ART. XXXVI.—*Rotatory Polarization Mechanically Produced*; by ARTHUR W. EWELL.

SOME four years ago Prof. A. W. Wright of Yale University suggested to the author that he investigate the optical effects of torsion upon transparent media, citing a reference of Verdet* to the unsuccessful attempts of F. E. Neumann and Drion to obtain rotatory polarization by twisting. The results of the author's investigation were given in a paper † published a year later in which it was demonstrated that jelly in a rubber tube, when subjected to severe twist, rotates in the opposite direction to the twist, the plane of polarization of polarized light which traverses the jelly parallel to the axis of twist, and that this rotation is a higher power of the twist than the first. In a paper ‡ published a year later it was shown that this rotatory polarization was quite uniform over the cross section of a jelly tube; that it was present in an annular jelly tube formed by pouring the jelly into the space between two rubber tubes both of which were cemented to glass caps; that a glass rod twisted when heated to softness showed no rotatory polarization when allowed to cool; and, finally, that glycerine, stirred or twisted in rubber tubes, even at zero degrees showed no rotatory polarization.

The present paper describes experiments performed during the past two years, in seeking definite quantitative relations between the rotation of the plane of polarization and the twist and twisting moment and what conditions besides the twist influence the rotation. A great number of experiments were made with several forms of apparatus which afterwards proved to be merely preliminary. Only the final form of apparatus and the experiments performed with it will be described.

MATERIAL USED.

Jelly.—The great majority of the experiments were made with jelly, which is by far the best material which the author has discovered, since it may be made very transparent, possesses in a marked degree all the features of an elastic solid and allows great distortion without serious injury to its optical or mechanical properties.

In the experiments described in former papers the jelly was composed of choice calves foot gelatine and water. The jelly used in the experiments described in this paper contained in addition glycerine, which makes the jelly tougher and constant,

* Verdet: *Optique* ii, p. 389.

† This Journal (3), viii, 89, 1899; also abstract in *Phys. Zeitschrift*, i, No. 18.

‡ Johns Hopkins University Circulars, June, 1900.

neither drying up nor putrefying. The proportion most suitable for the majority of the experiments was 1^g gelatine, 5^{cc} water, 5^{cc} glycerine in warm weather, and in winter a little more glycerine and water. A greater proportion of glycerine to water makes a slimy jelly, less of the nature of a rigid, hard body. The stiffest jelly which is still sufficiently transparent consists of 1^g gelatine, 2½^{cc} water, 2½^{cc} glycerine. The mixture of gelatine, water and glycerine was heated at about 100° C. until the gelatine was completely dissolved, which usually required nearly an hour.

Jelly Tubes.—Most of the experiments described below were performed with jelly in rubber tubes. A different form of tube was used from that of the earlier papers. A proper length of proper sized *pure* rubber tubing was chosen and a brass tube about 3^{cm} long, the external diameter of which was slightly larger than the internal diameter of the rubber tube, was slipped into each end of the rubber tube for about 1^{cm}, a circular glass disk slightly larger than the brass tube being pushed in ahead of the brass tube. The tube was then completely filled with the melted jelly through a slit in the side. With patient manipulation air bubbles could be avoided. The glycerine jelly was so tough that it was not necessary to close the slit with wax. The rubber tube was bound upon the brass tube with wire; unless otherwise stated, the internal diameter of the rubber tube was 1.14^{cm} and the external diameter of the brass tube 1.42^{cm}. This style of jelly tube, which will henceforth be designated as a "jelly tube," is greatly superior to the former style where square glass plates were directly cemented to the rubber tube, in that the brass tube could be grasped with far less distortion of the tube and afforded far more opportunity for adjustment than the glass plates.

Independent Jelly Cylinders.—These were constructed by pouring melted jelly into a test tube, allowing the jelly to set, then plunging the test tube into hot water which melted the jelly next the glass, allowing the cylindrical core to slip out. A suitable length was cut off with a sharp knife; short lengths of slightly smaller rubber tubing (internal diameter 1.76^{cm}) were slipped over the ends for about 7^{mm}. A glass disk was placed against each end inside the rubber tubing and a brass tube slightly larger than the rubber tubing (external diameter 1.90) was then pushed into the rubber tube up to the glass disk. The brass tubes were grasped in the clamps by which the twist was applied. These independent jelly cylinders were similar to what the jelly tubes would be with the central part of the rubber tube cut away. The jelly cylinder must be of exactly the popular diameter. If too large the rubber tubing is quite likely to cut it and if too small it is likely to slip in the rubber tubes.

APPARATUS.

The general apparatus in its final form is illustrated in outline in fig. 1. It is a biquartz polarimeter, without lenses and

with certain attachments for this particular work. The light passes from the frosted incandescent lamp, *a*, through the first Nicol, *c*, the biquartz, *d*, the jelly tube, *z*, and the second Nicol,

