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IV.—*On the Burning Mirrors of Archimedes, with some Propositions relating to the concentration of Light produced by Reflectors of different forms.* By JOHN SCOTT, Esq., Tain. (Plate III.)

(Read 6th January 1868).

As the reputed fact of ARCHIMEDES having burned the Roman ships engaged in the siege of Syracuse, by concentrating on them the solar rays, has not only been doubted but disbelieved by some of the most eminent scientific men, I shall briefly give the evidence on both sides.

The burning of the ships of MARCELLUS is mentioned by most of the ancient writers who refer to the machines which ARCHIMEDES employed in the defence of his native city, and their statements have been repeated by succeeding authors, without any doubts having been expressed until comparatively recent times. Our earliest authorities on the subject are DIODORUS SICULUS, LUCIAN, GALEN, DION, and PAPPUS. At a later period the architect ANTHEMIUS, of Tralles, in a fragment entitled *περι παραδοξων μηχανηματων* (Wonderful Machines), not only dwells particularly on the burning mirror of ARCHIMEDES, but adds, besides, that it was universally admitted in his time that ARCHIMEDES had destroyed the Roman fleet by means of burning mirrors. It is also mentioned by HERO, a writer on military engines—about the middle of the seventh century—and by EUSTATHIUS, ZONARES, and TZETZES, who flourished in the twelfth. The last two have transmitted to us passages extracted from the work of PAPPUS on the siege of Syracuse, which was then extant, but has since disappeared. We give that by TZETZES as the more definite and circumstantial :—“ When MARCELLUS had placed the ships a bow-shot off, the old man (ARCHIMEDES) contrived a hexagonal mirror. He placed at proper distances from the mirror smaller mirrors of the same kind, and which were moved by means of their hinges and certain square plates of metal. He placed it in the midst of the solar rays at noon both in summer and winter. The rays of the sun being reflected by this, a dreadful fire was excited on the ships, which reduced them to ashes at the distance of a bow-shot.”

In the sixteenth century mirrors similar to that of ARCHIMEDES seem to have engaged the attention of LEONHARD DIGGES, and of Baron NAPIER of Merchiston. At a subsequent period Father KERSHER took up the same subject, prosecuting it with such assiduity that he travelled to Sicily to examine the coast in the vicinity of Syracuse, and came to the conclusion that ARCHIMEDES might have opportunities of placing his mirror within 30 yards of the ships. He also mentions in his “*Magica Catoptrica*,” as the result of experiments which he had performed, that the superimposed rays reflected from five plane mirrors at the distance of

more than 100 feet, produced a heat which could scarcely be endured. Apparently convinced of the practicability of the achievement by means of plane mirrors, he entreates future mathematicians to prosecute the subject. BUFFON, following in his steps, completely established the fact that combustible materials can be set on fire at distances corresponding to the accounts we have of the mirror of ARCHIMEDES. This he effected by means of a combination of plane reflectors, consisting of ordinary looking-glasses, 8 inches by 6, attached to a single frame, each glass, as well as the supporting frame, being capable of motion in every direction. With forty of these glasses he set on fire tarred beech at a distance of 66 feet. A plank, smeared with tar and brimstone, was ignited at 126 feet by 98 glasses. A combination of 128, with a clear sun, inflamed very suddenly a plank of tarred fir at 150 feet, the conflagration springing up at once over a space of 16 inches in diameter—the whole reflected image of the sun at that distance. In addition to these experiments made about the beginning of April, others were exhibited with the summer sun, by which wood was kindled at 200 and 210 feet, and silver and other metals were melted at distances varying from 25 to 40 feet.

Let us now consider the evidence on the opposite side of the question. DESCARTES and others have treated the whole affair as fabulous, from the belief that the burning glass must have consisted of a single spherical or parabolic reflector. But since no mention is made of the kind of specula employed by ARCHIMEDES, such objections, after the successful experiments of BUFFON, necessarily become irrelevant. Another and quite different ground of doubt has arisen from the circumstance, that POLYBIUS, LIVY, and PLUTARCH make no mention of the destruction of the Roman fleet by means of burning glasses, although they describe somewhat in detail the ballistæ and other military engines constructed by ARCHIMEDES to resist the assailants. How far the silence of the forementioned writers should be taken as negative evidence, it is not easy to determine. It seems to indicate, at least, that the damage effected by the burning mirror was confined to comparatively narrow limits; for, if prominent in the defence, we naturally expect that it would have received special notice. But allowing that the fleet sustained no serious injury from this source, some fact must be left sufficient to account for the belief prevalent among ancient authors in favour of the achievement. Nothing less, I conceive, will suffice than to admit that ARCHIMEDES made an attempt to destroy the Roman fleet in the manner described. Such an admission, however, implies that he must have previously tried his combination of reflectors in private, and was able to ignite combustible substances at considerable distances. The mere attempt to bring an artillery so singular and subtle to bear on the fleet is in itself a conclusive proof that an experiment, similar to those exhibited by BUFFON in the Jardin-du-Roi at Paris, had been successfully performed 2000 years before within the walls of Syracuse. We can no more suppose the contrary than believe that any of the nations of modern Europe would send

into the field of actual warfare a novel piece of ordnance without subjecting it to a previous trial. Though on this hypothesis the element of success as an engine of war is questionable, the invention of the mirror as a fact in the history of science remains entire. Neither should it be forgotten, as perhaps a reason for the silence of POLYBIUS, LIVY, and PLUTARCH, that with the fall of Syracuse and the death of the illustrious inventor, all definite information relating to the scientific principles of the mirror seems to have perished—a result not improbable, when we consider that its application to the art of war would induce the original possessors to retain its construction, as far as possible, a secret in their own hands. Finally, it is an admitted axiom in estimating historical evidence, that the silence of one author respecting an event is never considered sufficient to invalidate a plain and consistent statement of that event made by another. That our conclusions should be formed in strict accordance with the principle enunciated, may be made apparent by striking and well-ascertained facts, some of which have been inaccurately recorded, and others altogether omitted by the most reliable contemporary historians. As an instance of the former, modern authorities maintain that the account given by LIVY of the route by which HANNIBAL conducted his army across the Alps cannot be reconciled with that by POLYBIUS, and an extract from Sir CHARLES LYELL'S "Principles of Geology" will show an historical omission equally inexplicable. Speaking of the first eruption of Vesuvius, he says, "The younger PLINY, although giving a substantial detail of so many physical facts, and describing the eruption and earthquake and the shower of ashes which fell at Stabiæ, makes no allusion to the sudden overwhelming of two large and populous cities, Herculaneum and Pompeii. In explanation of this omission, it has been suggested that his chief object was simply to give TACITUS a full account of the particulars of his uncle's death. It is worthy of remark, however, that had the buried cities never been discovered, the accounts transmitted to us of their tragical end might well have been discredited by the majority, so vague and general are the narratives, or so long subsequent to the event. TACITUS, the friend and contemporary of PLINY, when adverting in general terms to the convulsions, says merely cities were consumed or buried. . . . Suetonius, although he alludes to the eruption incidentally, is silent as to the cities. They are mentioned by MARTIAL in an epigram as buried in cinders; but the first historian who alludes to them by name is DION CASSIUS, who flourished about a century and a half after PLINY."

Returning to BUFFON'S combination of reflectors—when the focus had to be changed, a numerous staff of assistants required about half an hour to re-adjust the mirrors. After all, the superposition of the reflected light would be imperfect, each operator being liable to mistake the deviation of the image reflected by some other glass for that of which he had charge. PEYRARD has appended to his translation of the works of ARCHIMEDES a memoir of his own, in which he calls atten-

tion to these defects, and to remedy them proposes that a telescope and somewhat complex apparatus be attached to each reflector, that with fewer hands the adjustment might be rendered more accurate and speedy.

