THE MEASUREMENT OF INDUCTANCE AND CAPACITY BY MEANS OF THE DIFFERENTIAL BALLISTIC GALVANOMETER.

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THE differential galvanometer has long been used in the comparison of electric resistances, but apparently has not been employed to any great extent in the measurement of inductance and capacity. Believing that it deserves a more prominent place in electrical laboratories than has yet been given to it, the following descriptions of a number of methods for the determination of self-inductance, mutual inductance and capacity have been prepared. Some of these are null methods, others require the measurement of ballistic deflections of the needle of the galvanometer. Including three methods previously published by Kohlrausch and Niven, all have been tested by the present writer. He has not discovered in the literature of the subject any other methods which make use of the differential galvanometer.

The differential telephone¹ and differential transformer² have recently been employed in the measurement of inductance and capacity with great precision in the expression of results. The methods described are necessarily null methods, and make use of a system of conductors analogous to the Wheatstone quadrilateral, or a somewhat simpler, parallel arrangement of branches containing the coils of the telephone or transformer. The double condition of equality of amplitude and phase of alternating currents in the differential coils is required for silence in the telephone. Possibly in some of the null methods described in the present paper, the telephone may replace the galvanometer with gain in precision of measurements. Apparently in most of the methods the double condition can not be reached.

¹ Ho, *Electrical World*, 41, p. 884, 1903; Ho, *PHYS. REV.*, XIX., p. 166, 1904; Duane and Lory, *PHYS. REV.*, XVIII., p. 275, 1904.

² Trowbridge, *PHYS. REV.*, XX., p. 65, 1905.

In laboratories and testing rooms reasonably free from magnetic disturbances, a galvanometer of the form subsequently described may be employed. In localities where there are fluctuations in magnetic field, a galvanometer of the D'Arsonval type, differentially wound, may be found practicable. A differential galvanometer of this type was described at the meeting of the American Physical Society, held at St. Louis in September, 1904.¹ It appears that most if not all of the methods of measurement given in the present paper may be carried out with such a galvanometer, designed for ballistic use. The damping factor for the vibration of the coil should be made as small as possible, and its period should be rather long.

Working formulas given in this paper are easily deduced from equations of electromotive force, as exemplified in method 1. In nearly all cases current divides between parallel branches connecting points *A* and *B*. Expressions for difference of potential by way of the branches between *A* and *B*, during the period of variable currents, are equated, and integrated over this period. This leads to an equation connecting inductances or capacities, resistances, quantities of electricity passing through the coils of the galvanometer, and final, steady currents. Equality of final currents permits the cancelation of equal terms, and in the null methods the condition of equality of quantities passing through the galvanometer is introduced. In deflection methods difference between quantities is written as proportional to the deflection of the needle of the galvanometer. The proportional factor need not be known since it disappears by cancelation from ratios of deflections. When there are more than two branch circuits, ratios of steady currents are expressed as ratios of resistances. The charge of a condenser is written as capacity multiplied by difference of potential the latter factor appearing as current multiplied by resistance, when resistance is one of the parallel branches connects the terminals of the condenser.

Accuracy of results obtainable by the methods described in this paper rests upon a number of factors, some of which are apparent from inspection of working formulas. The writer was more con-

¹ J. C. Shedd, *Phys. Rev.*, XIX., p. 301 (abstract).

cerned to survey the whole field than to investigate the accuracy which may be obtained by any one method. Yet in this general survey, among separate measured values of the same quantity, agreement within two or three tenths of one per cent. was reached, not only in one method but among several. It appears that with care agreement within one tenth per cent. may be easily reached, especially among the null methods. With closer attention to detail, still better results may be expected. The question of accuracy of standards is not touched upon here, but merely the precision of measurements, that is, the agreement among individual results.

DESCRIPTION OF GALVANOMETER.