p , where it enters the observer's eye. The upper Nicol, p , is mounted in the tube, q , which rotates freely within the tube, w , attached to the gear wheel, o , and a graduated circle, t . The position of this Nicol is read with a pointer, v . The lower Nicol, c , and the biquartz, d , are fixed in a tube which is attached to the gear wheel, b . The upper gear wheel, o , and graduated circle, t , can be rotated in step with the lower gear wheel, b , the lower Nicol, c , and biquartz, d , by means of the rod, u , with its small attached gears. Thus the azimuth of polarization of the entering light, with reference to any particular radius of the jelly tube, may be varied without changing the relative position of the lower Nicol and the graduated circle of the analyzer. The circle, t , has a second graduation along the outer edge and the azimuth is read by means of the index, s . z represents a jelly tube of the form described above, m is the brass tube, e the glass disk and k the binding wire. The upper brass tube is clamped in the fixed clamp, n (the position of this clamp can be varied to suit different lengths of tube). The lower brass tube is fastened in the movable clamp, j . A graduated circle, v , is attached to the bottom of this clamp and the amount of twist applied to the tube is read with the index, h . The twist was generally produced by pulling the cords, x , by weights, or by hand, and then clamping the cords at A. The pulleys, y , had cone-bearings and turned with no appreciable friction. Their position could readily be varied. The lower clamp could be clamped in position by the clips, g , to the platform, e , which could be raised or lowered by the screws, f . i represents the cross section of a ring soldered to the platform, e , which fitted loosely within the lower clamp and served to keep it from being drawn too far to one side during adjustments.

Signs.—The right-handed screw notation is adopted.* If, advancing in any direction, the direction of rotation of the plane of polarization or the direction of twist of the body be related to the direction of advance as the direction of rotation of a right-handed screw is related to its advance, the rotation or twist is said to be positive while the opposite twist or rotation is called negative. The graduated circles being numbered in the usual direction, an increase in the reading means a negative rotation or twist. Twists and rotations are throughout expressed in degrees. Degrees are given to the nearest whole number. The accuracy does not warrant stating fractions, even in averages.

Double Refraction.—A double refracting body may rotate the plane of polarization of linear polarized light. Double

* Maxwell, Elec. and Mag., 3d ed., i, p. 25.

refraction and rotatory polarization may, however, be distinguished if the incident light is polarized in a number of azimuths, for simple double refraction will rotate the plane of polarization in one direction in some azimuths and in the other direction in other azimuths, for it has nothing in its nature to distinguish the two directions of rotation. In these experiments the jelly was always under some strain and, therefore, showed some double refraction. To eliminate the effect of the double refraction, which increased the proper rotation in some azimuths and decreased it in others, and also to obtain greater accuracy, readings were always made, by means of the above mentioned gearing, in eight symmetrically distributed azimuths, and *every position of the analyzer or rotation of the plane of polarization, given below, is the mean of eight such readings.*

Natural Rotation of the Jelly.—The natural rotation of the jelly (usually about $2^{\circ}.7$ per cm.) has been subtracted from the observed rotation in every case and the rotation stated is the difference due to the mechanical strain.

To obtain constant definite results it was necessary to determine what other conditions besides the twist influence the amount of the rotation. These conditions will now be considered in their approximate order of importance.

CONDITIONS INFLUENCING AMOUNT OF ROTATION.

Lateral Envelope.—Aside from qualitative experiments with glass, the only successful experiments described in previous papers were with jelly in rubber tubes. It was very important to determine whether independent cylinders gave this rotation when twisted. I will give in full the last set of readings with independent jelly cylinders which were made under what previous experience had shown to be the most suitable conditions. The lower clamp was fixed by the clips, *g*, after the twist was applied. As is shown later, it would have been better to have had it hanging free, but these independent cylinders could not support this weight under twist sufficient time for reading the rotation.

Independent jelly cylinders.

July 12, '02. Initial position of the analyzer for uniform color of the two halves of the biquartz = 94.

Length (between rubber clamps) = 5.1 cm . Diam. = 2.1 .

1' delay after applying twist before making readings to avoid relaxation disturbances (see below).

Twist	0	120	-120	120	-120	120	-150	150
	78	88	91	92	83	74	78	85
	79	86	69	91	63	89	69	86
Readings in	82	97	79	97	83	95	76	90
the 8 azimuths	83	89	76	86	75	98	79	92
	86	89	81	88	90	89	81	87
	82	90	70	94	70	89	79	85
	79	98	71	95	82	91	73	86
	78	91	78	92	72	96	73	83
Mean position	—	—	—	—	—	—	—	—
of analyzer	81	91	76	92	78	91	76	87
Rotation due								
to twist	-10		5	-11	3	-10	5	-6
Twist	-150	150	-120	120	-150	150		
	82	87	94	86	77	85		
	79	91	70	89	75	87		
	77	91	77	93	79	90		
	82	83	79	93	79	86		
	83	84	95	85	77	85		
	75	88	69	89	83	90		
	75	86	87	85	76	87		
	74	89	76	94	74	85		
Mean position of								
analyzer	78	87	81	89	78	87		
Rotation	3	-6	0	-8	3	-6		

The irregularity is not surprising when one considers the strains which are unavoidable in these independent cylinders and the dependence of the rotation upon certain of these strains, as shown later. These and similar observations, some of which are cited below, show qualitatively, that *simple jelly cylinders, without any envelope, acquire when twisted rotatory polarization in the opposite direction to the twist.* It might be suggested that possibly the rotation is due to a twist of the jelly within the rubber tubing. This is scarcely possible, for the rubber is so much more rigid than the jelly that the tubing which tightly grasps the jelly hardly twists at all.

A lateral envelope greatly increases the rotation for a given twist.

Dec. 9, '01. Independent jelly cylinder, length = 8.0, diam. = 2.0.

Twist	-170	190
Rotation	1	-1

Soft rubber tube slipped over jelly (internal diam. = 1.76).

Twist	90	-90
Rotation	-6	9

Dec. 28. Independent jelly cylinder. Length = 7.0 diam. = 1.8.

