

III.—LINGUISTIC MISUNDERSTANDINGS.

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PART II.

IV. THE ANTINOMIES OF TIME.

LET us now consider the words *finite* and *infinite* in reference to time. Here the analysis is more difficult, because the primary conceptions are more complex. For we can have no clear idea of time till we are first in possession of clear ideas of space, number, change and motion. The commonest method of measuring time nowadays is by clocks and watches. But in this way what do we really measure? We measure the *spaces* traversed by the hour, minute or second hands. The astronomer measures time by measuring the angular changes produced by the motions of the heavenly bodies, taking a complete angular revolution of the sun among the apparently fixed stars as his final and most convenient unit of reference. We call this unit a *year*, and to suit our convenience we arbitrarily divide it into other units called *days*, *hours*, *minutes*, and *seconds*. But here again what we really measure is *space*, the apparent space described by the sun, which measures time for us by its apparent motion round a great circle of the sky, just as the hour or minute hand of a clock measures smaller portions of time for us by its real motion round the clock dial. If we perceived no change we could have no notion of time. Our case when we are lying in bed motionless, with shut eyes but awake, is no exception. For in this case we are conscious of our successive ideas or sensations, and from these mental changes we form a rough estimate of the progress of time. These examples sufficiently show how much more complex is the conception of time than the conception of space.

We roughly divide time into three divisions, present, past, future; but how far does each of these extend? By a commonly accepted convention, we agree that the past and the future should each be considered infinite; the latter, reckoned from the present, being conveniently called *positive*; the

former *negative*. But what are the *limits* of the present? Here there is less agreement in linguistic usage. Some regard the present as a mere point in time, passing instantly into the past as soon as it is reached. From some points of view, and for some purposes, this convention has its utility; but they are exceptional cases. As a rule, it is more convenient to ascribe to the present an actual but limited duration, as when we speak of the present hour, the present year, or the present century. Thus viewed, the present presents no difficulty; but what about the infinite past, and the infinite future? These two infinities (direction apart) logically stand on the same footing, and thus ought to present no more difficulty than the infinity of space. Yet they do; at any rate, the former. A thing having once come into existence, be it an inanimate stone, or a sentient being, or the material universe, the conceptual supposition of its continuing to exist for ever in the future gives no shock to the reason. An endless as well as infinite future—infinite in the sense already defined—seems somehow less difficult to grasp, less of a self-contradiction, than a boundless as well as infinite spatial universe. But it is different with regard to a *beginningless* as well as infinite past. Many people, in thinking of an infinite deity, or of the finite or infinite universe, find this beginninglessness an impossible conception, and yet no less impossible the idea of anything, spiritual or material, springing suddenly and causelessly into existence out of an antecedent nothingness. These difficulties generally arise from reflexion and reasoning, for they do not seem to occur to very young children. These, I believe, as a rule, before they have heard of birth or death, take their own eternal existence, past and future, without beginning or end, simply and tacitly for granted. I knew a little girl who, when, at the age of four, she learnt for the first time that all must sooner or later die, her father, mother, brother, sister, and herself not excepted, gave way to a flood of tears, and for some days remained inconsolable. She could not resign herself to the bitter thought that for her, as for all near and dear to her, this happy life which she and they so thoroughly enjoyed must, after a limited but unknown length of time, come entirely to an end. What she thought of her past existence, before her parents had, as she supposed, “bought” her, I never heard, but I think it likely that she took for granted the beginninglessness of her past as she certainly did the endlessness of her future.

Why is an absolute void and nothingness in space an easier conception for us than an absolute void and nothingness in time? I think the reason is this, that in the measurement of

time we have the principle of *repetition* after a complete revolution; in the measurement of space we have not. In measuring the distance from A to B we move from A to B in a straight line and never go twice over the same spot. In the measurement of areas and volumes we adopt the same principle; we never count a unit of area or volume twice over. But in the measurement of time (which we cannot accurately accomplish without also measuring space) we are obliged to adopt the principle of revolution and repetition. The minute or second hand of a watch performs a complete revolution on the circular dial in an hour or minute respectively, and then repeats the revolution again and again *over the same space*. Similarly, the sun appears to describe a complete revolution round a great circle of the heavens in a year, and then repeats the revolution year after year, *round the same apparent circle*. And, *a priori*, before we have studied the origin and evolution of the heavenly bodies, we can see no reason why this should not have gone on eternally in a beginningless past, and why it should not also go on eternally in an endless future. As an *a priori* conception, this beginningless and endless eternity of matter or spirit seems an easier conception than that of a deity or of a material universe starting suddenly into existence out of a preceding eternal nothingness at some infinitely remote point in the past. That the universe should, at some infinitely future date, suddenly explode into its previous hypothetical nothingness is a hardly less difficult supposition. Yet neither supposition involves any logical or linguistic inconsistency. Sudden and startling presentations of inexplicable phenomena, which as suddenly vanish, are not unknown to our experience. Shooting-stars, meteors, fire-balls, etc., may be cited as examples which perplexed our ancestors; and, coming to modern times, how many of the spectators of a cinematograph performance understand the cause or principle of the unexpected marvels which so completely deceive their eyes while they excite their imagination or tickle their sense of humour? Suppose one of these spectators, as might very well happen, were suddenly to drop off into unconsciousness in the middle of a scene, and afterwards, as suddenly, come to himself at the very same point in the middle of the same scene in a repetition of the performance. He would be wholly unaware of the flow of time during his unconsciousness; he would never suspect that he had been unconscious; the rest of the second performance would appear to him the simple and natural continuation of the first. To such a spectator his blank interval of unconsciousness would count as zero in his measurement of time.