It is obvious that the improvement suggested by PEYRARD is only partial and its success doubtful. I conceive the Archimedean mirror to have been a compound reflecting apparatus, free from these defects, capable of being directed by one eye and guided by a single hand. From TZETZES we learn that the mirror was hexagonal, that like BUFFON's it consisted of a combination of reflectors, and that at proper distances from the outer mirror were placed other smaller ones of the same kind. This last peculiarity of the Archimedean mirror has no parallel in that of BUFFON; and yet the arrangement of the smaller mirrors, at proper distances from the larger, indicates that the relation of the two kinds to each other formed an essential feature of the combination. Although the above passage conveys no information respecting the nature of the specula, there is such a description of the connection of the parts as an intelligent observer might carry away, and yet be unacquainted with the scientific principles involved in the construction.

In the sequel (Arts 13 and 14) we show how larger and smaller specula, all of the same kind, can be so connected as to form a single compound reflector capable of concentrating on a single spot the reflected rays, and of darting them instantaneously in any direction, when they will produce the effects ascribed to the mirror of ARCHIMEDES. The results thus being the same, and the construction of the combination coinciding with the description given by TZETZES, we therefore infer that the real principle of the Archimedean mirror has been attained, and that the accounts which have come down to us respecting it are in the main authentic. This will be brought out more fully after the following general propositions have been considered.

For the historical facts contained in the preceding, I am chiefly indebted to PEYRARD's edition of the "Works of Archimedes," and to the article "Burning Glasses," in the *Encyclopædia Britannica*.

ARTICLE I.—Prop. *When the Light emanating from a Luminous Sphere of small angular diameter falls on a very small Plane Mirror; to find the Intensity of the reflected Light at any distance from the Mirror.*

Let GH (fig. 1, Plate III.) be the mirror, B the point of reflection, AC the small luminous sphere, and DE the plane on which the reflected light falls. It follows, as a consequence of the equality of the angles of incidence and reflection, that the angle which DE subtends at B is equal to that which AC subtends, DE being supposed perpendicular to the cone of rays reflected from B.

Whatever may be the form of the infinitesimal mirror at B, the perpendicular section of the reflected rays, at all finite distances from B, is a circle.

Let α = the angle ABC = DBE.

r = radius of the sphere AC.

c = the distance from B, measured along the slant side of the cone of rays, at which the diameter of the circle of reflected light is equal to unity.

I = the intensity of the light radiating from the sphere at its surface.

I' = intensity of the light at the mirror, on a plane perpendicular to the axis of the cone of rays falling on B.

$\therefore kI'$ = intensity of the reflected light at the mirror, k being a constant less than unity.

d = distance of the centre of the sphere from B.

d' = distance of DE from B, measured along the slant side of the cone of reflected rays.

δA = sectional area of the light incident on small mirror at B.

$$\text{Area of the circle DE} = \pi d'^2 \sin^2 \frac{\alpha}{2} \quad . \quad . \quad . \quad . \quad (1),$$

$$,, \quad = \frac{\pi d'^2}{4c^2} \quad . \quad . \quad . \quad . \quad (2),$$

because $c : d' :: \frac{1}{2} : \text{radius of the circle DE}$.

Since the intensity of the light, emanating from the surface of a luminous sphere and falling on a concentric spherical surface, is inversely as the square of the distance from its centre,

$$r^2 : d^2 :: I' : I \quad \therefore I = \frac{d^2 I'}{r^2} \quad . \quad . \quad . \quad . \quad (3).$$

$$\text{But} \quad \sin \frac{\alpha}{2} = \frac{r}{d} = \frac{1}{2c} \quad \therefore I = \frac{I'}{\sin^2 \frac{\alpha}{2}} \quad . \quad . \quad . \quad . \quad (4)$$

$$= 4c^2 I' \quad . \quad . \quad . \quad . \quad (5).$$

$$\text{Intensity of the reflected light at DE} = \frac{kI' \delta A}{\text{area of circle DE}} \quad . \quad . \quad . \quad . \quad (6),$$

$$,, \quad = \frac{kI' \delta A}{\pi d'^2 \sin^2 \frac{\alpha}{2}} \quad . \quad . \quad . \quad . \quad (7),$$

$$,, \quad = \frac{4c^2 kI' \delta A}{\pi d'^2} \quad . \quad . \quad . \quad . \quad (8).$$

When the plane on which the reflected light falls is not perpendicular to the

axis of the cone of rays, as de (fig. 1), the intensity of the light at e will exceed the intensity at d from the greater obliquity of the rays at the latter point. But if the angle DBE be small, and the plane de cut the axis of the cone at a considerable angle, the intensity will be nearly uniform over the whole ellipse de ; and the centre of the ellipse may be viewed as situated in the axis of the cone, because the elongated cone approaches nearly to a cylinder.

$$\therefore \text{intensity on } de = \frac{kI' \delta A}{\text{area of ellipse } DE} \quad (9).$$

In the case of the sun's light the above formulæ will give pretty accurate approximations, since $\alpha = 32'$ and $c = 107.4$ nearly (10).

ARTICLE 2.—Prop. *When a Cylindrical Beam of Solar Light is reflected from a Plane Mirror; to find the Intensity on a Plane Surface perpendicular to the direction of the reflected Beam, and at any distance from the Mirror.*

Since the rays which emanate from any single point in the sun's disc may be considered as perfectly parallel, however large the mirror, it follows that those from the centre of the disc will, after reflection from the mirror, form a perfectly cylindrical beam of parallel rays, and will cast on the given plane a circle of light of uniform intensity.

Let ABG (fig. 2) represent this circle, each point in its area is the centre of a circular image of the sun's disc reflected from a corresponding point, or infinitesimal area, of the mirror.

If O be one of these points, and the circle $ADBP$ the image of the sun's disc at the given distance, or, in other words, the base of a cone of rays whose apex is at the mirror, as shown in fig. 1, Art. 1, the illumination or intensity of the light at O will be that due to the superposition of all the images of the sun's disc whose centres fall within the area $ACBPA$; for none of the images of the sun's disc, whose centres fall without the above area, can extend so far as O . The point O is therefore illuminated by a portion of the incident beam equal in area to $ACBPA$, every increment of which, after reflection, gives rise to a conical pencil of rays, a part of whose base overspreads the point O .

Let I' = intensity of the solar light on a plane perpendicular to the direction of the incident beam.

Let $AO = r$, $FA = \rho$, $FO = z$, $AFO = \theta$, $AOC = \phi$, and $u_1 u_2 u_3 u_4 u_5$, &c., the sectional areas, at the mirror, of the respective pencils whose light overspreads the point O . Whatever may be the form of these small increments, the base of the cone of light to which they give rise will be a circle at all finite distances from the mirror, as shown Art. 1.

Therefore the intensity of the solar light at O from area $u_1 = \frac{kI'u_1}{\text{circle ADBP}} = \frac{kI'u_1}{\pi r^2}$,

$$u_2 = \frac{k\Gamma' u_2}{\pi r^2},$$

$$u_3 = \frac{k\Gamma' u_3}{\sigma r^2},$$

$$u_n = \frac{kI'u_n}{\pi n^2}.$$

$$\begin{aligned} \text{Whole intensity at O} &= \frac{kI'}{\pi r^2} (u_1 + u_2 + u_3 + \dots + u_n); \\ &= \frac{kI' \times \text{area ACBPA}}{\pi r^2} \end{aligned} \quad (1),$$

$$= \frac{kI'}{\pi r^2} (2 \times \text{sector AFC} - 2 \text{ triangle AOF} + 2 \times \text{sector AOP}),$$

$$= \frac{k\Gamma'}{\pi r^2} \left\{ \varrho^2 \theta + r^2 (\pi - \varphi) - \varrho z \sin \theta \right\} \quad (2),$$

$$= \frac{k\Gamma'}{\pi r^2} \left\{ \varrho^2 \theta + r^2 \left(\pi - \sin^{-1} \left(\frac{\varrho \sin \theta}{r} \right) \right) - \varrho z \sin \theta \right\},$$

because $\sin \varphi = \frac{\rho \sin \theta}{r}$,

also

$$\begin{aligned} z &= \varrho \cos \theta - r \cos \varphi, \\ &= \varrho \cos \theta - \sqrt{(r^2 - \varrho^2 \sin^2 \theta)}. \end{aligned}$$

When $\rho > r$ in the preceding expressions, and $\theta = 0$, then $\phi = 0$, $z = \rho - r$, and intensity at O = $\frac{kI'}{\pi r^2} \times \pi r^2 = kI'$ (5).