Twist	210	-220
Rotation	-10	11

Rubber tube slipped over:

Twist	-240	130
Rotation	30	-13

The rotation for a given twist is increased by increasing the envelope.

July 21, '02. Jelly tube. Length (of jelly in tube) = 3.0. Lower clamp free (i. e., suspended from jelly tube and not resting on the platform *e*).

Twist	-120	120
Rotation	8	-4

Wrapped tightly with thin rubber:

Twist	-120	120
Rotation	21	-30

Thin rubber wrapping replaced by a second rubber tube slipped over first rubber tube:

Twist	120	-120
Rotation	-26	11

Longitudinal Compression and Elongation.—It was early noticed that longitudinal compression increased the rotation of the plane of polarization for a given twist, while longitudinal elongation decreased it. Measurements of this effect with the final form of apparatus are given below. The lower clamp was twisted through an angle, read with the graduated circle, *v*, and pointer, *h* (see fig. 1) and clamped to the platform *e* by the clips *g*. The compression or elongation was applied by raising or lowering this platform by the screws *f*. The length of the jelly tube was taken as the distance between two pin points, pricked into the rubber tubing outside the glass disks *l*. Decrease in length we will take as positive and increase as negative.

I. Aug. 2, '02. Jelly tube C 5. Original length = 3.33. Twist of -90.

Change in length

(m.m.)	0	.5	1.2	0	-.5	-1.7	-3.9	-4.5	-2.7	0
Rotation	19	26	33	19	16	8	2	1	3	18

II. Aug. 4. Same tube. Twist = 90. (All rotations negative.)

Change in length..	0	.9	2	-.8	-1.7	-2	-4.9	-.1
Rotation	18	27	43	16	11	5	4	21

III. Aug. 5. Same tube. Twist = 80. (All rotations negative.)

Change in length	0	2	3.1	4	—1	—1	—4	—3
Rotation	4	9	22	4	2	1	1	2.

I. Aug. 5. Jelly tube D 1. Original length = 3.70. Twist = 120. (All rotations negative.)

Change in length	0	1.6	3	2.4	1.	—3	—1.2	—2.8	—5	—1
Rotation	9	25	118	75	36	18	10	5	2	9

II. Aug. 6. Same tube. Twist = — 90.

Change in length	0	1.4	2.5	1.2	.5	—1.6	—3.5	—5.4	—2.4
Rotation	14	35	74	33	14	6	5	1	6

The results for tube C 5 are plotted in fig. 2, and for D 5 in fig. 3, the abscissa being change of length, x , and the ordinate, rotation, y .

Apparently, when $x = \infty$, $y = \infty$; when $x = -\infty$, $y = 0$. This suggests an exponential curve of the general form

$$y = c_1 c_2^x \quad \text{or} \\ \log y = A + Bx$$

where c_1 and c_2 are empirical constants. In fig. 3a the ordinates are logarithms of the rotation and the abscissæ are changes of length as before. The closeness of these curves to straight lines indicates that the curves in figs. 2 and 3 are very approximately exponential curves.

If we let x' represent the actual length, i. e., in figs. 2 and 3, $x + a$ constant; when $x' = \infty$ $y = 0$, when $x' = 0$ $y = \infty$, which is satisfied by an equation of the form

$$y = k \frac{1}{x}$$

introducing a few values for x and y from any of the curves shows such an equation to be entirely unsatisfactory.

The above observations demonstrate that *the rotatory polarization of a jelly tube, subjected to a given twist, increases with longitudinal compression and decreases with longitudinal elongation, the change closely following an exponential law.*

Neglect of this condition was the chief cause of the great irregularities in the early work. When the lower clamp rested on the platform, e , it made an appreciable difference in the rotation which end of the jelly tube was first clamped, it being clamped in the one case when compressed and in the other when stretched by its own weight. The same tube was quite sure to give different results when taken out and replaced, for it could never be replaced under exactly the same strain. The first attempt to avoid this difficulty was to keep the measured length constant with clamps, but this gave little improvement.

In the final, vertical form of apparatus the lower clamp could be freely suspended from the jelly tube, simply guided by the ring *i*, and the twist was applied by the cords *x*, great care being taken to have them horizontal.

The following readings show that compression and elongation have no appreciable effect on the natural rotation.

Aug. 5.

Tube C 5.

Change of length	----	0	1.7	3.6	0	-1.1	-3.3
Position of analyzer	--	86	87	87	86	87	86

Tube D 1.

Change in length	0	1.1	2.1	-1.5	-2.9	-5
Position of analyzer ..	85	85	85	86	85	35

Independent cylinders would not endure elongation or compression under large static twist sufficient time to determine the rotation.

Hydrostatic Pressure.—The lateral envelope and longitudinal stress having so great an influence, it was of interest to find the effect of hydrostatic pressure. A jelly tube was clamped under suitable twist by screws passing through an outer, larger, iron tube. This tube with the jelly tube inside was placed inside a still larger iron tube with carefully packed glass windows at the ends. Hydrostatic pressure was applied with an hydraulic pump provided with a pressure gauge.

July 23d. Tube C 3. The pressure is expressed in pounds per square inch.

Pressure	0	80	80
Rotation	32	30	31

July 24th.

Tube C 4.

Pressure	0	120	0	160	0
Rotation	50	50	50	51	52

The above are typical examples of a series of observations which demonstrated that up to 160 lbs. per sq. in., *hydrostatic pressure does not affect the rotation.*

*Effect of Previous Strains.**—Relaxation is very conspicuous. If a jelly tube be subjected to prolonged twist the rotation decreases owing to the strain upon the jelly relaxing. For example :

June 25.

