Note on the Luminous Efficiency of a Black Body

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XXXVIII. *Note on the Luminous Efficiency of a Black Body.* By CHARLES V. DRYSDALE, D.Sc.*

AT the present moment, when the question of light production is so much to the front, it is of interest to ascertain the possibilities of various substances as incandescent bodies. Until comparatively recently nearly all artificial illumination has been derived from the incandescence of carbon in some form or other, and consequently from a substance which approximates to a perfectly black body. It is therefore of interest to ascertain the possibilities of such a body as a light radiator at various temperatures, and this can be readily obtained from the radiation formulæ.

The best known and most convenient law for calculation is that of Wien †,

$$R_{\lambda} = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where λ is the wave-length, T the absolute temperature, R_{λ} the intensity of radiation corresponding to the wave-length λ , and C_1 and C_2 constants. The value of C_2 according to Paschen & Wanner ‡ should be 14,400, and to Lummer & Pringsheim § 14,700 when λ is in microns, and T in degrees centigrade.

Wien's law is not quite as exact as that of Max Planck ||

$$R_{\lambda} = C_1 \lambda^{-5} / (e^{\frac{C_2}{\lambda T}} - 1),$$

but only differs from it by 1 per cent. when λT is greater than $C_2/4.6$ or about 3000. It may therefore be used for most calculations.

The energy radiated between any two wave-lengths is therefore

$$\int_{\lambda_1}^{\lambda_2} R_{\lambda} d\lambda = \frac{C_1}{C_2} T \left[\epsilon^{-\frac{C_2}{\lambda_2 T}} - \epsilon^{-\frac{C_2}{\lambda_1 T}} \left\{ \frac{1}{\lambda^3} + \frac{3T}{C_2 \lambda^2} + \frac{6T^2}{C_2^2 \lambda} + \frac{6T^3}{C_2^3} \right\} \right], \quad (2)$$

* Read January 22, 1909.

† *Berl. Ber.* 9th February, 1893.

‡ *Berlin Akad. Sitzungsberichte*, ii. pp. 5-11, 1899.

§ *Verh. d. deutsch. Phys. Ges.* 1899, p. 23.

|| *Ann. der Phys.* i. 1900.

and the total radiation

$$R = \int_0^{\infty} R_{\lambda} d\lambda = 6 \frac{C_1}{C_2^4} T^4,$$

corresponding to the Stefan law

$$R = \sigma T^4. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence

$$6 \frac{C_1}{C_2^4} = \sigma \quad \text{and} \quad C_1 = \frac{\sigma C_2^4}{6}.$$

Several determinations of the constant σ have been made, but that of Kurlbaum* is most generally accepted. His value is 5.32×10^{-12} watts per sq. cm. Combining this with Lummer & Pringsheim's value of 14700 for C_2 we have $C_1 = 41,400$.

At the same time by logarithmic differentiation

$$\frac{1}{R_{\lambda}} \cdot \frac{\partial R_{\lambda}}{\partial \lambda} = -\frac{5}{\lambda} + \frac{C_2}{\lambda^2 T}$$

from which we have the well-known relation

$$\lambda_{\max.} T = \frac{C_2}{5} \quad \text{or} \quad \lambda_{\max.} = \frac{C_2}{5T},$$

where $\lambda_{\max.}$ is the dominant wave-length.

Hence for numerical calculation we have the following formulæ:—

Total radiation

$$R = \int_0^{\infty} R_{\lambda} d\lambda = 5.32 \times 10^{-12} T^4 \text{ watts per sq. cm.} \quad (4)$$

$$R_{\lambda} = 41400 \lambda^{-5} e^{-\frac{14700}{\lambda T}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\lambda_{\max.} T = 2940. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\int_{\lambda_1}^{\lambda_2} R_{\lambda} d\lambda = 2.82 T \left[\epsilon^{\frac{\lambda_2}{\lambda_1}} e^{-\frac{14700}{\lambda T}} \left(\frac{1}{\lambda^3} + 2.04 \times 10^{-4} \frac{T}{\lambda^2} + 2.78 \times 10^{-8} \frac{T^2}{\lambda} + 1.89 \times 10^{-12} T^3 \right) \right]. \quad (7)$$

To obtain the radiant luminous efficiency we have only to

* *Ann. Phys. Chem.* lxx. 4. pp. 746-748, 1898.

calculate the value of the last integral between the spectral limits and divide by the total radiation R , and we have

Radiant efficiency η_R

$$= 5.3 \times 10^{11} \left[\epsilon_{\lambda_1}^{\lambda_2} - \frac{14700}{\lambda T} \left(\frac{1}{\lambda^2 T^3} + \frac{2.04 \times 10^{-4}}{\lambda^2 T^2} + \frac{2.78 \times 10^{-8}}{\lambda T} + 1.89 \times 10^{-12} \right) \right]. \quad (8)$$

The following results have been worked out by my Assistant Mr. A. F. Burgess, B.Sc., and are shown in the annexed table. In fig. 1 the values of the total and luminous ra-

Fig. 1.

diation and their logarithms are plotted against the ordinary temperature. Fig. 2 shows the relation between the intensity of radiation in watts per sq. cm. and wave-length for various

Fig. 2.

temperatures on the centigrade scale from formula 5. Fig. 3 shows the relation between the dominant wave-length and temperature from (6) and the radiant efficiency η_R from (8) for two different values of the spectral limits, one being taken from '38

Fig. 3.

to $\cdot 76\mu$, the other from $\cdot 4$ to $\cdot 7\mu$, the latter comprising practically the whole of the useful luminosity of the spectrum. The dotted curves on the same sheet correspond to Paschen & Wanner's value of C_2 and do not differ much from the others.