"Imagine something analogous to happen to our whole universe, sentient and non-sentient.¹ Suddenly all the laws of nature are suspended. All motion ceases. Gravitation is no more. . . . The rising and falling waves stop as if suddenly sculptured on a sea of ice. . . . The preacher in his pulpit stops in the middle of the word *firstly*; the orator in parliament in the middle of the word *closure*. . . . The brain functions no longer. All thought, all feeling ceases. . . . The universe still exists, if existence it may be called, as dead matter, but its life has departed. Then, after a hundred years (as in the well-known fable), or a thousand years, or a million years, its life returns as suddenly as it left it. The earth resumes its revolution on its axis; the planets resume their course round the sun; motion rebegins everywhere. . . . The preacher continues his sermon; the parliamentary orator his angry protest against the closure. Everything goes on as if nothing had happened; nobody knows or suspects that anything *has* happened—that the life of the whole universe has been arrested for a million or more years.

"Now, from the strictly logical standpoint, how should this hypothetical suspension of all the laws of our universe, physical and psychical, be regarded? What about our scientific formulæ. As regards all formulæ bearing on the question of time in general, and age in particular, would they not be more simply workable, as well as more reliable in their application, if we considered the whole period of cosmic suspension, however long, as non-existent? . . . How many bankers would be willing, or would be able if willing, to pay the amount of interest that would have become due after more than a million years? . . . Confusion and perplexity would meet us everywhere. The only possible solution from the practical standpoint is the one that would be unconsciously adopted: everybody would regard the whole period of suspension as non-existent, as absolute zero."

Now, just as two atoms shot at random in the universe may conceivably collide, though the chance of the collision be infinitesimal, so the preceding hypothetical event may conceivably happen, since it involves no linguistic self-contradiction and is not incompatible with any known data. *It cannot be proved false.* Just as much and no more may be affirmed of many of the speculative hypotheses seriously advanced by serious scientists as serious explanations of the origin and evolution of the universe.

¹This extract I quote (with some omissions) from my *Man's Origin, Destiny and Duty*.

But at present we are not discussing the possibilities of actual events, but the consistency of concepts, and the suitability of the terms in which we strive to express them. Can time exist—exist even as a clear concept—without the existence also of motion, or of the idea of motion? The illustration given will, I think, show that practical science at all event would, in its formulæ and calculations, have to ignore such existence.

V. VIRTUAL EQUALITY.

The so-called 'antinomies' of Kant appear to me to have sprung from a confusion between his clear apprehension of the primary or subjective meaning of the word *infinite* and his somewhat vague apprehension of the only meaning that can (in my opinion) be consistently attached to the word in exact mathematical researches. In practical mathematical researches it will be found that all valid formulæ containing reference to infinities or infinitesimals will retain their validity when these words convey the sense which I give them by express definition, namely, that the former is a number, magnitude, or ratio too large, and the latter a number, magnitude, or ratio too small, to be accurately or approximately expressed in the decimal or any other system of notation. Thus, every infinitesimal is the reciprocal of some infinity, and every infinity is the reciprocal of some infinitesimal; while every finite is the reciprocal of some other finite. On the other hand, the word *infinity*, in its primary or subjective sense, of *endlessness*, merely expresses the liberty claimed by the imagination of "beating the record," so to speak, whenever it chooses; that is to say, the liberty of surpassing any number, magnitude, or ratio, however large, by the conception of a number, magnitude, or ratio still larger. For example, if it be asked how many terms there are in the series A^1, A^2, A^3, A^4 , etc., in which A denotes any number or ratio, we may legitimately reply that the number of terms is infinite. Here however the word *infinite* does not denote a real number at all, nor any property that can be attributed to any real number, nor any class to which any individual number belongs; and the statement that the number of terms is infinite is only another way of saying that though there is a definite *first* term there is no definite *last* term; or, in other words, that there is no fixed limit beyond which the imagination may not continue the series. When, on the other hand, we say that the sum of the series $A^1 + A^2 + A^3 + \dots$ to infinity, assuming A to be a proper fraction between 0 and 1 (say, the fraction *one-half*) is $A/(1-A)$, what we really mean is that