Tube F 1.

Twist of -100 applied, rotation immediately after =	21
“ “ 2 min. “ =	18
“ “ 2 hrs. “ =	14

July 5.

Tube F 2.

Twist of 120 applied, rotation immediately after =	-21
“ “ 3 min. “ =	-17
“ “ 10 “ “ =	-17

If the rotation is read for a given twist and then a greater twist is applied in the same direction, on returning to the first twist the rotation has decreased owing to the increased relaxa-

* In the four species of relaxation effects the twist is counted from the original reading for zero twist. The variation in the rotation is of course due to change in the zero making the actual twist different from the stated apparent twist.

tion of the strain under the greater twist so that the actual final twist is less than the original. Similarly after a prolonged twist in the opposite direction the rotation for a given apparent twist increases, the actual twist being greater than before. Simply one of a great number of examples will be given.

June 24.

Tube F 2.

Rotation for twist of	120	after prolonged twist of	0	= - 16
"	"	"	"	150 = - 14
"	"	"	"	- 160 = - 18

If a twist has been maintained for a long time, on returning the tube to its original position for zero twist, an opposite rotation is sometimes observed owing to the jelly having relaxed so much that with the lower clamp in its original position for zero twist, there now is a considerable twist in the opposite direction. For example

June 24.

Tube F 2.

Original position of analyzer for no twist = 86. After standing two hours under a twist of -100 which gave a rotation of 13, on apparently removing this twist the position of the analyzer was 87.5, showing a small negative rotation.

To eliminate these relaxation disturbances, (1) a uniform short time (from $\frac{1}{2}$ to 2' depending upon the tube) should elapse between the application of the twist and the reading of the rotation, that a practically steady state may be reached; (2) the lower clamp should be free, i. e., suspended from the jelly tube as described above, and the twist maintained by the cords, that the position finally taken by the freely hanging lower clamp may be observed before and after each twist is applied.* It is necessary to wait a few minutes in determining the zero, for the approach to a position of equilibrium is asymptotic.

July 11.

Tube C 4.

Initial position of lower clamp for no twist = 67.

The first row gives the times after a weight of 180^g was applied to each cord, the second row the corresponding position of the lower clamp.

	30"	60"	90"	120"
	287	285	284	283
Weight removed				
	30"	60"	90"	120"
	61	63	64	64

* This method of determining the true zero, i. e., the position of the lower clamp for no twist, assumes that the rubber tube relaxes the same amount for a given twist as the jelly inside. While making observations described later upon the rigidity of independent cylinders and of rubber tubing, the relaxation of this rubber tubing for a given twist was found to be approximately the same as that of jelly for the same twist.

This asymptotic approach to equilibrium is similar to that observed with metals, only more conspicuous.

Relaxation disturbances can be avoided as described above. There is, however, another effect of the previous history. *A prolonged preceding twist increases the rotation for an immediately following less twist in either direction* but has little influence on a greater twist.

(Relaxation effects eliminated, observations in chronological order.)

July 16.

Tube C 3.

Twist	90	120	100	120	90	120
Rotation	-9	-25	for 10'	-29	-12	-27
Twist (cont.)	90	-100	120	90	90	
Rotation "	-11	for 10'	-26	-10	-7	

July 17.

Tube C₃.

Twist....	90	120	-100	90	120	100	90	120	90
Rotation .	-7	-24	for 15'	-11	-22	for 16'	-11	-24	-7
Twist (cont.)	120	100	120	90	-100	120	90	90	120
Rotation "	-24	for 18'	-27	-10	for 21'	-24	-9	-9	-26

This effect is not due to a simple disturbance of the jelly; a prolonged strain is necessary; alternate twists of short duration have little effect.

July 16.

Tube C₃.

Twist	90	16 alternate twists	90	90
Rotation	-9	of ± 120 in rapid succession	-9	-9

July 17.

Tube C₃.

Twist ...	90	120	16 alternate twists	120	90
Rotation	-9	-26	of ± 100 in rapid succession	-26	-8

As illustrated above, this effect of a previous twist rapidly dies away. Therefore to avoid this disturbance small twists should precede greater and in general some time should elapse between observations.

If we make alternate twists in succession, counting the twist from a constant zero, we notice both effects of previous history. A typical case is the following:

June 26.

Tube F 1.

Twist	100	-100	100	-100	100
Rotation	-19	10	-25	11	-28
Twist (cont.)...	-100	100	-100	100	-100
Rotation "	--	12	-28	13	-28

Theoretically the relaxation should make the odd rotations, -19 , -25 , etc., increase, while the even, 10 , 11 , etc., should slightly decrease. The increase of the latter is due to the other effect of a previous twist, which also causes the odd rotations to increase faster than they would by simple relaxation.

Under *prolonged stress* the rotation increases in contrast to the decrease with prolonged strain.

July 21. Tube C₄. Weight of -180° applied to cords at 10·18.

Time.....	10·20	10·30	10·35	10·37	10·45
Rotation....	11	14	+130° applied	-7	-13

July 22. Tube F 1. 180° applied at 11·13.

Time....	11·15	·20	·25	·30	·35	·50	12·08
Rotation	-25	-28	-33	-35	-38	-38	-180° applied
Time (cont.)..	12·08	·11	·21	·37			
Rotation “ ..	34	39	52	53			

The stress being constant, the actual strain is constant but the prolonged state of strain increases the rotation.