If, therefore, the circle ADPB falls wholly within the circle ABG, the intensity of the illumination on the circular space, whose centre is F and radius $\rho - r$, is constant and equal to that of the beam when it leaves the mirror, or the intensity is the same as if all the rays from the sun's disc were parallel to one another.

When $\rho = r$, then, $z = 0$, and F is the only point whose intensity $= kI'$.

Next when $\rho < r$ and $\theta = \pi$, then $\phi = \pi, z = r - \rho$, and intensity $= \frac{kI'}{\pi r^2} \times \rho \pi^2$
 $= \frac{kI' \rho^2}{r^2}$. (6).

This formula expresses the intensity throughout the circle, whose radius is $r - \rho$.

When $z = r + \rho$, the intensity vanishes.

The preceding limiting cases are also evident from the geometrical consideration of the problem taken in connection with equation 2; and the same principles can be applied to find the intensity, whatever the shape of the perpendicular section of the incident beam.

ARTICLE 3.—When there are n mirrors similar to the preceding, the light from each making an angle of incidence i , with the perpendicular to the plane on which it is thrown, the intensity of the central spot in each of the preceding cases becomes—

$$\text{In 1st case, } nkI' \cos i, \quad . \quad . \quad . \quad . \quad (1).$$

$$\text{In 2d case, } \frac{nkI' g^2 \cos i}{r^2}, \quad . \quad . \quad . \quad . \quad (2).$$

ARTICLE 4.—Prop. *A Small Luminous Sphere has its centre in one of the foci of a Prolate Elliptic Mirror, to find the Intensity on any Small Plane surface situated in the other Focus.*

Let f (fig. 3) represent the luminous sphere.

β = the angle which the small plane at F makes with FX: the axis of z coinciding with the axis of the mirror, and the plane of xz being perpendicular to the small plane passing through F; the axis of y , which is at right angles to the plane of xz , will therefore coincide with the small plane which passes through F.

r = FP.

θ = the angle PFZ.

ϕ = the angle which the projection of r on the plane of xy make with FX.

a = radius of the luminous sphere at f .

V = the angle which r makes with the normal to the small plane at F.

I = the intensity of the light at the surface of the small sphere.

Since the well-known differential of a volume $r^2 \sin \theta d\theta d\phi dr$ has for its perpendicular section, at the surface of the spheroid, $r^2 \sin \theta d\theta d\phi$; we may suppose the whole surface of the elliptic mirror to be divided into small areas, each of which receives from the sphere at f , and reflects to F, a pencil of light, whose perpendicular section, at any point P of the mirror, is $r^2 \sin \theta d\theta d\phi$.

Moreover, the intensity at P perpendicular to $Pf = \frac{a^2 I}{Pf^2}$ (by Art. 1, Equa. 3), and after reflection it becomes $\frac{a^2 k I}{Pf^2}$.

The cone of rays reflected from the increment of surface at P, in the direction PF, will have expanded at F into a circle (perpendicular to the radius vector), whose radius by similar triangles is $\frac{ar}{Pf}$ and area $\frac{\pi a^2 r^2}{Pf^2}$.

By Art. 1. Equa. 6, the intensity of the light on this circle at F = $r^2 \sin \theta d\theta d\phi \times \frac{a^2 k I}{Pf^2} \div \frac{\pi a^2 r^2}{Pf^2} = \frac{k I}{\pi} \sin \theta d\theta d\phi$.

and $\frac{kI}{\pi} \sin \theta \, d\theta \, d\phi \cos V =$ its intensity at F on the small plane, whose equation is $z \cos \beta - x \sin \beta = 0$,

$$\cos V = \frac{z}{r} \cos \beta - \frac{x}{r} \sin \beta = \cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi. \quad (1),$$

Substituting for $\cos V$ its value we obtain for the whole intensity at F, on the small plane,

$$\frac{kI}{\pi} \iint (\cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi) \sin \theta \, d\theta \, d\phi, \quad (2).$$

Denoting this integral by u ,

$$u = \frac{kI}{2\pi} \int (\cos \beta \sin^2 \theta + \sin \beta \sin \theta \cos \theta \cos \phi - \theta \sin \beta \cos \phi) \, d\phi + C.$$

Now $\cot \theta = \tan \beta \cos \phi$, when $V = 90^\circ$, that is, when PF coincides with the plane denoted by Equation 1,

$$\begin{aligned} \therefore \theta &= \cot^{-1}(\tan \beta \cos \phi) \\ \sin \theta &= \frac{1}{(1 + \tan^2 \beta \cos^2 \phi)^{\frac{1}{2}}} \\ \cos \theta &= \frac{\tan \beta \cos \phi}{(1 + \tan^2 \beta \cos^2 \phi)^{\frac{1}{2}}} \end{aligned}$$

Taking the integral between the limits $\theta = 0$, and $\theta = \cot^{-1}(\tan \beta \cos \phi)$,

$$\begin{aligned} u &= \frac{kI}{2\pi} \int \left\{ \frac{\cos \beta (1 + \tan^2 \beta \cos^2 \phi)}{1 + \tan^2 \beta \cos^2 \phi} - \cot^{-1}(\tan \beta \cos \phi) \sin \beta \cos \phi \right\} d\phi. \\ &= \frac{kI}{2\pi} \left\{ \int \cos \beta \, d\phi - \sin \beta \sin \phi \cot^{-1}(\tan \beta \cos \phi) + \int \frac{\sin \beta \tan \beta \sin^2 \phi \, d\phi}{1 + \tan^2 \beta \cos^2 \phi} \right\} \\ &= \frac{kI}{2\pi} \left\{ \int \cos \beta \, d\phi - \sin \beta \sin \phi \cot^{-1}(\tan \beta \cos \phi) - \int \cos \beta \, d\phi + \int \frac{\sec \beta \cdot d(\tan \phi)}{\sec^2 \beta + \tan^2 \phi} \right\} \\ &= \frac{kI}{2\pi} \left\{ \tan^{-1}(\cos \beta \tan \phi) - \sin \beta \sin \phi \cot^{-1}(\tan \beta \cos \phi) \right\} + C. \end{aligned}$$

and between the limits $\phi = 0$ and $\phi = 90^\circ$,

$$u = \frac{kI}{2\pi} \left(\frac{\pi}{2} - \frac{\pi}{2} \sin \beta \right),$$

$= \frac{kI}{4} (1 - \sin \beta)$, which is the intensity at F of the light reflected from that portion of the spheroid bounded by the planes passing through the co-ordinate axes of $+x$ and $+z$, $+y$ and $+z$, and the small plane produced.

Similarly $\frac{kI}{4}(1 + \sin \beta)$ gives the intensity of the light, from that portion enclosed by the planes passing through the axes $-x$ and $+z$, $+y$ and $+z$, and the small plane at F produced. Hence the intensity of all the light which can fall on the side of the small plane towards A (fig. 3).

$$\begin{aligned} &= 2 \left\{ \frac{kI}{4} (1 - \sin \beta) + \frac{kI}{4} (1 + \sin \beta) \right\}, \\ &= kI, \end{aligned} \quad (3).$$

Also $\frac{kI}{2} \sin \beta$ is the intensity at F due to the light reflected from the part of the mirror intercepted between the small plane produced, and the co-ordinate plane of xy .