(denoting the class, or any individual of the class, of infinities, H_1, H_2, H_3 , etc., by H , and the class, or any individual of the class, of infinitesimals, h_1, h_2, h_3 , etc., by h) if we denote the sum

$$A^1 + A^2 + A^3 + \dots + A^n$$

by S , then we shall have

$$\left(\frac{A}{1-A} - S \right)^h,$$

a symbolic statement which, in my notation, asserts that the difference between the greater ratio $A/(1-A)$ and the less ratio S is infinitesimal. Mathematicians commonly write

$$A + A^2 + A^3 + \text{ad infinitum} = A/(1-A);$$

but this equality never holds absolutely, for however far the series may be carried, the sum S is always less than $A/(1-A)$. Here the total number of terms in the series is the real though infinite number H , and the last term¹ of the series is A^n , which is necessarily an infinitesimal, since, by our data, A is a proper fraction less than 1. Hence also the statement $S = A/(1-A)$ asserts *virtual* and not actual equality; that is to say, there is a real difference between the two ratios asserted to be equal, but the difference is infinitesimal compared with (i.e., divided by) either. (See my paper on *Symbolic Reasoning*, No. viii., p. 509, in *MIND*, October, 1906.) This example illustrates the sense in which mathematicians commonly use the word *infinite*, though the lack of an exact and satisfactory definition of the word in the generality of textbooks renders their language sometimes obscure and their statements apparently inconsistent. In this and similar cases we may write $S = A/(1-A)$, provided it be understood that *virtual* and not absolute equality is asserted. As defined in my paper in *MIND*, two quantities or ratios, finite, infinite, or infinitesimal, are said to be *virtually* equal when the difference between them is infinitesimal compared with (or divided by) either, and in the infinitesimal calculus it would be convenient if we adopted the convention that the symbol of equality ($=$) between two quantities or ratios in the statement ($A=B$) only asserts that the two are either virtually or absolutely equal. Thus the statement ($x=x+dx$) asserts *virtual* equality, whether x be finite, infinite, or infinitesimal, it being understood that dx/x is an infinitesimal ratio.

¹ The symbol A^n may either denote the n th power of A (the exponent H being infinite by definition) or the statement that A is infinite. The context will always make clear in what sense it is employed in each case, for it is scarcely possible to mistake a statement for a ratio, or *vice versa*.

Prof. Keyser, in the *Hibbert Journal* of October, 1909, page 188, says that—

Mr. MacColl's conception of the *infinitesimal* is one that mathematicians have not been able to employ. As used by them, the term signifies, not a small quantity, but a variable that, under the conditions of the problem in which it occurs, may be made and kept small at will—a variable having zero for limit.

To this I reply that I believe the reason why mathematicians have not, so far, employed my conception of the *infinitesimal*—a conception which they all possess, however differently they may express it—is that my allied and complementary conception of “virtual equality” had never occurred to them. Restricting, as they do, the symbol of equality ($=$) to *absolute* equality, they could not consistently make the assertion ($a=a+x$) even when x is infinitesimal compared with a , so that, to preserve logical accuracy, they are obliged to be continually appealing to the round-about notion of a *limit*, an appeal which my proposed convention as to ‘virtual equality’ and the meaning of the symbol ($=$) would render needless and irrelevant, without sacrificing one iota of logical accuracy.

In explaining the principle of a ‘derivative’ or ‘differential coefficient,’ modern writers on the infinitesimal calculus find it necessary to lay down various cautions to prevent beginners from misunderstanding the real meaning of their symbolic formulæ and operations.

Let me quote the following from an excellent work (*An Elementary Course of Infinitesimal Calculus*, by Horace Lamb, F.R.S.) which I have often recommended to pupils:—

The symbol dy/dx is to be regarded as indecomposable, it is not a fraction, but the limiting value of a fraction. The fractional appearance is preserved merely in order to remind us of the manner in which the limiting value was approached.

Now, I agree that dx and dy should each separately be regarded as indecomposable, the letter d having no meaning apart from the letters x and y , which denote real quantities or ratios; but there is no necessity for so regarding the whole complex symbol dy/dx . I see no reason at all why we should not, like Leibnitz, regard dy/dx as a real fraction whose value depends upon the real values of its numerator and denominator dy and dx . The following simple example will, I feel sure, make this clear to every reader of MIND, whether he be acquainted with the infinitesimal calculus or not:—

Let $y=x^2$, and let dx be infinitesimal compared with x , so that dx/x is an infinitesimal ratio. Also let dy denote the

increment received by y (that is, by x^2) in consequence of an infinitesimal increment dx received by x . Otherwise expressed, let $dy = (x + dx)^2 - x^2$. We get

$$dy = 2x \, dx + (dx)^2.$$

Hence, $dy/dx = 2x + dx$. So far, the sign of equality has denoted *absolute* equality. Now, since (by definition) dx is infinitesimal compared with x , it must also, *a fortiori*, be infinitesimal compared with $2x$, so that we get $2x + dx = 2x$. Here the sign of equality denotes *virtual* and not *absolute* equality. Thus, finally, we get $dy/dx = 2x$, an equality which is again *virtual* and not *absolute*. From this point of view there is no reference to a limit, as the conception of a limit is not needed. In this case, dx and dy are two really existing infinitesimals, and my assertion that the ratio of the latter to the former is *virtually* equal to $2x$ (as I define the word *virtually*) is really equivalent to what mathematicians mean when they say that $2x$ (technically called the "differential coefficient of y with regard to x ") is the *limit* to which the fraction $\delta y/\delta x$ approaches as the increment δx (which they never speak of as an *infinitesimal*) approaches zero.