Influence of Temperature.—As would naturally be expected, the *rotatory polarization decreases with rise of temperature*, owing to decrease of strain with increased relaxation and a more fluid condition of the jelly. Throughout this work it has been noticed that the softer and more fluid the jelly the less the rotation.

Aug. 6. Tube E 1. Twist of 90° .

Temperature	23°	36	37
Rotation	-31	-25	-10

Tube E 6. Twist of -90° .

Temperature ..	23°	38	38	23	Repeating	23
Rotation	26	15	3	4	Twist	21

The jelly tube was loosely surrounded by an electric heating coil. A thermometer between the jelly tube and the coil gave the temperature.

From the above we should expect that with a given jelly tube, the lower clamp being freely suspended from it and the cords which apply the twist being horizontal, the position of the lower clamp for zero twist being read before and after each twist, sufficient time being allowed for the jelly tube to approximately reach zero twist, a uniform, short time being allowed after the twist is applied before the rotation is read, and a considerable time intervening between a large twist and a succeeding small twist, the temperature being approximately

constant—under these conditions we should expect the rotation for a given twist to be constant and definite. Such is approximately the case, as is illustrated by the following observations, where these precautions have been carefully observed :

RELATION BETWEEN TWIST AND ROTATION.

July 21, A. M. Tube D 3. Length of jelly = 3.0cm . 1' delay after twist was applied and commencement of readings of rotation.

Twist	-90	90	-110	110	-120	120	-130	130
Rotation..	1	-4	2	-13	2	-18	2	-26
Sum		5		15		20		28

July 21, P. M. After tube has been removed from the instrument, turned end for end and replaced.

Twist ..	-60	60	90	-90	110	-110	120	-120	130	-130
Rotation	-1	1	-6	0	-13	0	-21	1	-26	1
Sum ...		2		6		13		22		27

July 22, A. M. After tube has been taken out, turned end for end, and replaced.

Twist ..	-60	60	-90	90	-110	110	-120	120	-130	130
Rotation	0	0	2	-4	2	-11	3	-17	4	-22
Sum		0		6		13		20		26

On account of change of zero due to relaxation the actual twists of the jelly, when the rotation was read, were 56, 83, 101, 109, and 118 respectively.

July 22. Tube D 4. Length of jelly = 3cm . 2' delay between twist and reading of rotation.

Twist	60	-60	90	-90
Rotation....	-8	2	-39	9
Sum.....		10		48

The actual twists were 56 and 83.

July 22. Tube F 1.* Length of jelly = 4cm . 1' delay between twist and reading.

Twist	-60	60	-90	90	-110	110
Rotation..	1	-1	3	-1	6	-3
Sum		2		4		9
Twist (cont.)	-120	120	-135	135	-150	150
Rotation "	8	-4	11	-6	19	-15
Sum "		12		17		34

* These readings differ widely from those given for this tube on previous pages. The explanation is that in the earlier observations the lower clamp rested on the platform *e* and the jelly tube was longitudinally compressed relatively to what it was here when stretched by the freely hanging lower clamp.

The actual twists were 57, 85, 104, 112, 125, 139 respectively.

Aug. 9. Tube 11. Length of jelly = 3^{cm}. $\frac{1}{2}$ ' delay between twist and reading.

Twist	----	-60	60	-90	90	-110	110	-130	130
Rotation	-	0	0	1	-1	1	-2	4	-5
Sum	-----		0		2		3		9
Twist (cont.)	-----	-150	150	-170	170	-190	190		
Rotation "	-----		4	-13	6	-29	16		-57
Sum	"	---		17		35		73	

The actual twists were 57, 86, 104, 123, 142, 160, 178.

To illustrate the accuracy of the work, the observations upon tube (43) will be given in full.

Aug. 11. Tube 43. Length of jelly = 3^{cm}. $\frac{1}{2}$ ' delay between twist and reading.

Apparent twist	0	90	-90	120	-120	150	-150	170	-170
Corrected twist	0	84	-84	112	-112	140	-140	158	-158
	79	83	73	85	68	90	58	100	50
	81	81	77	86	71	97	61	101	50
	82	85	79	87	74	89	63	94	53
	83	84	81	83	75	90	65	98	56
	82	82	77	84	69	92	63	95	53
	81	83	79	87	73	94	60	100	54
	80	81	73	85	68	93	61	100	54
	80	81	74	81	69	92	60	103	50
Means	-----	81	83	77	85	71	92	61	100
Rotation	-----		-2	4	-4	10	-11	20	-19
Sum	-----			6		14		31	
								48	

The natural rotation of all the above tubes was about 2·7 per cm.

Dissymmetry in Rotations.—The dissymmetry in the two directions of rotation for apparently the same numerical twist has been very conspicuous. This is partly due to uncertainty of the true position of the lower clamp for zero twist. A slight error will make the actual twist applied in one direction differ from that applied in the other direction by twice this error, and the difference in the rotations will be still greater. The principal cause is probably dissymmetry in the rubber tube. In most of the cases where the dissymmetry was marked the moment was measured which was required to produce a given twist, and in every case it was found that a greater moment was required for a twist in the direction which gave the greater rotation, i. e., for a given twist more force was

applied to the jelly tube, probably in the form of greater constraint upon the jelly by the rubber tube. From the previous results upon the effect of an envelope, the greater rotation would be expected.

For example :

July 21.

