man is rational, but it is not admissible to say Humanity is Rationality. Hence Locke asserts that all our affirmations are in concrete (*Essay*, bk. iii., ch. viii., § 1). Indeed such terms as Greenness, Rationality, which correspond to ideas of qualities abstracted from subject, time, and place, in which alone they can occur, if taken strictly cannot be either Subjects or Predicates, they are as incapable of entering into relations as Leibniz's *Monads*, and are supposed as immutable.

The fixity of the content of abstract ideas as entertained by any mind is guaranteed (so far as it is guaranteed) by definition of the names applying to the ideas—definition of the sort which sets forth conventional connotation, or (in some cases) extensional definition, *e.g.*, Green is the colour of grass, glaciers, sea-water, emeralds, malachite, etc. This holds of those relatively abstract ideas which are called General as well as of the Lockian Abstract Ideas—of green, man, rational, as well as of greenness, humanity, rationality. In the case of the general ideas, however

¹ Lotze, trying (in his *Logic*, bk. i., ch. ii.) to fit the ordinary categorical form *S is P* to the law of Identity (*A is A*), fails in the most absolute way, because he is trying throughout to work solely and entirely with Abstract Ideas; he is committed to a thorough-going conceptualism. After an elaborate discussion, he arrives in § 54 at the statement that "the impossible judgment *S is P* resolves itself into three others, *S is S*, *P is P*, *S is not P*". *S is P* "taken just as it stands" is "a contradictory and self-destructive form of expression". Quite so, if *S* and *P* are taken in connotation only.

(man, house, etc.), abstraction is made not from all denotation but only from denotation definitely fixed as to time and place, and from all intension except that which is selected and fixed in the definition (= connotation).

In these General Ideas we have a necessary instrument of thought. They fix, for thought, the living transient qualities of real existences. They can, it must be assumed, be applied in a variety of ways without being thereby altered—like the cubes of a child's letter game, or building blocks, or the men on a chess-board, or the cards of a pack, or the stones in a heap, or the letters of the alphabet, or the notes of the musical scale. Their very function and essence is to remain unchanged throughout an endless variety of applications. They are the only things which in so far as they are what they are meant to be, are the same (*i.e.* are unchanged), *semper ubique et pro omnibus*.

In use, in application, we have to restore the existential, denotational, time-and-space aspect, unique in every case, which (as well as much qualitative detail) falls into abeyance when an attribute as occurring in a Subject is abstracted and takes on the fictional provisional form of a Lockian Abstract.¹ And unless there were concretely related things from which general ideas of relations might be derived, it is difficult to see how definite concrete applications of such ideas could be made, why the idea-relations should ever fit actual things. In: This lake is sky-blue, Some rubies are rose-red, The Pacific Ocean is Atlantic-Ocean-colour, the notions of sky-blue-ness, etc., derived from Attributes of definite Subjects, are applied to Attributes of specified Subjects, and the denotation, which has been evanescently in abeyance, is restored. The notions of both quality and relation do not suffer any intrinsic alteration by being thus applied, in fact if they were changed by application it would not be they that were applied.² A general notion (unlike a living actual, continuously existing thing) if changed ceases to be itself, it vanishes and is replaced by something qualitatively different. And its connotation determines its whole being, so intensional change would involve numerical change.

And this characteristic of (relative) immutability (and consequent validity for thought) is none the less real because the origin and history of general ideas is to be sought in developing human experience, and connotational definition is both subject to revision and

¹ Compare, *e.g.*, the way in which quantification of the Subject-name in an A proposition falls into abeyance when the proposition is converted *per accidens*.

² Cf. Mr. F. C. S. Schiller's admission that in the sciences "respect for the sacredness of technical meanings is one of the first obligations inculcated on the student and constitutes a motive which only urgent scientific necessity and due notice avail to set aside" (MIND, October, 1907, p. 605).

has generally been preceded (in the natural growth of thought) by extensional definition; while also general ideas as psycho-physical occurrences probably always occur in close combination with other "mental elements" from which the practised mind is able to make abstraction. The general idea is in fact a kind of selective instrument—like a magnet to iron-filings—the corresponding general name being (as has been said) a sort of handle of the thing named.

Let us consider for a moment here an illustration discussed by Mr. Joachim in *MIND* for July, 1907, p. 414. "The note A," he says, "whether struck on this piano or that, or now or yesterday, is *the same* [=?] note. But it is not 'precisely and numerically the same,' if that means—as I presume it does—*the same to the exclusion of all difference*." It is (Mr. Joachim goes on to explain) not "exactly alike but not identical," nor is it "numerically identical," but it is identical-in-difference [=?].