Let it be clearly understood that an assertion of virtual equality, such as $(x + dx = x)$, not merely asserts that dx is *negligible* compared with x , that practically it may be omitted because of its extreme smallness in comparison, but that it *must* be omitted in all possible calculations, because (by express definition) no arithmetical notation, and *a fortiori*, no instrument however delicate, can ever take account of its existence.

If a regular polygon of M^M sides (in which M denotes a million) be supposed inscribed in a circle, the difference by which its perimeter falls short of the circumference is certainly negligible, and more than negligible, compared with either as regards all practical calculations, but the ratio is not *infinitesimal*, because, though inconceivably small, it is still arithmetically expressible; that is to say, it can be expressed approximately by certain conventional collocations of the ordinary digits. In this case, therefore, we cannot consistently assert that the perimeter is *virtually* equal to the circumference. And if for M^M we substitute its millionth power M^{MM} , or any other huge but arithmetically expressible number, the result will be the same; the excess of the circumference over the inscribed perimeter, though utterly negligible, will, from our very definitions of the terms, be *finite* and not *infinitesimal*. But if we suppose a regular inscribed polygon of H sides, then the difference between the perimeter

and the circumference would be infinitesimal compared with either (since H is, by definition, infinite); and for this reason, we can here assert that the perimeter is *virtually* equal to the circumference, and express this virtual equality in the form ($P = C$), in which P stands for *perimeter*, C for *circumference*, and the symbol ($=$) for an assertion of *virtual equality*.

Leibnitz founded his infinitesimal calculus on the notion of infinitesimals, which he merely regarded as extremely minute quantities, without clearly indicating in what respect an infinitesimal differs from a very small finite. Newton founded his calculus on the notion of the 'ultimate ratios' of vanishing quantities; that is, the ultimate ratio of the increment δy to the increment δx when the latter (and consequently, as a rule, the former), by continual decrease, reaches the limit zero; in which case $\delta y/\delta x$ takes the form $0/0$. Both were right in their respective conceptions, but they expressed those conceptions awkwardly and in apparently self-contradictory language, which led the logicians, and many even of the mathematicians, of the day to question the legitimacy both of their reasoning and of their symbolic operations. Modern mathematicians have adopted Leibnitz's notation as more convenient than Newton's, but they have completely rejected the conception of negligible infinitesimals on which Leibnitz founded his notation, on the ground that it is logically inadmissible. And logically inadmissible the conception undoubtedly is so long as the symbol of equality ($=$) is restricted to *absolute* equality; for it is clear that A cannot be *absolutely* equal to $A + h$ so long as h has any real value however small. But my convention, that the symbol of equality shall only denote an equality that may be either absolute or virtual (as I define the word *virtual*), entirely removes from Leibnitz's symbolic formulæ and operations the reproach of inconsistency. Modern mathematicians, following Newton's conception, have secured for it a certain measure of consistency, but at a heavy and needless sacrifice of brevity and simplicity. They have replaced Newton's conception of an 'ultimate ratio' of the form $0/0$ by the more consistent idea of a *limiting* ratio, which they express in the form dy/dx . They regard dy and dx , however, not necessarily as infinitesimals or other small quantities, or indeed as necessarily quantities at all. They merely insist that the composite symbol dy/dx shall denote the exact limiting ratio which it is employed to represent. They thus studiously avoid Leibnitz's notion of *infinitesimals* by dispensing even with the word. Instead of speaking of infinitesimals, they nearly always speak of *limits*.

If x and a be real ratios (finite, infinite, or infinitesimal)

and virtually equal, it generally follows that any function of x (if a real ratio) is virtually equal to the same function of a . When two infinities H_1 and H_2 are virtually equal, their difference $H_1 - H_2$, or $H_2 - H_1$, though necessarily infinitesimal compared with (or divided by) either, may be finite, infinite, or infinitesimal compared with any finite. All that the definition of "virtual equality" requires is that the fractions $(H_1 - H_2)/H_1$ and $(H_1 - H_2)/H_2$ shall each be either a positive or negative infinitesimal.

If x and a (whether finite, infinite, or infinitesimal) be virtually equal, the fraction $(x^n - a^n)/(x - a)$ is virtually equal to na^{n-1} . Mathematicians usually express this by saying that na^{n-1} is the *limit* to which the fraction $(x^n - a^n)/(x - a)$ indefinitely approaches as the variable x approaches the finite constant a . But by my proposed convention as to "virtual equality" and the meaning of the symbol of equality ($=$), the ratio a need not be finite. The proposition holds good universally, whether x , a , n (individually or collectively) be finite, infinite, or infinitesimal.

Let $A_1, A_2, A_3, \dots, A_n$ be any ratios in ascending order of magnitude, and such that A_1 is virtually equal to A_2, A_2 to A_3, A_3 to A_4 , and so on. Then, if n be *not* infinite, the smallest ratio A_1 is virtually equal to the largest ratio A_n , so that we can write $A_1 = A_2 = A_3 = \dots = A_n$.