The galvanometer used in this work was constructed by Mr. Stamper, the mechanic in the department of physics of the University of California. Two coils are contained, one in the upright standard of the instrument, one in a hinged portion, so arranged as to permit free access to the needle and suspending fiber. Each coil was wound by laying two wires side by side, and keeping them together from beginning to end. This insured nearly equal values of magnetic effect, of self-inductance and of resistance of the windings. When the four windings, constituted as described, and numbered 1 and 2 in the hinged part of the galvanometer, and 3 and 4 in the fixed standard, were connected in series, 1 and 3 opposing 2 and 4, the deflection of the needle was 12 minutes of arc when the current flowing was 0.15 ampere. It was considered desirable to secure more exact magnetic balance. To this end a small accessory coil of three turns of wire was mounted at the back of the galvanometer and connected in series with windings 1 and 3. This coil turns on a horizontal axis and is easily adjusted so that there is no deflection of the needle when the four windings are connected as previously described and traversed by steady current many times greater than is used in the measurement of inductance or capacity. The coils are connected externally by double conductors so that possible displacement of wires during the progress of a measurement may have no effect upon the magnetic balance. The self-inductances of the coils of the galvanometer are assumed equal in the deduction of working formulas. In the instrument described

they were found to be close enough in value for the immediate purpose of the investigation, without further adjustment. If necessary a pair of small coils, mounted according to the plan of the Ayrton and Perry variable standard of inductance, may be placed at a safe distance from the needle of the galvanometer and connected in series with the coil having the smaller self-inductance. The resistance of the four windings of the galvanometer, connected in series, is 12.9 ohms at 16° C. Adjustment of the branch circuits to equality of resistances is reached by the plan described under the first of the following methods of measurement.

The suspended system or needle of the galvanometer consists of an ellipsoidal magnet, 6.5 mm. in diameter, mounted on a sliver of bamboo. A small mirror is carried at the upper end of the bamboo stem and just below the mirror is a thin brass disk, 16 mm. in diameter, turned with a heavy rim to secure large moment of inertia with small mass. The whole is hung from a silk fiber. The purpose of the brass disk is to increase the period of vibration of the needle, for convenience in reading turning points on the telescope-

scale in the deflection methods. Ballistic deflection of the needle for given magnetic impulse of current passing is thereby reduced in the same proportion as the period is increased, so that in practice one has to strike a balance between convenience of use and sensitiveness. To bring the needle to rest rapidly and effectively, just before a ballistic deflection is to be measured, a second accessory coil of 3.7 cm. diameter and 15 turns of wire is mounted back of the instrument and made part of an independent circuit through a battery cell and a double key, placed within reach of the observer's hand. One key closes the circuit through 150 ohms resistance mounted on the base of the key; the second key shunts out this resistance when a stronger, temporary, controlling field is desired at the needle. To vary the controlling field further the plane of the accessory coil may be turned about a horizontal axis. Thus when the needle is vibrating through large angles the stronger field, applied at the proper time, rapidly reduces the swing. When the needle is nearly at rest a weaker field is desirable.

When the most favorable conditions for securing accuracy of results are considered, it must be at once evident that the galvanometer here described is adapted to a limited range of measurements, because its resistance is small. Notably in the measurement of capacities and of inductances associated with large resistances, a more sensitive galvanometer would be required. In neither of the cases cited would increased resistance of the galvanometer, due to the use of a greater number of turns of finer wire, be objectionable, while the deflection of the needle when there is a given difference between the quantities of electricity passing through the two coils, would be greatly increased. If necessary, further increase of sensitiveness may be reached by known methods, such for example as the use of an astatic needle.

METHODS OF MEASUREMENT.

All of the methods to be described are essentially comparisons between any two of the four quantities, resistance, self-inductance, mutual inductance and capacity. Incidentally measurements of period of vibration of the galvanometer needle and of ballistic deflection are involved. About half of the methods are null

methods which seek for adjustment securing balance between impulses of opposing force-moments exerted upon the needle, the impulses integrated over the period of variable current, and both impulses of extremely short duration. Since four quantities may be combined, two and two, in ten different ways, a convenient classification of methods is suggested. These classes are the comparison of two resistances; two self-inductances; two mutual inductances; two capacities; resistance and self-inductance; resistance and mutual inductance; resistance and capacity; self-inductance and mutual inductance; self-inductance and capacity; mutual inductance and capacity. Comparison of resistances with the differential galvanometer, using steady currents, is so familiar that no further reference will here be made to it. The methods described in this paper will then fall into the remaining nine classes. They are doubtless not all which might be devised, but will serve as illustrative examples, covering fairly well the usual range of measurements.