Such explanation of the point in dispute as is possible is, I believe, to be found by appeal to the distinction already referred to between abstract scientific, logical, conventional connotation (which is set out in definitions), and what may be called 'historical' actuality (the full concrete denotational and intensional entity). If we grant (as I understand Mr. Joachim to affirm) that as logically fixed, limited and defined "the note A" has always precisely 'identical' pitch, then "the note A," as far as connotation is concerned, is under all circumstances precisely 'identical'.¹ If this is so, if it is a certain absolute pitch (abstracted not from denotation but from all other items of intension, from the instrument of production, and every other circumstance whether like or unlike, connected or disconnected, relevant or irrelevant) that is connoted by *the note A*—there is and can be only one note A in the sense in which there can be only one letter A (*a*), one number 2, four conic sections, three species of triangle, one relation between the diameter and circumference of a circle.

Any actually produced (or even imagined) "note A" which as regards the element of pitch is "precisely identical" with any other pure "note A," is yet never, in actual occurrence, in all other respects (timbre, loudness, time, place, etc.), recognised as exactly like any other; such occurrences would be indistinguishable the one from the other—if there *were* two, we could not know it. As it is, while there is by supposition only one pure note A (= a note of a given pitch), every production (repetition, occurrence) of that one note is different in *some* respects from every other, so there need be no difficulty about distinguishing the various produc-

¹ It may however, I suppose, be plausibly maintained that the connotation of the note A refers not to *absolute* pitch but to *position* in the musical scale. A piano, e.g., may be tuned 'flat,' and 'concert pitch' has varied. Further, it may be said that on a piano of seven octaves, there are (7×3) note A's (if we may count sharps and flats as well as naturals).

tions—by voice, harp, piano, violin, etc. Similarly if with one seal I impress the same unique device on a hundred documents, or write the letter *A* or the word *And* all over a page, or the motto *Per aspera ad astra* over and over in various kinds of script, there is still as far as meaning, value, or relevant quality is concerned only the one device, the one letter *A*, the one word *And*, the one motto *Per aspera ad astra*. But the occurrences in writing are many, and each distinguishable from all the rest.¹

In an analogous way the meaning of any significant assertion *A* is *B*, is not altered by any other assertion, nor its truth if it is true² by any assertion whatever nor by the concomitant mood or impressions of the assertor's mind or any other fact. Unless *A* at Edinburgh can deny precisely and numerically *the same* assertion (or truth) which *B* at Oxford affirms (not of course the same speaking or writing of it), there could be no such thing as agreement or disagreement in thought, no argument, no refutation—in fact no such thing as general knowledge common to many minds, or even general knowledge in any one mind. No doubt if we consider the whole concrete case of *B* at Oxford affirming and *A* at Edinburgh denying, it must be admitted that though what the one affirms is what the other denies, yet the 'identity' of the thesis is an 'identity' which has different concomitants in the one case from what it has in the other—there is, therefore (if we choose so to express the condition of affairs), a complex 'identity'-in-difference. The intensional or qualitative 'identity'-in-difference of e.g. an isosceles and a right-angled triangle, requires (as all classification does) the comparison of things numerically different which are partly similar and partly dissimilar.

But this difference in the time and place, etc., at which the 'thesis' is apprehended, has no effect upon its meaning; these relations of time and place are relations of the persons affirming or denying and concern the history of the thesis, but are of a kind which (though not irrelevant to its history) is quite irrelevant to its meaning or to its truth, if it is true. The meaning, whatever it may be, is no more affected by them than the meaning of *A* is at Edinburgh, or *B* is at Oxford is affected by being asserted (1) at Cambridge and (2) at St. Andrews. The differences between *A* and *B*, and Oxford and Edinburgh, are no doubt profound, and of interest and importance for the unique individuals and places concerned, and those who have to do with them, but they are not of consequence for the meaning of any general statement main-

¹ In all that is said above, it is of course not meant to deny that a definition may, from time to time, be altered, or that terms may be ambiguous. In fact it is the main object of these notes to try and clear up to some extent such an ambiguity.

² Might it be said that an assertion is true if it asserts co-existence of qualities which are co-existent in the 'Universe of Discourse' to which reference is made?

tained or denied by A or B, the intrinsic character of which remains unaffected by variations of time and place, and the personality of its assertors. If this were not so, everything, as in Alice's shop in Wonderland, would be perpetually turning into something else the moment one touched it. One could not even juxtapose two assertions in thought without thereby altering both. The "solid ground" would "give way beneath our feet" at every step.

Suppose I assert that *all fronds of the Mountain Buckler fern are erect*. The meaning of the assertion is fixed and definite, and if true, it is true once for all. Quite obviously, my outlook, location, state of health or temper, and so on, cannot affect its truth or meaning in any way. And if I go on to say that the fronds are, also, lance-shaped in form, pinnately divided, that the pinnæ stand opposite (generally), that they are narrow and tapering and are pinnatifidly divided—do any of these affirmations, or all of them, in the least alter or modify the meaning of my original statement that all the fronds are erect? It must be admitted that no one of them alters the meaning of any other; but what is very importantly modified is my knowledge of, and power of 'identifying,' the thing itself, the actual object in time and space, the Subject of Attributes to which all these successive characteristics are assigned. All the successive predicates are related as joint characteristics of the whole which they qualify; they are related not as modifying each other, but as modifying it.