From this theorem it follows that in mathematical researches involving n affirmations of *virtual equality* (however large the finite number n may be), no error can possibly enter into the final result through the repeated omission of infinitesimals in successive affirmations of virtual equality. The theorem is a simple corollary from the easily proved formula $(Fh)^k$, which asserts that the product of a finite and an infinitesimal is an infinitesimal. The formula holds however large the finite F may be—even if it denote the millionth power of the millionth power of a million.

VI. MATTER AND MIND.

Among the antinomies discussed by metaphysicians are the arguments for and against the possibility of the real existence of space, time, and the material universe, apart from the existence of a human or superhuman mind to perceive them. To enter seriously and fully into such a discussion would be a formidable undertaking. No one can do so profitably without first making sure that he and his readers attach the same meanings to the words he employs. Otherwise he enters a labyrinth of ambiguities from which it is scarcely

possible for him to find an exit. And the readers who venture to follow him commonly share his fate. The words *real* and *existence* especially need defining, and definitions of them are not easy. We all understand these words in various senses according to the context; and often also, even with the context to guide us, we wofully *misunderstand* them. In common parlance we speak of real existence and of unreal (or imaginary) existence, and, logically enough, we regard these two classes of existence as mutually exclusive. Yet, on close inspection, it is not easy to find the exact line of demarcation. Should the abstractions *truth* and *error* be considered unrealities because they have no form, weight, or substance? Hardly, though their existence certainly depends upon that of the persons who understand or misunderstand them. Is an unreal or imaginary existence a contradiction in terms, a linguistic inconsistency? What is the difference between an unreal or imaginary existence, such as that of a fairy, and an absolute non-existence? May we reply that unrealities, like fairies and fictitious characters in novels, exist in the mind, and must therefore have at least a *subjective* existence? "In the mind?" What does the preposition *in* here mean? Is the mind (or soul) then a substance of some kind, material or immaterial, in which another substance, real or imaginary, can exist? Or are we merely talking figuratively and—rather vaguely—because the ideas which we strive to convey are too vague for exact expression? Is the mind the same as the soul? And if not, what is the difference? Can the distinction be clearly shown by an exact definition of each? Materialistic philosophers—or those who call themselves such, for these words also are ambiguous—sometimes speak of the soul as a "function of the brain," and sometimes as an "emanation from the brain". What do they mean? If they were pressed hard for definitions or explanations, and answered frankly, I think they would be forced to own that they did not know—that—to put it bluntly—they had been talking nonsense. In mathematics the word *function* has a clear and definite meaning. As a rule, when we can correctly say that y is a function of x , we can also say correctly that x is a function (though generally a different function) of y . If in this or some analogous sense, the soul can be said to be a function of the brain, can we, following the mathematical analogy, say that the brain is also a function of the soul? Idealists might plausibly maintain this view, but not materialists, as it is directly opposed to the latter's fundamental conceptions. Again, taking the other materialistic view, how can the soul be an

"emanation," or *flowing*, from the brain? Can the soul, even as an analogy or metaphor, be likened to a liquid or gas flowing or escaping from a reservoir? The points of unlikeness are surely far more numerous than those of resemblance. And of the points of unlikeness the most striking is the fact that the mind or soul is *conscious of its own existence*, while the flowing gas or liquid is *not*.

In connexion with this question of the soul, I may be allowed to say a few words in reference to an objection raised by Prof. Taylor in his kind and appreciative criticism of my *Man's Origin, Destiny and Duty*, in *MIND*, July, 1909, pp. 451-453. In that book I define the soul as "*that which feels*," and argue that, wherever it may be situated, there is no proof that it is in the brain or nervous system, as these, judging from observation and experiments, appear (like the rest of the body) to be mere insensible channels through which some unknown force is transmitted (we know not whence) to the soul, causing sensations, and *from* the soul—generally by exertion of the *conscious will*—producing actions. In regard to this view, which he does not seem wholly to reject, Prof. Taylor writes:—

But the view is still retained that this subject [the soul or real subject of consciousness] is extended and occupies a region in the physical space of ordinary perception. Thus Mr. MacColl's contention against the ordinary materialist takes the form of maintaining that the 'soul' must be *somewhere*, but the *somewhere* need not be "in the brain": it may be millions of miles away. Now, I should prefer to ask whether the question "Where is the soul?" has any meaning at all. Is it more reasonable to ask whether e.g. my belief that $2+3=5$ is in my brain or ten millions of miles away, than to ask what is the distance between Piccadilly Circus and the middle of next week? . . . The seat of consciousness is removed to a distant and possibly extra-stellar point, but the question still remains whether there would be any sense in saying that thought and sensation are "at" this point? Does a thought or feeling take up any extension at all? I think the author would see, on further reflexion, that the unity of our mental life, on his theory, would commit us definitely to the view that the soul literally is a mathematical point, and such a view is surely as unintelligible when that point is said to be millions of miles away as when it is said to be "in" the brain. The real absurdity surely lies in assigning presence "at" a point to the self at all.