In the same manner it can be shown, that the intensity of the light concentrated at F, on the opposite side of the small surface, and reflected from the remaining portion of the spheroid, is also equal to kI .

These results are independent of α , the radius of the luminous sphere, and are equally true for all spheroids which have F for one of their foci, wherever the other may be situated.

It appears then, in conclusion, that the light emanating from a small luminous sphere, with its centre in one of the foci of a prolate elliptic mirror, produces at the other focus a nucleus of radiant light and heat, equal in intensity to the radiation at the luminous surface diminished by the quantity lost by reflection.

Again, putting $\beta = 0$ (in Equation 2), we obtain intensity

$$= \frac{kI}{\pi} \iint \sin \theta \cos \theta \, d\theta \, d\phi, \quad (4).$$

Integrating between

$$\phi = 0, \text{ and } 2\pi;$$

intensity

$$= 2\pi \times \frac{kI}{\pi} \int \sin \theta \cos \theta \, d\theta,$$

$$= 2kI \int \sin \theta \cos \theta \, d\theta;$$

and between θ° , and zero,

$$= kI \sin^2 \theta, \quad (5).$$

This expresses the intensity at the focus of the light reflected from a segment of the spheroid intercepted between the vertex and a plane perpendicular to the axis; and the intensity produced by a zone intercepted between two planes, perpendicular to the axis of revolution, is

$$kI (\sin^2 \theta - \sin^2 \theta') \quad (6).$$

ARTICLE 5.—The preceding proposition is true, independently of the size and form of the luminous body in the focus f (figs. 3 and 4).

Since radiant light and heat diminish in the inverse ratio of the square of the distance, it follows that the quantities received from circular areas of equal angular magnitudes are equal, whatever their absolute magnitudes, when the intensities at the radiating surfaces are equal. Taking this principle in connection with the fact, that a luminous surface appears equally bright when viewed at any angle, the light emanating from CD, part of the surface of DCE, will therefore have at F the same intensity as if it had proceeded from the small sphere AB (fig. 4). But the light reflected from P to F can only emanate from some part of the surface DC, which lies within the cone described by PA, revolving about Pf. Hence the

intensity at F of the light radiating from DC, and reflected at P, is equal to that which would have resulted from the sphere AB; and the same is true for every increment of the elliptic mirror. Therefore, the total illumination at F, from the luminous surface DCE, is equivalent to that from the small sphere AB.

From this and the preceding article, we infer that a zone or segment of an elliptic reflector may be used as a pyrometer. For if such a zone, contained between two planes perpendicular to the axis, be placed before an opening in a furnace, the place of the focus f falling within the heated body, the heat reflected to F may be reduced, by diminishing the breadth of the zone, until it can be measured by a Fahrenheit thermometer; and I, the intensity of the total radiation from any point f within the furnace, can be determined in degrees of Fahrenheit by Art. 4, Equa. 6.

ARTICLE 6.—Prop. *When a Parabolic Reflector has its axis directed to the centre of the Sun, to find the intensity of the converging Rays which fall on a small Plane Disc at the Focus.*

Let α = angular diameter of the sun, which is about $32'$,

c = the distance at which the reflected image of the sun expands into a circle equal to unity in diameter, being about 107.4 ,

r = PF (fig. 5),

I' = intensity of the sun's rays at the earth's surface,

I = intensity at the surface of the sun,

$u, k, \beta, \theta, \phi$, and V = the same as in proposition (Art. 4).

Then $\frac{\pi r^2}{4c^2}$ = area of the circle, which the light reflected at P occupies at F perpendicular to PF.

The intensity of the light reflected from the increment of surface at P on this circle by Art. 1, Equa. 6.

$$\begin{aligned} &= r^2 \sin \theta \, d\theta \, d\phi \times kI' \div \frac{\pi r^2}{4c^2}, \\ &= \frac{4c^2 kI'}{\pi} \sin \theta \, d\theta \, d\phi; \end{aligned}$$

and on the small disc at F, $= \frac{4c^2 kI'}{\pi} \sin \theta \, d\theta \, d\phi \cos V$.

$$\cos V = \cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi,$$

$$\therefore u = \frac{4c^2 kI'}{\pi} \iint (\cos \beta \cos \theta - \sin \beta \sin \theta \cos \phi) \sin \theta \, d\theta \, d\phi, \quad (1).$$

Integrating as in Article 4, we obtain for the intensity of the light reflected from the corresponding sections of the parabolic mirror

$$c^2 kI' (1 - \sin \beta), \quad . \quad . \quad . \quad . \quad (2).$$

$$\text{and} \quad c^2 kI' (1 + \sin \beta), \quad . \quad . \quad . \quad . \quad (3).$$

Hence the total intensity at F (fig. 5) on the side of the plane disc towards

A, resulting from the light reflected from the segment of the paraboloid cut off by the plane of the disc produced,

$$= 2\{c^2kI'(1 - \sin \beta) + c^2kI'(1 + \sin \beta)\} = 4c^2kI' = kI \quad (4),$$

I being the intensity at surface of the sun.

But $c = 107$ nearly, therefore the numerical value of this equation

$$= 45796kI'$$

nearly, which is a degree of concentration several times that of the most powerful burning glass ever constructed.

Again putting $\beta = 0$ (in Equation 1),

$$u = \frac{4c^2kI'}{\pi} \iint \sin \theta \cos \theta d\theta d\phi;$$

and integrating as in Article 4, Equation 4,

$$u = 4c^2kI' \sin^2 \theta; \quad (5),$$

which gives the intensity at the focus of the light reflected from a segment of a paraboloid, intercepted between the vertex and a plane perpendicular to the axis; and the intensity produced by a zone, intercepted between two planes perpendicular to the axis, is

$$4c^2kI'(\sin^2 \theta - \sin^2 \theta') = kI(\sin^2 \theta - \sin^2 \theta') \quad (6).$$

Equation 5 shows that the concentration at the focus varies as $\sin^2 \theta$: it is a maximum when $\theta = 90^\circ$, and is independent of the parameter of the parabola. It may therefore be inferred that a reflector employed to detect the heat of the lunar rays should be as large a segment of a paraboloid as possible; and the same condition is essential in improving to its utmost limit the space-penetrating power of the reflecting telescope.

Again, suppose the parabolic mirror to extend to infinity, it can also be shown that the light concentrated at the focus on the other side of the small disc is equal to $4c^2kI' = kI$. What has been proved respecting the intensity at the focus is approximately true for every point on the plane of the small disc not farther from F than $\frac{p}{8c}$, the quantity $\frac{p}{8c}$ being the radius of the sun's image reflected from the vertex of the paraboloid, and p the parameter of the generating parabola. Thus, in every position in space, when the axis of a parabolic mirror, whose extent of surface is not less than that cut off by a plane passing through the focus, is directed to the sun, a circle of radiant light and heat is formed equal in intensity to the radiation at the solar surface minus the quantity lost by reflection.

ARTICLE 7.—The intensity at the focus of a parabolic reflector is independent both of the form and distance of the luminous body.

Let BAC (fig. 6) represent a section of the parabolic reflector, and GH that of a luminous surface of uniform intensity; it can be shown, as in the case of the spheroid (Art. 5), that the concentration at F produced by the only rays which can fall on it, namely, those emanating from GH parallel to the axis AF, is equal to the intensity at the luminous surface GH, minus the quantity lost by reflection. It is evident that the section of the luminous body must not be less than CB.