Tube D 3.

Negative rotations greater.

80°	pull on cords	gave a twist of	64
-80°	"	"	-65
200°	"	"	178
-200°	"	"	-183

Here we see greater rigidity for positive twists, which produce negative rotations.

Tube D 4.

80°	gave	47	} giving a similar result.
-80°	"	-49	
150°	"	91	
-150°	"	-96	

The dissymmetry is not due to the natural rotation, as the excess is as likely to be for one direction as for the other.

As neither rotation is more important, we will study the average, or sum of the two rotations. The observations with tube D 3 are plotted in fig. 4. The observations upon tubes D 4 and F 1 are given in fig. 5 and for tubes 11 and 43 in fig. 6. The abscissæ are twists and the ordinates rotations (sums).

The curves resemble exponential curves of the general form $y=cx^n$ or $\log y=\log c+n \log x$. To determine whether they are such, and if so, of what power, I have plotted in fig. 6a the logarithms of the twists and rotations. All of these curves approximate to straight lines, at least for the greater twists, and the inclination of these straight lines is very close to four, giving the very interesting result that, when other conditions are kept constant, *the rotation is closely proportional to the fourth power of twist.*

The curves of the sums of two rotations will have the nature of each one separately, for if the law of one be $y_1=k_1x^n$ and of the other $y_2=k_2x^n$, the equation of the sum will be the same power of the twist, or, $y_1+y_2=(k_1+k_2)x^n$.

The law could not be tested with independent cylinders since they would not sustain the weight of the lower clamp under twist long enough to determine the rotation.

Plane Parallel Light.—Since no lenses are used, the light passing through the jelly will not be perfectly plane parallel. To make sure that the phenomena are not caused or influenced by this fact (for instance, through reflections from the interior of the rubber tubing, which however is quite dark), a jelly tube was clamped under twist and examined with strictly plane

parallel light from an arc light lantern, which passed in succession through polarizer, biquartz, jelly tube and analyzer.

Twist.

The position of the analyzer for the untwisted jelly was 35. When the jelly was given a certain unmeasured positive twist

Twist.

the position of the analyzer was 59 showing a rotation of -24 . When the jelly had a certain negative twist the analyzer read 10 showing a rotation of 24 .

Glass.—A glass rod was secured from the John A. Brashear

Co., Ltd. of Allegheny, Pa., 11.9^{cm} long, 0.506^{cm} in diameter, with carefully polished ends, perfectly transparent and show-

ing no trace of strain when examined between crossed Nicols. Circular wood blocks were carefully fitted and glued around the ends. These were fixed in the clamps of the apparatus

described above. The amount of twist was measured by means of a mirror attached to the lower wood block and a distant telescope and horizontal scale. Successive twists in both directions were applied up to a twist of $3^{\circ}1$ produced by weights of 2030^g applied by the cords to the opposite sides of the lower clamp of 4.58^{cm} diameter.* On applying this twist in the opposite direction the glass rod broke. No evidence of rotatory polarization was observed. Observations reported in a previous paper† indicated that twist produces rotatory polarization in glass, but glass is obviously a very unsuitable material, since we have seen above that at least in jelly tubes the rotation is closely proportional to the fourth power of the twist and therefore appreciable results would hardly be expected with glass, which only permits the smallest twists.

The *rigidity* of these jellies was carefully studied.

Relation between Twisting Couple and Twist.

Mar. 11. Independent jelly cylinder. Length = 5.6, diameter = 2.0. Moment of couple = weight applied to each cord $\times$ 4.58, the diameter of the lower clamp upon which the cord was wound. The same couple was applied in both directions.

Moment	22.8	45.8	96	169	238	306	376	466	561
—Twist	9		40	73	102	129	147	179	207
+Twist	11	21	44	73	102	128	152	186	209

These measurements are plotted in fig. 7, I.

Independent cylinders could not endure such twists if the cylinders were vertical supporting the lower clamp; a modified form of apparatus was used in the above measurements, in which the cylinder was horizontal and the movable clamp rested upon friction wheels similar to those of an Atwood's machine.

July 21. Independent jelly cylinder; (approximate) length = 8, diam. = 2.0.

Moment		22.6	48.7	75	101
+Twist		33	69	110	145
—Twist		32	71	111	148 (broke)

These measurements are plotted in fig. 7, II.

These results show that the rigidity remains very nearly constant for twists of the general magnitude of those which produced the rotations, and before *the rotation is in practically the same relation to the twisting moment as to the twist.*

It would have been interesting to have measured the rigidity of jelly in rubber tubes but this did not prove feasible. The soft rubber tube is so much more rigid than even the stiffest

* These figures give as the rigidity of this glass 2.97×10^{11} .

† This Journal (3), viii, 89, 1899.

transparent jelly that practically all the moment applied was applied to the former.

July 22.

Jelly tube 44.

Moment applied	366	—366
Twist	47	—49

Identically same tube empty.

Moment	366	—366
Twist	59	—55

Same tube filled with water.

Moment	366	—366
Twist	59	—57

The unavoidable uncertainty in such measurements is so large a fraction of the difference that no extended observations were made.

Rigidity of Rubber Tubing.—Since the rubber envelope plays so important a part, the relation between couple and twist was investigated for a rubber tube similar to those filled with jelly, but filled with water that the interior might exert a similar resistance to compression as jelly without offering any rigidity.

April 9, '01.

4^{cm} of water.