Now, I admit at once that these are serious objections to my theory or hypothesis that the soul (whether material or etherial, or composed of some other substance entirely imperceptible to our present human senses) may possibly have, at any given moment, some definite though unknown form or size, and may occupy, like a planet, sun, or atom, some definite though unknown position in space. Great however as are the difficulties that lie in the way of this hypothesis,

those that confront the opposite hypothesis, the hypothesis that the soul (the sentient entity by my express definition) has no spatial existence, seems to me more formidable still. But here again, perhaps, Prof. Taylor and I, like so many other sincere controversialists, do not always attach quite the same meanings to the same words. Even if my hypothesis led to the conclusion he supposes, that, as regards size, the soul corresponded to the conception of a mathematical point—a conclusion which can hardly follow from my premisses, since these leave its size and form unknown and indefinite—the conclusion would involve no inconsistency. For, from my point of view—which I admit however to be different from that of mathematicians in general—a point may have any size whatever, provided the unit of reference be infinite in comparison. If any portion of matter, whether an atom, an electron, or something else still smaller, be infinitesimal compared with any nameable finite unit, be it the volume of a drop of water or that of the earth, then, and not otherwise, it may be regarded as a *point*. And this infinitesimal point may also consistently be conceived of as infinite in comparison with another point still smaller; and so on *ad infinitum*. It is all a matter of ratio or comparison, and depends entirely upon our arbitrary unit of reference. A ratio h_1/h_2 between two infinitesimals, like a ratio H_1/H_2 between two infinities, may be finite, infinite, or infinitesimal; but a ratio F_1/F_2 between two finites *must* be finite, from our very definition of the word. Assuming the soul to have a spatial existence, we have no data at present for determining its size, form, or position at any given moment, or whether these be fixed or variable. If any man chooses to assert that his soul (spatially considered) is at this moment finite, or that it is infinite, or that it is infinitesimal (these words being understood as I define them), I can neither verify nor disprove his assertion, though the first hypothesis—I cannot in the least explain why—seems to me the most likely. “But why consider the soul spatial at all?” Prof. Taylor would ask. My reply is that otherwise I must regard it as belonging to the class of entities which most grammarians lump together under the name of *abstractions*, such as *hunger*, *hardness*, *battle*, etc., and that such abstractions are but disguised predicates which cannot be separated from some non-abstract subject understood. There can be no hunger without a hungry person or animal; there can be no hardness without some hard substance; and there can be no battle without some sentient beings (human or non-human) who struggle for mastery. Similarly, I cannot conceive of a *thought* apart from a *thinker*,

or of a *feeling* or sensation without a soul or *feeler*. The last word is here used in a somewhat novel sense, but the context explains it. And as, by express definition, I class thoughts and mental emotions in the category of sensations, it follows that the soul (including mind and spirit) is the thinker as well as the feeler. This extension of the meaning of the word *feeling* or *sensation* may not be in accordance with the usage of psychologists or physiologists, but it is, I think, in accordance with the usage of all of us in ordinary speech ; for don't we all employ such expressions as "I feel sure," "I feel the force of the argument," etc.? Besides, words are after all mere symbols, like the mathematician's x , y , z , to which we may give any convenient meaning that suits our purpose, provided the context leaves no doubt as to what that meaning is. Can an idea, or emotion, or sensation, or occurrence be consistently spoken of as occupying any definite position in space? Yes, provided the speaker's or writer's meaning can be inferred clearly from the context. No one misunderstands the meaning of such a remark as "her thoughts are far away with her absent children," and nobody in this case is under the delusion that the thinker is in one place, and her thoughts far away in another. Don't we speak of the site of such and such a battle, though the abstract conception of a battle has in itself no form or position apart from those of the combatants? The conclusion arrived at by some modern psychologists, that "the thoughts themselves are the thinkers," seems to me as much a linguistic inconsistency as would be the statements that "the combats themselves are the combatants," that "the receipts themselves are the receivers," and that "the speeches themselves are the speakers". The statement (which I have quoted from memory) that "the thoughts themselves are the thinkers," is, if I am not mistaken, due to Prof. James. I do not suggest that it expresses Prof. Taylor's opinion, but it seems to me that his opinion that the soul cannot consistently be spoken of as occupying any spatial position necessarily leads to the conclusion expressed by the quoted statement.

VII. MATTER AND MIND.—*Continued.*

But then it may be asked, "If the thought itself is not the thinker, where is the thinker?" Superficially considered, the question sounds absurd. "*There* is the thinker," it may be replied—"that one-armed and one-legged man, sitting on that bench in the park, with that far-off look in his eyes." "Yes, but how much of him constitutes the thinker?" Since