When dealing with inductive resistances, the usual precautions are to be observed. Coils must be sufficiently separated to avoid mutual magnetic effect, and their positions and distances should be such that no effect is perceivable at the galvanometer. A reversing key or commutator is placed in the battery circuit, instead of a simple key merely to open or close the circuit. In this connection Lord Rayleigh¹ remarks that: "A reversal has two advantages over a simple make or break. In the first place the effect is doubled and is therefore more easily measured; and in the second the battery is more likely to work in a uniform manner, the circuit being always closed except for a fraction of a second at the instant of reversal." It may be added that if the damping of the needle differs sensibly between open and closed circuit through the galvanometer, the correction for it will be considerably simplified.

DESCRIPTION OF METHODS.

Method 1, Comparison of Self-Inductances.—(Fig. 1.) A battery cell and commutator *K* are arranged to send current from *A* to *B* through a divided circuit. On one side are the inductive resistance

¹ Phil. Trans. R. S., 1882, Part II., p. 678.

L_1 (or L_2), a box of non-inductive resistance coils, and windings 1 and 3 of the galvanometer, represented in the figure by a circle; on the other side a resistance box, and windings 2 and 4 of the galvanometer, represented by the second circle. B is a contact key which may be moved at will along a german silver wire CD , 50

Fig. 1.

centimeters in length, to secure adjustment to exact equality of the total resistances, R_1 and R_2 , on the two sides. Resistance may be included in the battery branch if necessary for ballast. (a) With inductance L_1 in circuit, balance the resistances for zero deflection of the galvanometer

needle, using steady current. Then R_1 is equal to R_2 . (b) Bring the needle of the galvanometer to rest, reverse the commutator K and measure the ballistic deflection d_1 . (c) Insert the inductance L_2 in place of L_1 and adjust the resistance in series with it to balance the same resistance R_2 previously used. (d) Reverse the commutator as before and measure the ballistic deflection d_2 . The ratio of inductances is then

$$\frac{L_1}{L_2} = \frac{d_1}{d_2}. \quad (1)$$

To deduce this expression, write the difference of potential between A and B for any time during the period of variable currents,

$$R_1 i_1 + L_1 \frac{di_1}{dt} + I' \frac{di_1}{dt} - M \frac{di_2}{dt} = R_2 i_2 + I' \frac{di_2}{dt} - M \frac{di_1}{dt}, \quad (2)$$

in which I' is the self-inductance of galvanometer windings 1 and 3, or 2 and 4, M is their mutual inductance, and i with indicating subscript is current intensity. Integrating over the period of variable currents,

$$R_1 Q_1 + L_1 I_1 + I' I_1 - M I_2 = R_2 Q_2 + I' I_2 - M I_1, \quad (3)$$

in which I_1 and I_2 are final, equal, steady currents, and Q_1 and Q_2 are quantities of electricity passing through the coils of the galva-

nometer during the period of variable flow. Canceling equal terms and dropping unnecessary subscripts,

$$Q_2 - Q_1 = \frac{L_1 I}{R}. \quad (4)$$

Similarly, from operations (c) and (d),

$$Q_2' - Q_1' = \frac{L_2 I}{R}. \quad (5)$$

The differential quantities are proportional to the ballistic deflections d_1 and d_2 , the proportional factor involving the period of vibration of the needle of the galvanometer and constants of the coils. The deflections need not be corrected for damping, and are measured either in radians or directly in centimeters on a scale, provided that the telescope remains at a fixed distance from the galvanometer.