ARTICLE 8.—Prop. *When the Axis of a Mirror in the form of a Right Cone is directed to the centre of the Sun, to find the Intensity of the reflected Light on any point in a Plane placed perpendicular to its Axis.*

Let CAD (fig. 7) represent a section of the mirror,

O the point on which we wish to determine the intensity of the reflected light.

Every small increment of the mirror gives rise to a cone of rays which casts an ellipse of light on the plane at F, the major axis of which passes through the point F. The light of all these ellipses, whose centres fall within a certain distance of the point O, will overspread it and increase the intensity at that point.

If P (fig. 8) be the centre of one of these ellipses NOM, considerably magnified, whose circumference passes through O, then P is a point in the curve within which must fall the centres of all the ellipses whose light can overspread O.

To find the equation to this curve,

let $FP = \rho$

$FO = z$

α and β = the co-ordinates of P, referred to rectangular axes whose origin is at O,

θ = the angle PFO.

Now $a^2y^2 + b^2x^2 = a^2b^2$ is the equation to the ellipse NOM, the centre being the origin.

When referred to the axes OX and OY; by substituting,

$$y = (y' - \beta) \cos \theta - (x' - \alpha) \sin \theta,$$

$$x = (y' - \beta) \sin \theta + (x' - \alpha) \cos \theta,$$

$$\tan \theta = \frac{\beta}{z + \alpha}, \text{ we obtain}$$

$$(\alpha^2 \cos^2 \theta + b^2 \sin^2 \theta)(y' - \beta)^2 + (\alpha^2 \sin^2 \theta + b^2 \cos^2 \theta)(x' - \alpha)^2 - 2(\alpha^2 - b^2)(y' - \beta)(x' - \alpha) \sin \theta \cos \theta = a^2 b^2.$$

Putting $x' = 0$, $y' = 0$, there results the equation to the required curve, which is

Let i = angle of incidence on the mirror,
 R = perpendicular distance of B from the axis,
 r = perpendicular distance of any point in the curve similar to HPG (fig. 9),
as represented by FQ,
 v = FA = FB, because BFA is an isosceles triangle (fig. 7),
 u = intensity at O,
 I' = intensity of the sun's light at the earth's surface;
then $2i$ = angle BFA,
 $90^\circ - i$ = angle BAF,
 $a = b \sec 2i$,
 $\pi ab = \pi b^2 \sec 2i$ = area of ellipse, semi-axes a and b ,
 z = FO, in this case less than b .

Since the rays reflected from C and D (fig. 7) fall upon the same point O, the circumference of the circle described by O in its revolution about the axis is illuminated by the light reflected from the two annuli described by C and D. Besides the point O is situated so near the axis, that the perpendicular distances of C and D from the axis may be considered as equal to one another. Thus, to find the concentration which results from the converging of the rays to the axis, we have,

$$2\pi r : 4\pi R :: I' : \text{intensity at distance } r \text{ from F, } \left(= \frac{2RI'}{r} \right),$$

the intensities being estimated on planes perpendicular to the rays. Wherefore the intensity on the plane at F of the reflected light

$$= k \times \frac{2RI'}{r} \times \cos 2i = \frac{2kRI' \cos 2i}{r},$$

which would give the intensity on the increment at Q (figs. 9 and 10), if the sun's rays were perfectly parallel. But instead of this light being confined to the increment at Q, it is spread over an ellipse whose area = $\pi b^2 \sec 2i$, and hence the intensity at O due to this increment

$$= \frac{2kRI' \cos 2i}{r} \times \frac{r dr d\theta}{\pi b^2 \sec 2i} = \frac{2kRI' \cos^2 2i}{\pi b^2} dr d\theta;$$

and the same is true for a corresponding increment on the other side of the axis GH (figs. 9 and 10),

$$\begin{aligned} \therefore u &= \iint 2 \times \frac{2kRI' \cos 2i}{r} \times \frac{r dr d\theta}{\pi b^2 \sec 2i} \\ &= \frac{4kRI' \cos^2 2i}{\pi b^2} \iint dr d\theta \end{aligned} \quad (7).$$

This, taken between the limits $r = \rho$, and $r = 0$; and $\theta = \pi$, and $\theta = 0$, gives the total intensity

$$u = \frac{4k RI' \cos^2 2i}{\pi b^2} \int_0^\pi \rho^2 d\theta.$$

Now, from Equa. 3, $\rho = z \cos \theta + a \left(1 - \frac{z^2}{b^2} \sin^2 \theta\right)^{\frac{1}{2}}$, z being less than b ,

$$\therefore u = \frac{4k RI' \cos^2 2i}{\pi b^2} \int_0^\pi \left\{ a \left(1 - \frac{z^2}{b^2} \sin^2 \theta\right)^{\frac{1}{2}} + z \cos \theta \right\} d\theta.$$

Expressed in terms of elliptic functions,

$$\begin{aligned} u &= \frac{4k RI' \cos^2 2i}{\pi b^2} \left[a E_{\frac{z}{b}}(\theta) + z \sin \theta \right] + C. \\ &= \frac{4k RI' \cos^2 2i}{\pi b^2} a E_{\frac{z}{b}}(\pi) \end{aligned} \quad (8).$$

But by Art. 1, $R \operatorname{cosec} 2i : c :: b : \frac{1}{2}$, ($R \operatorname{cosec} 2i$ being = FB, fig. 7),

$$\therefore b = \frac{R \operatorname{cosec} 2i}{2c},$$

$$= \frac{R}{2c \sin 2i},$$

and

$$a = b \sec 2i = \frac{b}{\cos 2i}.$$

Substituting in Equation 8, we get

$$\begin{aligned} u &= \frac{8ck I' \sin 2i \cos 2i}{\pi} E_{\frac{z}{b}}(\pi), \\ &= \frac{4ck I' \sin 4i}{\pi} E_{\frac{z}{b}}(\pi) \end{aligned} \quad (9).$$

An expression which may be put in the following form :—If a circle be described with F as a centre (figs. 7, 8, or 9) and $2b$ as a diameter, and an ellipse with O for one of its foci and the same diameter as a major axis; the circumference of the circle will be to the circumference of the ellipse as the intensity at the axis to the intensity at O.

When $z = 0$, then $E_{\frac{z}{b}}(\pi) = \pi$

$$\text{and} \quad u = \frac{4ck I' \sin 4i}{\pi} \times \pi = 4ck I' \sin 4i, \quad (10),$$

which is the expression for the intensity at the axis, and shows that in the same conical mirror it is constant at every point in the axis; whereas, in conical mirrors

of different inclinations, there are two which produce a maximum effect at the axis, viz., when $4i = 90^\circ$ and 270° (in Equa. 10), that is, when BAF (fig. 7) $= 67\frac{1}{2}^\circ$ or $22\frac{1}{2}^\circ$.

When

$$z = b \text{ (Equa. 9), } E_z(\pi) = 2,$$

and

$$u = \frac{8ckI' \sin 4i}{\pi} \quad . \quad . \quad . \quad (11).$$

If $k = \frac{1}{2}$, the maximum intensity at the axis expressed numerically is

$$u = 4ckI' \sin 90 = 4 \times 107 \times \frac{1}{2} I' = 214 I'.$$

When $k = \frac{1}{2}$ and $4i = 2^\circ 36'$, it can be shown by Equa. 10 that $u = 10I'$ nearly, a heat sufficient to ignite wood and other combustibles. This can be effected at a distance of 130 feet with a segment of the reflector 18 inches broad, and having a mean diameter of 6 feet.

Case 2. When the point O is situated at a considerable distance from the axis, z being much greater than b , the distances CO and DO will now differ perceptibly from one another and from $FA = FB = v$ (fig. 7).

Let R and R' represent the distances of the points C and D respectively from the axis, then

$$R = fC \sin 2i = (v + z \cot 2i) \sin 2i,$$

and

$$CO = fC - fO = FA + Ff - fO = v + z \cot 2i - z \operatorname{cosec} 2i = v - z \tan i.$$

Similarly for the point D,

$$R' = (v - z \cot 2i) \sin 2i,$$

and $DO = v + z \tan i$.