he can still think, and think as well as when he had two arms and two legs, the missing arm and leg formed no indispensable portions of the thinker, any more than his hair, or his nails, or his clothes. He is still the "thinker" without them. How far can we carry on this slicing away of non-indispensable portions of the material body and still leave the "thinker"? What is very curious is that that one-armed and one-legged thinker will tell you that when the weather is damp he still feels pains in his fingers and toes, not merely in those of the arm and leg which he still possesses, but also in those of the arm and leg which have been amputated and no longer, so far as he is concerned, exist. Point out to him the absurdity—at least from the linguistic standpoint—of this statement, and he will own it; but yet he will assure you that if the evidence of his eyes, aided by that of his sense of touch, did not convince him of the contrary, he would be under the illusion that he still possessed the missing members also; for that the sensation of pain, apparently felt in the no longer existing toes and fingers, is exactly similar now, after the amputation, to that which, in damp weather, he had previously felt before it. The hasty physiologist will say that, in spite of the apparent direct evidence of his senses, a man really feels pain *in his brain*, the seat of all pain and pleasure, mental or physical, as of all other kinds of sensation or consciousness. The evidence for this conclusion was never quite convincing, and modern experiments and observations tend more and more to discredit it. The brain, it is true, is an important medium through which the soul receives sensations, and generally the possibility of thought, sensation or consciousness, depends upon its physical condition—just as generally—not always—the sensation of warmth depends upon the physical condition of the atmosphere. In the same condition of the atmosphere one person may experience an uncomfortable sensation of heat, while another, standing near him, may shiver and suffer from the very different sensation of cold. Generally speaking, the sensation of seeing depends—in part at least—on the condition of the eyes, that of hearing on that of the ears, and that of smelling on that of the nose; but if recent experiments made by eminent doctors and physiologists can be trusted, the soul—which is the sentient entity by definition, and therefore the real person—can, in certain exceptional cases, see, hear, and smell through other channels than the organs through which those sensations are usually transmitted. If then the eyes, ears, and nose are not absolutely indispensable organs for the transmission of their special sensations to the soul, why should the brain be an

absolutely indispensable organ for the transmission to the soul of sensations in general, thoughts and mental emotions included? So far, if I am not mistaken, Prof. Taylor and I hold the same views; we neither of us believe that the brain or any other material part of the body feels or thinks; but we differ as to the propriety of assigning spatial position anywhere, either within the body or without the body, to the mysterious entity that does.

I may remark that, in their essential principles, the arguments which led some philosophers to deny the spatial existence of the soul lead others to deny, not only the spatial existence of the soul, but the real existence, except as a concept, of space itself, and as a necessary corollary, the real existence of time, with which the existence of space seems to be inseparably connected. Thus, the whole objective and material world is resolved by them into Berkeley's subjective immaterial "ideas," or, as I find it more convenient (because more widely suggestive) to call them, *sensations*. I prefer the word *sensations*, because some philosophers deny 'ideas' to the lower animals, whereas *sensations*, and therefore *souls*, as I define the word, belong, as they admit, to both. Now, the question at issue is this: Does matter exist except as a mere sensation, or as a mere collection of sensations? The true answer seems to me to be this, that matter, in its common conditions of solids, liquids, and gases, is a very prominent *cause* of certain sensations, but should not be identified with the sensations themselves; which, whether elementary or compound, should be considered as its *effects*. It leads to self-contradiction to identify a cause (whether known or unknown) with its effects. Just as we infer a cause from its effect or effects, as, for example, the existence of a lion from its roaring or footprints, so we infer the existence of matter (whatever its ultimate constitution) from one or more of the sensations which it produces on the subjective or sentient portion of us which I call the *soul*.

To make my meaning plainer, let me give a homely illustration similar to one of Berkeley's, but somewhat differently presented and developed. I see before me something which, from its appearance, I infer to be an apple; but as the evidence afforded by the sensation called *seeing* is not always reliable, when unconfirmed by other sensations, I approach, touch, lift, and smell it. Let us suppose that the fresh sensations thus obtained strongly confirm my former conclusion, but do not convince me entirely, as, after all, the supposed apple might be a small turnip shaped and painted by a clever practical joker to resemble an apple, and rubbed by him with apple *essence* so as to deceive the sense of smell. As a final test

therefore I cut a small slice and bite and chew it. If the new sensation of taste thus added to my former data confirm my former conclusion, may the conclusion be now regarded as absolutely certain? Not quite, for in spite of the evidence afforded by all those sensations, it might conceivably happen that a fresh sensation, namely, the discovery of a hard substance like a peach-stone at its centre, would prove to me that this was a new kind of fruit which, while it possessed many of the properties usually connoted by the word *apple*, possessed others which the word, as hitherto understood, did not connote. What in such a case should we do? Extend the meaning of the word *apple* so as to include the new fruit? or invent a fresh name, such as a *peach-apple*, or *apple-peach*, to distinguish it? Neither course would involve any logical inconsistency, but the latter would be more convenient. Now, in this case, should the word 'apple-peach,' or 'peach-apple,' denote the total collection of sensations? Should it not rather denote their cause?—or, to speak more accurately, the more salient portion of the infinite number of their causes? For, be it remarked, a cause is never single. The number of causes producing any sensation is really infinite. We could not, for example, have the sensation of seeing without the vibrating ether, nor the sensation of smell without the air to convey the effluvium, and so on for other causes, of which a few are more or less known, but of which the infinite majority must, to our very limited faculties, remain for ever unknown and unsuspected.