$$\frac{Q_2 - Q_1}{Q_2' - Q_1'} = \frac{d_1}{d_2} = \frac{L_1}{L_2}. \quad (6)$$

Method 2, Comparison of Self-Inductances.—This is a null method due to Professor C. Niven.¹ Coils whose inductances are L_1 and L_2 (L_1 greater than L_2), are joined in series with non-inductive resistances and connected one on each side of the galvanometer circuit. Referring to the similar arrangement of Fig. 2, L_1 is placed nearer the point C than its series resistance, L_2 nearer the galvanometer, and R_2 is omitted. If each non-inductive resistance is made equal to the resistance of the opposing coil, there will be no permanent deflection of the needle of the galvanometer when steady current is applied. Further, points between the coils and the series resistances are at the same potential, and may be joined by a cross-wire without disturbing balance for steady currents. If the resistance of this cross-connection is adjustable, a value S will be found by trial, such that there is no ballistic deflection of the needle of the galvanometer, when current is reversed by means of the commutator K . If R_1 is the resistance between point B and the connection joining L_1 and its series resistance, the ratio of inductances is

$$\frac{L_1}{L_2} = \frac{2R_1 + S}{S}. \quad (7)$$

¹ Niven, Phil. Mag. (5), Vol. 24, p. 234, 1887; Gray, Abs. Meas. in El. and Mag., Vol. 2, p. 463, 1893.

Method 3, Comparison of Self-Inductances.—This is a null method for which the scheme of connections is indicated in Fig. 2. If the resistance of one of the coils, R_1 , is considerably greater than that of the other, the non-inductive resistance in series with R_1 may be omitted.

Fig. 2.

The resistances are first to be balanced for steady currents; then adjusted for zero ballistic deflection when commutator K is reversed. Since balance for steady currents is to be maintained, this will require double adjustment of resistance R_2 , in parallel with R_1 , and one of the series resistances. With inductances of ordinary magnitude, it will be found that near the final adjustment of R_2 small changes in series resistances will suffice to maintain balance for steady currents. These changes may be made by shifting contact B along the german silver wire CD . If there is difficulty in deciding upon the best value of R_2 for zero ballistic deflection, the usual method of interpolation between values giving small right and left deflections may be employed. The ratio of inductances is

$$\frac{L_1}{L_2} = \frac{(R_1 + R_2)^2}{R_2^2}. \quad (8)$$

This equation may be easily deduced from an equation of electromotive forces by the plan followed in the establishment of equation (1).

Method 4, Comparison of Mutual Inductances, Null Method.—(Fig. 3.) Four coils, whose self-inductances are denoted by L_1 , L_2 , L_3 and L_4 , and mutual inductances by M_{12} and M_{34} , are connected, together with non-inductive resistances and the differential galvanometer, as indicated in the figure. (a) Balance the resistances for zero deflection of the needle of the galvanometer, using steady currents. (b) Connect the coils so that the magnetic fields of coils 3 and 4 shall coincide in direction, while the fields of coils 1 and 2 are opposed when current flows from A to B . Adjust the resistances in series with coils 1 and 4 so that there may be no ballistic deflec-

tion when the commutator K is reversed. (c) Connect the coils so that the magnetic fields of both pairs shall be opposed in direction, and again adjust for zero ballistic deflection. Interpolation may be employed to obtain the best values of resistances. If R_1 denotes the total resistance from A to B by way of coil 1, and R_4 denotes the total resistance by way of coil 4 in adjustment (b); and R_1' and R_4' denote corresponding values in adjustment (c); then the ratio of mutual inductances is

$$\frac{M_{12}}{M_{34}} = \frac{R_1 R_1' (R_4 + R_4')}{R_4 R_4' (R_1 - R_1')}. \quad (9)$$

Fig. 3.

With other magnitudes of mutual inductances and self-inductances than those employed by the writer, other connections than those described under (b) and (c) may be required. Corresponding changes in the signs of the working formula will then be necessary.