Significant statements (which are always abstract in a sense already explained, a sense which does not exclude concreteness) may be perfectly true as far as they go, though they can never comprise perfect and complete truth about historical reality or any part of it. That the interior angles of an isosceles triangle are equal to two right-angles, is an abstract truth, the meaning of which is not affected by the further abstract truth that the angles at the base of an isosceles triangle are equal to each other, or that any triangle is half of a parallelogram. The meaning of the assertion, This is an engraving of a picture by Gainsborough, is not modified or changed by the further assertion, The picture is a portrait of Lady Mulgrave, it is one of the artist's masterpieces, it is supposed to be now in America.¹ But though any one of these statements does not alter or modify the meaning of the others, each one does modify the hearer's knowledge *concerning the object which is spoken of*. Or take the three propositions: (1) The book I am reading is *A Blind Bird's Nest*; (2) It is a charming story; (3) It is by Mary Findlater, who is a sister of Jane Helen Findlater. The meaning of (2) and (3) does not in any way modify the meaning of (1), but (2) and (3) do add to the knowledge about the book I am reading which was conveyed by (1).

¹ It became known last January (1908) that Gainsborough's portrait of Lady Mulgrave had been since the Price sale in M. Groult's collection in Paris.

Similarly in furnishing a room the carpet, curtains, tables, chairs, pictures, and so on, are *what they are*, but the addition of them *to the room* makes a profound difference to its appearance and comfort. Or suppose again that the walls of my room are papered with a "chrysanthemum pattern" paper,—full of pink, blue, and yellow tints, and that I cause a plain pink frieze to be replaced by a plain blue one but do not change the paper, the paper is still by supposition the same paper—the same in denotation and in intrinsic quality—but the room looks very different indeed. And so with, *e.g.*, the outlines and colouring in a picture—each line and touch is what it is, but the picture, the whole, is modified by each. Every individual, every thing, every part of the universe, is unique, and has inexhaustible qualities and relations, but in as far as it can become an object of knowledge, it must be determined by the attribution of general ideas.

And in the constructions of memory, imagination, and inference, the same holds good—the general ideas are like 'dressed' stones which may be put together without mortar or cement, and can be rearranged for one mental construction after another without suffering any intrinsic change. They are thus 'precisely identical' amid changes of relation and of circumstance which affect *not* the general ideas, *but* the subjects of attributes which they qualify. Similarly with, *e.g.*, the letters of the alphabet, the notes of the musical scale, arithmetical and mathematical symbols (as already observed).

Whatever is in time or space has, it would appear, important relations with all other things in time and space. Among the attributes of anything in the world must be relations of some sort to everything else in the world. Tennyson's apostrophe to the "flower in the crannied wall" seems to present this point of view—completeness and coherence of knowledge is the ideal here suggested to us, to which we may by degrees approximate more nearly, but which we are always indefinitely far from attaining. No one thing (part of the world) can be *completely* known unless everything else (all the rest of the world) is completely known; we cannot quite fully know a part without knowing the whole. But it is only by help of general ideas, common names, fixed connotations, clear definitions, that we can make any steps at all towards the complete fully-systematised historical knowledge which is our goal; we cannot know the whole without knowing the parts. We must begin from contact with this part or that of the vivid but bewildering fulness of reality, and the most extreme and (seemingly) artificial exercise of abstracting thought has, as its one purpose, the bringing of that rich reality into orderly and intelligible connexion.

It is interesting to observe that certain categories of connexion, *e.g.*, classing and classification, may be applied to concrete things without in any way altering the existence and intrinsic attributes

of those things (though with most potent effects from the point of view of thought and its connexions)—no interaction between the actual things, no change in them, no joining and no separating, is produced by ranking them as like and unlike—in fact, if it were so, the relation would be self-destructive. But the relation between (1) thought and knowledge and (2) the things classified is profoundly modified.—A South African lion is entirely unaffected by being classed with his Asiatic namesake and distinguished from the Cape ‘tiger’ (panther), though to bring about contiguity in space between two such co-existing individuals, is to introduce historical relations of vital importance for the creatures themselves.

A steady recognition of the difference between the two fundamentally important uses of Identity, Identical above discussed (which might perhaps be provisionally distinguished as (1) Historical, (2) Abstract), might perhaps do something towards clearing up past controversies and preventing misunderstandings. The dispute in recent numbers of *MIND*, for instance, on the nature of truth, might have light shed upon it if it were recognised that (as I suppose) it is the historical identical in the most extended sense, the whole to which qualifying attributions are made, on the predominant interest and importance of which Mr. Joachim is concerned to insist, while it is an abstract ‘identical’ that is the ideal which Mr. Russell and Mr. Moore seem to have in view—the fixed meaning of immutable abstract ‘entities’.

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