8. On the Harmonic Polar of a Conic

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oscillation of the body about a parallel axis through O is the same as that about the given axis, for in each case AO is the length of the equivalent simple pendulum.

Line of percussion.—If the body, initially at rest, be struck by a blow P along a line through O perpendicular to AO and in the plane of oscillation, there will be no shock felt at A, that is, the whole of the impulse will be spent in producing motion. For, taking moments about A, we have (§ 93) $P(a+b) = M(k^2 + a^2)\omega = M(ab + a^2)\omega$, therefore $P = Ma\omega$, which is the momentum of the body supposed collected at G.

If an impulse had been applied in any other manner there would have been an induced impulse on the axis such that the resultant impulse would be in magnitude and line of action equal to the above given impulse P.

Hence the line of action of P is called the *line of percussion*. The mass of the equivalent simple pendulum which would be equally moved by the impulse P is $\frac{Ma}{a+b}$.

For, call its mass M_0 , then $P = M_0(a+b)\omega$, therefore

$$Ma\omega = M_0(a+b)\omega, \therefore M_0 = \frac{Ma}{a+b}.$$

Similarly, if O were the centre of suspension, the mass of the equivalent simple pendulum at A is $\frac{Mb}{a+b}$.

Therefore we have the following important theorem:—The compound pendulum moves exactly as if it were condensed into two heavy particles rigidly connected by a massless framework, one particle at A (of mass $\frac{Mb}{a+b}$) and the other at O (of mass $\frac{Ma}{a+b}$), the two together being equal to M, and having G for their centre of gravity.

The controlling equation between a and b is $ab = k^2$.

If any blow be struck along any line in the plane of oscillation, its effect can be graphically resolved (as in Fig. 5) into two components, one along the line of percussion producing motion, and the other having its line of action through A, and causing an impulse on the axis.

Let the line of action of the blow be from C to B, meeting the line of percussion in B, then the lines of action of the components acting at A and O are AB and OB. If the triangle of impulses be completed, as ABC, the *magnitudes* of the components can be measured. A. LODGE.

If the line of action of the blow is parallel to the line of percussion, the components at A and O can be obtained by the method of moments. A. LODGE.

8. On the harmonic polar of a conic.

Let O be the fixed point, TL the polar of O, Q'OQL any chord through O. Draw the diameter COR. Also draw the tangent RT' (which is parallel to TL), and RW an ordinate of CP, the diameter which bisects QQ'.

Then from similar Δ^s LVT, RWT',

$$VL : VT = RW : WT'$$

$$\begin{aligned} \text{and} \quad & VO : CV = RW : CW \text{ from CVO, CWR,} \\ \therefore & VO \cdot VL : CV \cdot VT = RW^2 : CW \cdot WT', \\ \therefore & VO \cdot VL : PV \cdot Vp = CD^2 : CP^2 \\ & = QV^2 : PV \cdot Vp, \\ \therefore & VO \cdot VL = QV^2, \end{aligned}$$

and since V is the mid-point of QQ', therefore Q'OQL is an harmonic range. J. J. MILNE.

9. On a construction in the *Aperçu Historique*.

Many readers will have admired Chasles' elegant construction for the axes of an ellipse of which two conjugate diameters are completely given. However, the method of discovery is still more elegant and highly instructive. It is a matter of some surprise that it does not appear in didactic treatises; but Chasles was perhaps not at his best in explaining it. It appears in the notes to the *Aperçu Historique* (p. 359), and the author's object is to show that geometry need not be abandoned because some points of a figure may, under changed circumstances, become imaginary.

If CP, CD be conjugate radii of a hyperbola (and its conjugate), and if through P a parallel be drawn to CD, and along it PL, PL' be taken each equal to CD, then we know that CL, CL' are the asymptotes, and that the axes are the bisectors of the angle LCL'. Now the radius of the original hyperbola is not CD but $CD\sqrt{-1}$. If then in the case of the ellipse we take $PL_1 = PL'_1 = CD\sqrt{-1}$, CL_1, CL'_1 will be the imaginary asymptotes of the ellipse (I use the subscript ₁ to denote imaginary points; M. Gaston Tarry, I think, suggests the use of coloured letters). But how to construct the bisectors of the angle L_1, CL'_1 ? A *coup de maître* solves the difficulty. The bisectors are the tangent and normal to a conic having L_1, L'_1 for foci and passing through C: but this is a *real* conic, whose real foci are along a perpendicular through P to PL_1 (or CD) at distances from P equal to CD. Hence, for the ellipse, take PK, PK' perpendicular to CD (i.e. along normal at P) and equal to CD; the bisectors of the angle KCK' give the directions of the axes. No doubt Chasles was led to this construction by the then prevailing hazy impression of the geometrical interpretation of $\sqrt{-1}$.

R. W. GENESE.

10. On pairs of imaginary roots.

Elementary proof that if, in an equation

$$f(x) \equiv \dots + p_{r+1}x^{r+1} + p_r x^r + p_{r-1}x^{r-1} + \dots = 0,$$

p_r^2 is less than $p_{r-1}p_{r+1}$, the equation must have a pair of imaginary roots.

Whatever A and B may be, the equation

$$Ax(p_r x - p_{r-1})f(x) + B(p_{r+1}x - p_r)f(x) = 0$$

will be free from the term involving x^{r+1} ; and if A and B be properly chosen, it will also be free from the term involving x^r . The new equation, having two consecutive terms wanting, must have at least two imaginary roots. But its roots will be the same as those of the old equation, together with those of the quadratic

$$Ax(p_r x - p_{r-1}) + B(p_{r+1}x - p_r) = 0.$$