Now, the analysis which we have here applied to a word denoting a particular kind of matter may also be applied to the term *matter* in general. The first question to be settled is: What property or properties do we attribute to *every* kind of matter, apples included? Scientists in general agree that whatever entity comes under the designation of *matter* must possess at least one property, the property variously named *weight*, *ponderability*, or *attraction*. In other words, every particle of matter in the universe attracts and is attracted by every other particle of matter. We notice this property, this tendency to mutual approach, according to a certain observed law, in several substances, we infer it in others, and we agree that all the substances, known or unknown, which possess it shall be called *matter*. But what do we mean here by the words 'notice' and 'infer'? 'Noticing' and 'inferring,' like all other percepts and concepts, come (according to my definition) under the general name of *sensations*, so that, otherwise put, *matter* is, as in the former analysis, the name of the most prominent of the many causes of these sensations.

But there is one perplexing entity which this definition of

matter appears to exclude. I speak of the hypothetical ether. I say "hypothetical," not because I doubt the existence of some space-occupying substance whose vibrations and other properties cause the sensation of seeing, as well as other sensations which (or whose causes) we connect more or less with such vague words as *heat*, *electricity*, *magnetism*, etc., but because the various properties commonly attributed to the ether are difficult to conceive as coexisting in the same substance; and also because no two scientists quite agree as to the list of properties which would constitute a self-consistent and satisfactory definition of the substance which the word *ether* is supposed to represent. Prof. Haeckel, in his *Riddle of the Universe*, speaks of the ether as "imponderable matter," which (if ponderability is the one quality by which we distinguish matter from other entities) is a clear contradiction in terms. We may consistently call the ether an imponderable *substance*, or an imponderable *entity*, and conceive of it as occupying a portion (finite or infinite) of abstract space, but we cannot consistently speak of it as imponderable *matter*.

Pushed to their extreme logical limit, the arguments against the spatial existence of the soul—of the entity that feels—would be equally valid against the spatial existence of the ether, or even of matter itself. If matter or the ether be space-occupying *causes* or *transmitters* of sensations, why should not the soul be similarly a space-occupying *receiver* of sensations? Why also should not one soul similarly transmit (more or less modified) the sensations received, or similar sensations to another soul, human, superhuman, or infrahuman? As to the exact volume of space (fixed or variable) occupied at any given moment by any individual soul, from an infinitesimal to an infinite, nobody has any data for asserting; but then nobody has any absolutely sure data for making a similar assertion about the space or volume occupied at any given moment by any individual material body, or even by the whole material universe. Suppose the volume or space, compared with some imaginary fixed and constant unit of reference, occupied by the whole material universe at this moment were rapidly diminishing, but that all our units of distance, area, volume, time, and forces were also changing in corresponding proportions. We should for ever, generation after generation, remain ignorant of the appalling circumstance. When our mile had become an inch we should still call it a mile, and our new inch would have the same ratio to the new mile as our old inch had to the old mile. When our year had become a second, our new second would be reduced in proportion, and so would the

duration of our thoughts, actions and lives. When our bodies had become microscopic, in proportion to their present size, we should still consider them of the same size, weight, and volume as we consider them now; and, relatively to the respective new units of size, weight, and volume, the same numbers and fractions would represent them. Yet, when our material universe had thus shrunk (as regards volume) to an atom or an electron, with our stars, our planets, and ourselves, all inside it as at present, and at the same relative distances, we should still be logically obliged to consider it as real as we consider it now, and, for exactly the same reasons. On the other hand, the other mathematical, abstract, empty, soulless spatial universe of nothingness beyond would, as now, have any dimensions we chose to assign to it, subject, as now, to the condition that (solely for the convenience of symbolic reasoning and calculation) we should assume those dimensions to be infinite.

But, it may be asked, if the whole material and ethereal universe, and *a fortiori* our own bodies, might thus eventually become infinitesimal in comparison with our present units without our ever suspecting it, does not this reasoning arrive at virtually the same conclusion as that of the idealists, who maintain that matter, space, and time are mere conceptions of the mind, and that, except as mental conceptions, they have no real existence? To this I can only reply that as regards the empty, abstract, airless, etherless space of the mathematician, the conclusion seems to me correct, and that, since time, even as a concept, cannot well exist apart from space, the conclusion may be correct as regards time also; but I cannot quite see how we can consistently speak of *matter* as a mere concept. The question of size is wholly irrelevant. An ant, or even a microbe, is every whit as real as a whale or an elephant. We must define our words so as, if possible, to avoid self-contradiction, and if matter be defined as a *cause* or *transmitter* of a certain defined class of sensations, this cause or transmitter must (like the soul or *receiver* of sensations) be as real as the sensations themselves. This follows from three fundamental assumptions with which neither science nor logic can well dispense without linguistic inconsistency. They are: first, that every effect must have a cause or combination of causes; secondly, that an effect can never be its own cause, nor a cause its own effect; and, thirdly, that if any effect be considered real, its cause or causes must be considered real also. The words cause and effect are here used in their customary scientific sense, without any implication, affirmative or negative, as to the real existence of one single *First Cause*.