Moment	325	555	784	1015	1245	1488
+Twist	20	40	59	80	104	
—Twist	24	43	59	81	103	128

These readings are plotted in fig. 7, III. *The rubber tube closely follows Hooke's law within this wide range.*

Does the rigidity have any of the peculiarities of the rotatory polarization?

Rigidity and Previous History.—The rigidity of the jelly does not seem to be influenced by a previous prolonged twist. Aug. 4. Independent jelly cylinder. Length = 7.8. Cross section = 3.60cm^2 .

Moment of couple.....	31.3	—31.3	—31.3	31.3	31.3
Twist.....	48.5	—44	—45	47.5	48

Twist of 80 applied for 25'. Immediately after—

Moment.....	31.3	31.3	—31.3	—31.3
Twist.....	48	47	—45	—45

Twist of —80 applied for 50'. Immediately after—

Moment.....	—31.3	31.3	31.3	—31.3	31.3
Twist.....	—45	46	48	—45	46

Other series of observations gave the same result.

Rigidity and Longitudinal Extension.—Independent jelly cylinders would endure a prolonged twist if stretched simply by the weight of a light lower clamp, but would not hold any additional weight for stretching the jelly. Therefore the vibration method of determining the rigidity was adopted, and after some failures, considerable success was obtained. The apparatus used was very similar to the mechanical part of the apparatus described above except that the lower clamp, which was freely suspended from the bottom of the jelly cylinder, was as light as possible and had a steel wire projecting below to which could be clamped at right angles either a wood bar or a brass bar. Brass bobs could be attached to the ends of the latter.

The length of the independent jelly cylinder was measured between the relatively inflexible rubber clamps at the ends. The cross section was obtained as follows: Two points were marked on the jelly with aniline ink. The distance between these marks was measured when, first, the jelly was free from any attachments; second, when stretched by the lower clamp; third, with the wood bar additional; fourth, with the brass bar in place of the wood bar, and fifth with the brass bar and bobs. The jelly cylinder, free from any attachments and with carefully cut ends, was placed in a graduated glass sufficiently filled with benzine (which is inert towards the jelly) to cover the jelly. From the change in level of the benzine the volume of the jelly was determined, which with the measured length gave the cross section. The proportional change of length for each weight added being known, the cross section for each weight was determined.

$$n = \frac{8\pi I l}{r^4 T^2}$$

I =moment of inertia of mass, attached to lower end of cylinder; T =complete period of vibration, which was measured

with a stop-watch reading to fifths of seconds; l and R are the length and radius respectively of the jelly cylinder.

Data.

Lower clamp; weight 41 $\frac{1}{2}$, moment of inertia = 50 $\frac{1}{2}$ cm².
 Brass bar; weight = 87, length = 20.1, radius = 0.375, mom. of inertia = 2940
 Wood bar; " = 11, " = 17.7, " = 0.25, " = 287
 Bobs; " = 42, " mean distance from axis = 9.05, " = 3450

Aug. 5. Independent jelly cylinder.

Total length of jelly immersed in benzine	13.95
Volume	51.90
Cross section	3.72
Distance between rubber clamps, no attachments	15.00
" " aniline marks "	10.50
" " " clamp + wood bar attached	10.98
" " " brass "	11.37
" " " " + bobs "	11.83

Wood bar attached.

$$I = 337 \quad l = \frac{10.98}{10.50} 15.00 = 15.7 \quad \pi r^2 = \frac{10.50}{10.98} 3.72 = 3.55$$

Times of 10 vibrations: 7.2, 7.4, 7.4, 7.6, 7.4. Mean $T = 0.75$
 $n = 1.8 \times 10^5$.

Brass bar attached.

$$I = 2990 \quad l = 16.2 \quad \pi r^2 = 3.44.$$

Times of 20 vibrations: 40.6, 40.6, 40.4, 40.4, 40.6. Mean $T = 2.03$
 $n = 2.5 \times 10^5$.

Brass bar + bobs attached.

$$I = 6440, \quad l = 16\frac{1}{2}, \quad \pi r^2 = 3.30.$$

Times of 10 vibrations: 37.6, 37.6, 37.6 (broke)
 $n = 7.6 \times 10^5$.

The rigidity was also successfully measured statically for a shorter length of this same cylinder when stretched by clamp + wood bar.

$$n = \frac{2Ml}{\pi r^4 \theta}$$

where M = moment of couple required for the twist θ .

$l = 10.8$ $\pi r^2 = 3.55$ $M = 13.980 \cdot 1.74$ (1.74 = diameter of part of lower clamp upon which the cord supporting the weight of 13 $\frac{1}{2}$ was wound). Mean value of $\theta = \frac{44}{57.3}$.

$$n = 1.8 \times 10^5 \quad (\text{exact agreement}).$$

Aug. 8. Independent jelly cylinder, (made with a different gelatine from the cylinder of Aug. 5). Omitting details, when the length was 10.40 the rigidity was 1.06×10^5 and when the length was 10.83 the rigidity was 1.06×10^5 .

These results indicate that the rigidity of jelly increases with longitudinal extension.

Dispersion.—No success was attained in measuring the dispersion, i. e., the variation in rotatory polarization for light of different wave lengths.