Fig. 4 shows the comparison between the results so calculated and the experimental determinations of Prof. Féry*,

Fig. 4.

who has found the amount of light radiated per watt of total radiation at various temperatures. Taking the Carcel as 10 c.p., and the proportion of light to total energy from the calculated curve, we find that the luminous energy per c.p. or the mechanical equivalent of light comes out at about $\cdot 075$ watt per candle @ 1600° , which is not far from the most probable result of other determinations. It should be mentioned that the employment of Planck's formula instead of Wien's would have given lower rather than higher values for the efficiency.

* *Rayonnement Calorifique et Lumineux*. Thèses présentées à la Faculté des Sciences. Paris 1902.

These curves show at a glance the enormous extent to which the luminous efficiency is dependent upon temperature, and how extremely low it is at ordinary temperatures. At 1500°C . the efficiency is only of the order of 1 per cent. or less, while at 2000°C . it is still only about 3 per cent. In order to get anything like an efficient source of light by pure temperature radiation therefore, temperatures of from 4000° to 6000°C . would be required, and even then the efficiency would be only of the order of 30 to 50 per cent. This strongly points therefore to the necessity for working in the direction of selective radiation or luminescence.

It is perhaps worthy of note that the temperature of maximum efficiency is in the neighbourhood of the temperature of the sun as found by Prof. Féry, viz. 5500°C ., which is due to the close agreement of the dominant wave-length of the sun with that which produces the maximum luminous effect on the retina. This correspondence is doubtless to be attributed to the influence of natural selection on the eye, as Prof. Féry has himself pointed out.

Temp. $^{\circ}\text{C}$.	λ max. μ .	Total Radiation Watts/sq. cm.	Luminous Radiation Watts/sq. cm.		Luminous Efficiency per cent.	
			$\cdot 4 - \cdot 7 \mu$.	$\cdot 38 - \cdot 76 \mu$.	$\cdot 4 - \cdot 7 \mu$.	$\cdot 38 - \cdot 76 \mu$.
1000 ...	2.31	13.97	8.653×10^{-4}	2.525×10^{-3}	.0062	.0181
1500 ...	1.66	52.57	.1358	.2766	.258	.526
2000 ...	1.29	142.01	2.534	4.264	1.784	3.003
2500 ...	1.06	314.5	17.47	25.96	5.554	8.253
3000 ...	.90	610.5	69.39	98.24	11.367	16.092
3500 ...	.78	1078.1	195.5	257.7	18.137	23.919
4000 ...	.69	1773.4	452.2	562.8	25.498	31.737
4500 ...	.615	2760.8	849.5	1060.3	30.771	38.405
5000 ...	.56	4112.5	1454.0	1789.0	35.355	43.499
5500 ...	.51	5909.0	2280.2	2779.6	38.589	47.041
6000 ...	.47	8237.3	3339.8	4051.0	40.544	49.180
6500 ...	.435	11194	4639.6	5606.3	41.446	50.081
7000 ...	.405	14883	6173.5	7442.5	41.477	50.003
7500 ...	.38	19428	7931.7	9536.2	40.827	49.087
8000 ...	.355	24921	10079	11920	40.449	47.835

DISCUSSION.

The SECRETARY asked the Author if he would state exactly what he meant by the term "luminous efficiency." The ratio of luminosity to energy radiated was different in different parts of the spectrum.

Dr RUSSELL thought that the physiological effect on the eye ought to be considered. If the paper had been entitled "The radiation efficiency, for rays contained within the limits of the visible spectrum, of a black body" it would perhaps have been described better. The sensibility of the eye to light rays varied not only with the wave-length but also with the intensity of the rays and the time during which they had been acting on the retina. If we considered only the objective stimulus of the luminosity, the Author's implied definition might be, he thought, accepted. But if we consider, as we ought, that the subjective sensation produced in the normal eye has to be taken into account when judging luminous efficiency, another definition is required and the problem becomes far more complicated.

The AUTHOR, in reply, said that the term "luminous efficiency" was not very generally understood. There were three modes of defining it, of which two were due to Prof. Nichols of Cornell University, who distinguished between "total efficiency" and "radiant efficiency." The former of these quantities was the ratio of the luminous energy radiated between the spectral limits to the total energy consumption of the source of light, and the latter the ratio of the luminous energy as above to the total radiation from the source. As had just been pointed out, however, neither of these definitions took any cognizance of the very different luminosities of the various spectral colours, and this had been realized by Dr C. Guilleaume, who had proposed to correct the luminous radiation by multiplying each ordinate of the radiation curve by a factor depending on the luminosity at that ordinate, which was unity at the point of maximum effectiveness (0.54μ). The Author had proposed (Proc. R. S. 1908) the term "reduced" total or radiant efficiency, for the efficiency expressed on this basis, which was evidently the most rational one. The term "radiant luminous efficiency" was, however, recognized and had been employed correctly in the paper.