The ellipses of light which overspread O reflected from each increment of the space around C have for their minor and major axes respectively,

$$b = \frac{v - z \tan i}{2c}, \quad a = \frac{v - z \tan i}{2c \cos 2i};$$

and the corresponding quantities with respect to the space around D are

$$b' = \frac{v + z \tan i}{2c}, \quad a' = \frac{v + z \tan i}{2c \cos 2i}.$$

It can be shown, as in *Case 1*, Equa. 7, that the concentration at O, due to the light reflected from the space around C, is

$$\begin{aligned} &= \frac{2k RI' \cos^2 2i}{\pi b^2} \iint dr d\theta, \text{ and taken between } \xi_1 \text{ and } \xi_2, \\ &= \frac{2k RI' \cos^2 2i}{\pi b^2} \int (\xi_1 - \xi_2) d\theta, \\ &= \frac{4k RI' a \cos^2 2i}{\pi b^2} \int (1 - \frac{z^2}{b^2} \sin^2 \theta)^{\frac{1}{2}} d\theta, \end{aligned}$$

because (fig. 10)

$$\xi_1 = FP = z \cos \theta + a (1 - \frac{z^2}{b^2} \sin^2 \theta)^{\frac{1}{2}},$$

$$\xi_2 = FP' = z \cos \theta - a (1 - \frac{z^2}{b^2} \sin^2 \theta)^{\frac{1}{2}},$$

$$\xi_1 - \xi_2 = 2 (1 - \frac{z^2}{b^2} \sin^2 \theta)^{\frac{1}{2}}.$$

Taking the integral between the required limits $\sin \theta = \frac{b}{z}$, and $\theta = 0$ we obtain

$$\frac{4k RI' a \cos^2 2i}{\pi b^2} (A - H)_{\frac{z}{b}} \quad . \quad . \quad . \quad (12),$$

$(A - H)_{\frac{z}{b}}$ denoting the difference between the asymptote and the infinite hyperbolic arc whose major axis is unity, and eccentricity $\frac{z}{b}$, a finite quantity, though A and H are severally infinite. But, if the distance from the centre to the focus be equal to unity, the transverse axis is $\frac{z}{b}$ and $(A - H)_{\frac{z}{b}} = \frac{z}{b} (A - H)$, which expressed in complete elliptic functions of the second order gives

$$(A - H)_{\frac{z}{b}} = \frac{z+b}{b} E_{e_1} - \frac{z}{b} E_e,$$

e being put for $\frac{b}{z}$, the relation between the moduli e and e_1 , being

$$e_1 = \frac{2\sqrt{e}}{1+e} = \frac{2\sqrt{bz}}{z+b};$$

and e' and e'_1 denoting the corresponding quantities when b' is put for b .

Substituting in Equation 12, we get for the light reflected from the space around C,

$$\frac{4k RI' a \cos^2 2i}{\pi b^2} \left\{ \frac{z+b}{b} E_{e_1} - \frac{z}{b} E_e \right\};$$

and similarly the light reflected from the space around D,

$$= \frac{4kR'I'a \cos^2 2i}{\pi b'^2} \left\{ \frac{z+b'}{b'} E_{e_1} - \frac{z}{b'} E_e \right\}.$$

Hence the total intensity at O is

$$u = \frac{4kR'I'a \cos^2 2i}{\pi b^2} \left\{ \frac{z+b}{b} E_{e_1} - \frac{z}{b} E_e \right\} + \frac{4kR'I'a' \cos^2 2i}{\pi b'^2} \left\{ \frac{z+b'}{b'} E_{e_1} - \frac{z}{b'} E_e \right\},$$

and substituting for $a, b, a', b', R,$ and R'

$$u = \frac{4c(v+z \cot 2i)kI' \sin 4i}{\pi(v-z \tan i)} \left\{ \frac{v+(2c-\tan i)z}{v-z \tan i} E_{e_1} - \frac{2cz}{v-z \tan i} E_e \right\} \\ + \frac{4c(v-z \cot 2i)kI' \sin 4i}{\pi(v+z \tan i)} \left\{ \frac{v+(2c+\tan i)z}{v+z \tan i} E_{e_1} - \frac{2cz}{v+z \tan i} E_e \right\}, \quad (13),$$

the values of the moduli in terms of $c, v, z,$ and i being

$$e_1 = \frac{2\sqrt{2cz(v-z \tan i)}}{v+(2c-\tan i)z}, \quad e = \frac{v-z \tan i}{2cz}, \quad e'_1 = \frac{2\sqrt{2cz(v+z \tan i)}}{v+(2c+\tan i)z} \text{ and } e' = \frac{v+z \tan i}{2cz}.$$

Since the value of u will not be altered by substituting for v and z any two quantities having the same ratio, it follows that the intensity of the reflected light is uniform along the line which joins A and O (fig. 7).

ARTICLE 9.—Corollary. The value of k , the fraction which expresses the relation between the intensities of the reflected and incident rays, may be found by means of a conical reflector, thus:—

Let R = distance from the axis of a small zone described by AB (fig. 11).

r = distance from the axis at which the reflected light or heat becomes, by convergence to its axis, equal in intensity to the incident.

Then $kI' : I' :: r : R,$

$$\therefore kRI' = rI', \text{ and } k = \frac{r}{R}.$$

This result, calculated on the assumption that all the rays emanating from the sun are parallel, will not deviate perceptibly from the truth, except when r is small compared with R .

The value of k may also be found by using the combination of n plane mirrors. By Article 3, Equation 2, intensity

$$= \frac{nkI'g^2 \cos i}{r^2},$$

but if the direct rays of the sun fall on the same plane with the light reflected by the combination, the intensity will be

$$\frac{nkI'\rho^2 \cos i}{r^2} + I'.$$

Bringing the combination nearer the plane on which the light is thrown, the intensity of the n mirrors can be made equal to the above equation by diminishing the value of r .

Hence
$$\frac{nkI'\rho^2 \cos i}{r'^2} = \frac{nkI'\rho^2 \cos i}{r^2} + I',$$

$$\therefore k = \frac{r^2 r'^2}{n \rho^2 (r^2 - r'^2) \cos i}.$$

As these results are independent of the absolute value of I' , the equality of temperature may be detected by a Fahrenheit thermometer, or any more delicate means of indicating equal temperature.

When k is accurately known, this combination, or the conical reflector, may obviously be used to ascertain the intensity of the solar beams at different hours of the day and different periods of the year, and will thereby furnish data for estimating accurately the heat or light absorbed by the atmosphere. The light lost by the solar rays in penetrating the atmosphere being known, the intensity of the radiation at different parts of the solar disc may be found by (Article 6, Equation 6), if a zone or segment of a parabolic reflector can be constructed having a focal length of 70 or 80 feet.

ARTICLE 10.—Prop. *When two Conical Mirrors have a common Axis, their Surfaces being either perpendicular or parallel, if the rays incident on the exterior Reflector parallel to the Axis meet after reflection the interior one, they will be again reflected parallel to the Axis in a beam of increased intensity.*

Let AB (figs. 12 and 13) be the common axis of two conical reflectors described by the revolution of the lines MN and CD about the axis AB, CD being either perpendicular or parallel to MN.

When AB (fig. 12) is directed to the centre of the sun, the rays which fall on the surface described by MN will make with it the angle

$$SKN = MKH = KHD = FHC = DCB,$$

because $DCB = SKN$, therefore FH is parallel to AB (Euclid, 1–27). The intensity of the finally reflected beam at H is to that incident at K as the perpendicular distance of K from AB is to the perpendicular distance of H from

AB. Since every cylindrical annulus of rays incident on the exterior mirror forms after reflection from the interior a cylindrical annulus of equal thickness, the intensities must be inversely as the mean radii, the loss from reflection being neglected, or, if taken into account, equal to k^2 times the preceding intensity nearly.