The biquartz could not be used since this is cut for some particular color, in this case to rotate the mean yellow through 90° , giving rise to the "sensitive tint" when placed between parallel Nicols. With simply two Nicols the dispersion was evident. If the analyzer and polarizer were crossed and the jelly introduced, on slightly turning the analyzer the field became green, showing the extinction of the longer waves, and on turning farther in became red, showing the extinction of the shorter waves. When a twist was applied this dispersion was still evident and the positions for the extinction of the different colors seemed farther apart, but no quantitative measurements could be made. When the jelly was twisted the coloration of the fields was irregular. Attempts were made to test the light emerging from the analyzer in both a direct vision spectroscope and an ordinary spectroscope. As the analyzer was rotated, successive parts of the spectrum did not perceptibly fade out, but near certain positions the whole spectrum gradually diminished in brightness until a minimum was reached, and then increased in brilliancy. The position for this minimum could only be determined roughly, but it essentially agreed with the portion of the plane of polarization of yellow light as determined with the biquartz.

Aug. 6. Tube 31. 3^{cm} of jelly.

Position of analyzer; with biquartz = 225, without biquartz for greatest darkness = 226, with the addition of a direct vision spectroscope, for greater general darkness of spectrum = 225.

Twist of 100.

Position of analyzer; with biquartz = 243, without biquartz = 242, with *d.v.s.* = 241.

Twist of -140.

Position of analyzer; with biquartz = 207, without biquartz = 207, with *d.v.s.* = 205.

Quartz plates introduced produced a dark band in the appropriate part of the spectrum, showing that the apparatus was properly adjusted.

Probably dispersion is present and very likely it follows the law of inverse squares of the wave length. The biquartz locates with great sensitiveness the mean position of the plane of polarization of yellow light. Owing to the complicated strains introduced with the twist, there is probably always yellow light polarized in all neighboring azimuths, of only little less intensity and similarly for other colors. Moreover, the jelly appears much more transparent for yellow light than

for other colors. These facts, coupled with the irregularities in location of the plane of polarization of yellow light in different azimuths, as illustrated above where full sets of readings are given, make it natural that the dispersion should not be very clear.

THEORY.

With hesitation I suggest the following imperfect theory. In a section of a twisted cylinder, perpendicular to the axis, along any line, whether straight or curved, there is nothing to distinguish one direction of motion from the opposite direction of motion. Therefore for two rays circularly polarized in opposite directions, traversing a twisted cylinder parallel to the axis, the conditions are identically the same in the wave front. Therefore the only difference can be in the lines along which the two opposite disturbances are propagated. The successive points disturbed by a positive circularly polarized ray lie at any instant on a negative spiral and vice versa. Is there anything to distinguish two opposite spirals parallel to the axis of a twisted cylinder? If the side of a spiral farthest from the axis of the cylinder has the same direction of twist as the adjacent part of the cylinder, the side of the spiral nearest the axis must have the opposite direction, and so there would be no distinction between the spirals were it not for the important fact that in a twisted cylinder the stresses and strains increase as we leave the axis radially, and therefore the outside of a spiral has a predominating influence.

Suppose the twist of a cylinder is positive, counting direction the same as for the rays. Any tangential (perpendicular to a radius) square in the cylinder with two of the edges perpendicular to the axis, will be distorted into a rhombus, the axes of strain being the diagonals. Therefore there will be stretching along the outside of the positive spiral and compression along the outside of the negative spiral. We may expect compression to decrease the velocity, for, in general, the denser a body, the slower light is transmitted through it. If this be so, we should expect to see the ray corresponding to the positive spiral to be transmitted faster than the ray corresponding to the negative spiral, or the cylinder having a positive twist and the positive spiral, corresponding to a negative circularly polarized ray, a negative circularly polarized ray will have the greater velocity and the plane of polarization will be rotated in a negative direction. Or, torsion should produce rotatory polarization in the opposite direction to the twist, which agrees with the experimental facts.

The effect of pressure upon the refractive index of jelly was investigated. Pressure was applied to jelly between two glass plates, forming a prism, by varying the angle of the plates. No

conclusive results were obtained owing to the great double refraction, introduced by the compression. Jelly is very incompressible.*

I do not see how in the above theory to account for the effect of the envelope unless the envelope affords a constraint preventing the jelly from yielding under the strains produced by the twist, the more perfectly these strains are maintained the greater being the rotatory polarization.

Consider the above mentioned rhombus into which the square has been distorted. The distortion will bring the diagonal along which there is compression nearer the axis than the other diagonal. Therefore longitudinal compression will increase the compression along this diagonal more than it will decrease the elongation along the other diagonal, and therefore we should expect the rotatory polarization to be increased, which agrees with the experimental facts. Similarly we should expect elongation to decrease the rotatory polarization, which it does. On this theory we are not surprised that hydrostatic pressure produces no effect, since it is perfectly symmetrical and homogeneous.

This rotatory polarization might be expected in a general way according to many theories of rotatory polarization which require rotatory polarization where there is such dissymmetry present as in a twisted body.

Conclusions.

The following are the most important facts added to those established in previous papers.

1. For a given twist, the rotatory polarization is very much increased by an envelope about the jelly.

2. The rotatory polarization of jelly twisted in a rubber tube is very much increased by longitudinal compression, and decreased by longitudinal elongation, closely following an exponential law.

3. It is not affected by hydrostatic pressure.

4. It is, to some extent, dependent upon the previous history of the jelly.

5. When these disturbing influences are kept constant the rotatory polarization is quite definite for a given twist and,

6. The rotation is closely proportional to the fourth power of the twist.

7. The rigidity of the jelly increases with longitudinal elongation.

Finally, a theory has been suggested for explaining some of the above facts.

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* P. von Bjerkén, Wied. Ann., 1891, p. 817.