ARTICLE 11.—The preceding annulus of rays may be thrown upon a circular area whose diameter is equal to the breadth of the zone, which forms a section of the annulus. To effect this, we have only so to increase the angle DCB, that the rays may meet the axis at the required distance, as shown in fig. 14.

In like manner, the parallel rays, from any extent of reflecting surface may be thrown upon the area whose section is Ff (fig. 15) by constructing one or both of the conical mirrors of frustums having the required extent of curved surface and the requisite inclination, the same axis being common to all. But when the breadths of the annuli are small compared with the distance of the focus Ff from the reflectors, the diameter of the circular area mentioned must be increased by the diameter of the sun's image for that distance.

Cor. When the number of lines CD, DE, EG, &c. (fig. 15), is indefinitely increased, their lengths being diminished, CDEG becomes part of a parabola.

ARTICLE 12.—Prop. *If two Parabolic Reflectors have a common Focus, the Solar Rays which are made to converge by reflection from the exterior Mirror will again form a beam of parallel Rays by reflection from the surface of the interior one.*

Let MKN and DHC (fig. 16) be sections of two confocal parabolic reflectors of which AB is the axis of the exterior and CE that of the interior, f being their common focus.

When AB is directed to a point in the sun's disc, the rays which fall on the exterior mirror parallel to the axis AB, in converging to f , will meet the surface of the interior mirror, and be reflected parallel to its axis f CE, as indicated by the course of the rays SKHF. Thus the solar beam of light which falls on the exterior and larger mirror is again reflected into a beam of parallel rays, and the intensity of the final beam will be greater than that of the incident, as explained in the preceding article. Moreover, since the axis f E of the interior mirror may make any angle with AB the axis of the exterior, the final beam may be thrown in any direction.

ARTICLE 13.—Prop. *The Rays which converge to the focus of the exterior Parabolic Mirror may be thrown by a second reflection on a given circular space, by constructing the interior reflector in the following manner :—*

Let DH and H'D (fig. 17) be parts of parabolas, whose common focus is f , their axes being respectively fC parallel to RG , and fC' to QG .

By causing these to revolve about the line fA , a surface will be described, such that the rays converging to f and falling on DH, will be reflected parallel to fC , and fall on the plane FF'.

In like manner, the rays converging to f and falling on H'D, will be reflected parallel to fC' , and will intersect the axis of revolution at G , and fall upon the plane FF'.

If this surface be substituted for the inner reflector DHC (fig. 16), the rays reflected from the outer mirror whose axis is directed to the sun, when converging to f , will meet the inner reflector described by H'QDH, and be reflected (as indicated in fig. 17) so as to intersect its axis of revolution at G , and fall upon the plane FF'. And this is true, whatever angle fG makes with the axis of the exterior reflector.

Cor. When the number of parts in H'QDRH are indefinitely increased, and their lengths diminished, it evidently becomes the arc of a hyperbola whose foci are f and G .

ARTICLE 14. The convergence of the solar rays upon a given area can also be effected by combining a number of exterior reflectors, each with its corresponding interior, as indicated in fig. 16, the axes of all the exterior parabolic reflectors being directed to the centre of the sun's disc, while the axes of the interior are directed to the centre of the given spot, on which the light has to be cast. Neither is it necessary that the respective reflectors should be complete symmetrical paraboloids: the exterior may consist of a series of large plates, each forming a part of a paraboloid of revolution, with a corresponding plate cut from a less paraboloid for its inner reflector. If the axes of all the exterior plates be in the same straight line, such a combination may have a common focus, each interior having that diameter of its generating parabola, which passes through the centre of the plate directed to the spot on which the light is required to fall. The practicability of such a combination is evident, from fig. 16, where K may be viewed as the centre of one of the exterior plates, and H that of its corresponding interior, having the diameter of its generating parabola, which passes through H , directed to the plane on which the light is concentrated. The exterior plates may be joined together to move as one piece, and in like manner the interior. This combination is capable of casting the finally reflected beam in a direction

making any angle with the axis of the exterior plates; and by a readjustment of the inner plates, the distance at which the rays finally meet may be varied at pleasure.

ARTICLE 15. It is stated that the Archimedean burning mirror was hexagonal. Let us consider if the combinations we have been illustrating can be made to conform to that figure. The term hexagonal may have reference either to the appearance of the mirror as a whole, or to the form of each individual reflector. Figs. 14 and 15 will correspond to the former; for if the external and internal conical frustums be each divided into six equal segments, with sufficient space between the segments to admit of free motion, the combination, viewed at a distance, will resemble a hexagonal polygon. Assuming the other meaning to be the correct one, we have only to suppose the form of the parabolic plates, which constitute the exterior and interior reflectors previously explained, to be hexagonal. From this would result two advantages:—they could be so formed that the different six-sided figures would unite together without leaving any interval, and the section of the beam cast by each on the required spot, approximating to a circle, would approach more nearly to the maximum effect with a given section of solar light.

ARTICLE 16.—Prop. *As the Expansion of the Sun's image is in proportion to the distance from the Point of Reflection, no greater accuracy is required for the construction of curved surfaces, capable of producing Combustion at distances of 150, 200, and 300 feet, than for those of a focal length of only a few inches.*

The expansion being about 1 foot in diameter for every 108 feet of focal distance, it follows that a reflector is sufficiently accurate for a burning glass, if it can concentrate the rays which fall on each part of its surface from the centre of the sun's disc, within a circular area, whose diameter is the same multiple or part of 1 foot which its focal length is of 108 feet.

The same principle may be exhibited in another and more definite form. Burning-glasses, which produce at the focus an intensity equal to parabolic ones, may be constructed of plane reflectors arranged as tangent planes to a paraboloid of revolution.

Since the increment of intensity is $\frac{kI'\delta A}{\text{area of circle DE}}$ (Art. 1, Equa. 6, fig. 1), if we take a portion of the surface of the paraboloid subtending an angle at the focus not greater than the sun's disc, the denominator of the foregoing fraction may be considered constant; and the intensity at the focus reflected from such an extent will be

$$\frac{kI' \text{ area of circle DE}}{\text{area of circle DE}} = kI'.$$

But a plane mirror which is a tangent to the paraboloid at the same spot, and of just sufficient area to reflect the circular beam of light whose section is equal to the circle DE, will also produce at the focus an intensity equal to kI' (Art. 2, Equa. 5); that is, the concentration at the focus is the same, whether a circular beam of the section mentioned be reflected from the surface of the paraboloid or from its tangent plane; and the same will evidently apply to any polygonal beam capable of being inscribed in the circular.

Hence a burning mirror, scarcely inferior in its effects to a parabolic one, may be formed of plane hexagonal reflectors, their sizes, of course, depending on the distance of the focus. For example, as the sun's image overspreads an area of 1 inch in diameter at a distance of 9 feet, a burning mirror of that focal length may be formed of plane hexagonal pieces, each side about half an inch; whereas at 108 feet distance, the sides of the plane hexagonal plates need not be less than half a foot, and so on in proportion.

Plates of the latter size being greater than those with which BUFFON performed his experiments, we infer that his combination, at distances exceeding 100 feet, would be little inferior in power to a parabolic segment of equal focal length, and capable of reflecting exactly the same sectional area of the solar beams.

Again, what has been proved true of plane mirrors, tangents to a paraboloid of revolution, must be equally true of a series of tangential circumscribing conical frustums. In all these cases, however, it is probable that the advantage in practice will remain with the parabolic figure, from the light at its focus having a greater area of maximum intensity.

From these results, as well as from independent calculations, we conclude that refracting burning-glasses may be constructed, by placing at some distance from an axis a series of acute-angled conical zones, or wedge-shaped pieces of glass (fig. 18), built up like the compound lens of BREWSTER, which will produce combustion at as great distances as BUFFON'S combination of reflectors.

ARTICLE 17. That the practicability of the Archimedean mirror may be made still more apparent, we shall now apply Equations 1 and 2, Article 3, to find the numerical intensity of the light in the focus of BUFFON'S combination. This was attempted by PEYRARD, but his conclusions are vitiated by the false premises from which he set out. He assumed that the intensity is uniform at every part of the luminous image reflected by a plane mirror,—a supposition proved incorrect by Art. 2. In our calculation we shall suppose that each of BUFFON'S mirrors, which were 8 inches by 6, produced an effect equivalent to a circular beam of 6 inches diameter, when it leaves the mirror.

Taking $k = \frac{1}{2}$, and $\cos i = 1$, we obtain by substitution, in the equations of Art. 3, the following results :—

$$\text{On 23d March, heat produced by 40 glasses at 66 ft. French} = \frac{40 \times \frac{1}{2} \times 6^2}{\left(\frac{66}{9}\right)^2} \text{ I}' = 13.3 \text{ I}'$$

Do.	do.	98	„	126 ft.	=9 I'
3d April, 4 P.M.,	do.	112	„	138 ft.	=8.57 I'
10th April, after 12 noon,	do.	128	„	150 ft.	=8.29 I'
10th April, 2.30 P.M.,	do.	148	„	150 ft.	=9.95 I'
11th April, 2.30 P.M.,	do.	21	„	20 ft. = $21 \times \frac{1}{2} \text{ I}' = 10.5 \text{ I}'$ (by Art. 3, Eq. 1)	
Do.	2.30 P.M.,	do.	12	„	20 ft. = $12 \times \frac{1}{2} \text{ I}' = 6 \text{ I}'$

In the first experiment, on the 23d March at noon, tarred beech was ignited with the 40 glasses; but the mirror not being mounted on a stand, acted at a disadvantage. On the same day, when 98 glasses ignited a plank smeared with tar and brimstone, the mirror is said to have been still more disadvantageously placed. The experiment on the 3d April was at 4 o'clock P.M., with the mirror mounted, and placed on its stand. The sun being weak, a slight inflammation was produced on a plank covered with threads of wool.

On the 10th April, with a clear sun, the 128 glasses very suddenly kindled a plank of tarred fir. At half-past 2 o'clock on the same day, the combination of 148 glasses was tried on a plank of beech tarred in part, and covered in some places with shreads of wool. The inflammation, which was very sudden, commenced on those parts of the wood which were uncovered. Beech previously charred was the material ignited with 21 glasses, and little combustible materials were the substances set on fire by 12 glasses, on the 11th April.

An inspection of the results in the preceding table shows that if $k = \frac{1}{2}$ be correct, wood done over in the manner mentioned can be ignited by a heat varying from eight to nine times that of the direct mid-day rays of the sun at Paris in April, and finely divided combustible substances by a heat considerably less, as proved by the experiment with 12 mirrors. But if we assume $k = \frac{2}{3}$, which is probably nearer the truth, the heat required to produce the same effect will vary between ten and eleven times the sun's mid-day heat. If the number in the right-hand column of the preceding table be multiplied by $\frac{1}{k}$, it gives the minimum number of plane mirrors capable in each case of producing ignition—that is, the number of mirrors which come under Equa. 1, Art. 3.

Supposing $k = \frac{1}{2}$, we find the numbers, for the first five experiments adduced, to be respectively 26.6, 18, 16.58, 17.14, and 19.18. The sizes of these mirrors will, of course, depend on the distance of the focus, and the angle at which they receive the incident light. When the distance is about 108 feet, each of them should have an extent sufficient to reflect a beam of solar light, not less than one foot in diameter, and their dimensions vary in the same ratio for other focal lengths.

As these minimum combinations have been calculated on the assumption

that the centre of the luminous circle reflected by each of the mirrors can be directed with precision to a given point, which in practice is not attainable, their number, or the size of each, must be somewhat increased, to compensate for defective adjustment. After making such an allowance, it will appear that at distances not exceeding 150 feet, between 16 and 20 plane mirrors, each $2\frac{1}{2}$ by $1\frac{1}{2}$ feet, may be substituted for the numerous combinations of BUFFON, the adjustment of which required so much time and trouble.

What has been shown respecting the power of the solar rays to produce combustion after one reflection, can easily be extended to the case in which the light undergoes two reflections, the intensity of the final beam being then reduced to about one-fourth that of the direct solar rays. To compensate for this diminution, the exterior reflectors must have about twice the area of BUFFON'S combination. Taking, for example, the 128 plane mirrors which kindled combustibles at 150 feet, the reflecting surface of the combination is $\frac{8 \times 6 \times 128}{144} = 42\frac{2}{3}$ square feet, and the sectional area of solar light, which we supposed it to reflect, was $\frac{\pi}{4} \times 6^2 \times 128 \div 144 = 25$ square feet nearly. It seems then, that about 80 feet of reflecting surface, or an extent capable of reflecting 50 square feet of solar light, will be more than sufficient to inflame such a material as tarred wood, &c., at the distance of 150 feet, after having undergone two reflections,—an extent of surface not too great to be united in one compound mirror, constructed in the manner explained in the foregoing articles.

Considering the scepticism which has prevailed respecting the Archimedean achievement in the most favourable circumstances, we are the less surprised to find that some recent authors, in quoting the passage from TZETZES, omit the statement which refers to the burning of the Roman ships in *winter*. Instead of ignoring these winter attacks, let us examine them in the light which BUFFON'S experiments supply. That performed with 112 mirrors, at the distance of 138 feet, was at 4 o'clock on the afternoon of the 3d April, at which time the altitude of the sun in the sky of Paris would nearly correspond to his meridian altitude at mid-winter in the more southern latitude of Syracuse; and as the difference between the meridian altitude of the sun at the summer and winter solstice amounts to above 46° , it must be admitted that this additional fact corroborates in a striking manner the evidence already adduced.

Having now shown how compound burning mirrors can be constructed corresponding in every respect to the description which TZETZES gives of the one invented by ARCHIMEDES, and that every statement in the passage is in accordance with well-established facts, we conclude that his narrative is no fiction, but, on the contrary, a true account of a real mirror, capable of producing all the effects ascribed to it. While this ancient discovery can be tried, after the lapse

of two thousand years, by the light of modern science, and pass with credit through the ordeal, the pretended discoveries of comparatively modern times, when subjected to the same test, fall to pieces.

An instance may be given without digressing from the subject of our paper. THOMAS DIGGES, who republished in 1591 a work by his father, LEONHARD DIGGES, entitled "Pantometria," would make us believe, in the preface to this edition, that he had seen his father at sundry times fire gunpowder and discharge ordnance at a distance of half-a-mile or more, by means of the sun's beams. Had he been aware that to accomplish such a feat would require at least four thousand square feet of reflecting surface, we may venture to affirm that he would not have overstepped so far the Archimedean range.

We may observe, in conclusion, that the experiments of BUFFON, taken in connection with the preceding deductions, are calculated to produce a strong conviction that, in clear and comparatively warm climates, the sun's rays may be made, at a small expense, to supersede in some respects the fires employed in culinary operations. Further, when it is considered with what ease a combination of plane mirrors, or a series of conical reflecting zones, can be constructed, capable of producing a heat exceeding that of the most intense furnace (Art. 16), we infer that the solar beams may also be turned to account by the chemist and metallurgist. For these purposes, one reflection only is required, as the reflected light can be made to fall always on the same spot, by directing the axis of the reflector to the centre of the sun's disc, and causing it to follow the sun's motion in the heavens, by revolving round a fixed axis parallel to that of the earth.