Method 5, Comparison of Mutual Inductances, Deflection Method.—Connect coils 1 and 2, whose mutual inductance is M_{12} , in series on one side of the divided circuit of the galvanometer; and coils 3 and 4, whose mutual inductance is M_{34} , on the other side. (a) Insert non-inductive resistance in the branch of the circuit having the lesser total resistance, and balance for steady currents. (b) With the magnetic fields of both pairs of coils coinciding in direction when current is flowing, reverse the commutator K and measure the ballistic deflection d_1 . (c) Reverse the terminals of coil 3, so that its field may oppose that of coil 4, and measure the ballistic deflection d_2 , obtained by reversal of the commutator. (d) Interchange the terminals of coil 1 so that its field may oppose that of coil 2, and measure the ballistic deflection d_3 , obtained by reversal of the commutator. The required ratio of mutual inductances is

$$\frac{M_{12}}{M_{34}} = \frac{d_2 - d_3}{d_2 - d_1}. \quad (10)$$

The signs in this equation depend upon the order in which the terminals are interchanged in operations (b), (c) and (d).

Method 6, Comparison of Capacities, Null Method. — Equal, non-inductive resistances are connected as indicated in Fig. 4. The sliding contact B is adjusted for final balance of resistances, shown by zero deflection with steady current. Condensers 1 and 2, whose capacities are C_1 and C_2 , are joined as indicated, one side of each connected to a traveling plug, so that each condenser may be put in parallel with any required portion of the corresponding resistance. If R_1 and R_2 are the resistances thus bridged between point B and the traveling plugs, when the ballistic deflection of the galvanometer is zero, the ratio of capacities is

$$\frac{C_1}{C_2} = \frac{R_2^2}{R_1^2}. \quad (11)$$

Method 7, Comparison of Capacities. — This is a null method for which the scheme of connections is indicated in Fig. 5. Condensers

Fig. 4.

Fig. 5.

1 and 2 are placed as shown with non-inductive resistances in series, to avoid too heavy discharge currents through the coils of the galvanometer. A battery of several cells is connected to a resistance of twenty thousand ohms or more, through a commutator K . Traveling plugs A_1 and A_2 permit the bridging of resistances R_1 and R_2 as shown. By trial the plugs are placed so that the ballistic deflection of the needle of the galvanometer is zero when the commutator is reversed. The ratio of capacities is

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}. \quad (12)$$

Method 8, Comparison of Capacities.—This null method is a modification of the last. Double contact keys are arranged to charge the condensers 1 and 2 when the keys are depressed; and to discharge each condenser through one coil of the galvanometer when they are simultaneously released. The lower contact points of the keys are connected, one to each of two traveling plugs A_1 and A_2 , placed as in method 7, and the upper contact points are connected to resistances in the branches of the galvanometer circuit. The discharge currents pass through the coils of the galvanometer when the keys are released, unite and pass through a common wire to the plates of the condensers which are not in connection with the keys. These plates are also joined to one end of the high resistance as in method 7. This arrangement avoids in a measure the trouble arising from faulty insulation of either condenser.

Method 9, Comparison of Capacities, Deflection Method.—Referring to Fig. 2, replace each inductive resistance by one of the two condensers whose capacities C_1 and C_2 are to be compared, and omit the shunt resistance R_2 . A battery of several cells will in general be required, instead of the single cell shown in the figure. (a) Reverse the commutator K and measure the ballistic deflection d_1 . (b) Remove both condensers from the branches of the circuit, leaving only non-inductive resistances in series with the coils of the galvanometer. Join the condensers in parallel with each other and with one of the resistances R_1 . Adjust the other resistance and the sliding contact B to obtain zero deflection when steady current is flowing. The total resistance from A to B by either branch is now R . (c) Reverse the commutator K and measure the ballistic deflection d_2 . The ratio of capacities is

$$\frac{C_1}{C_2} = \frac{2R^2d_2 + R_1^2d_1}{2R^2d_2 - R_1^2d_1}. \quad (13)$$

The same battery is used throughout and its internal resistance is negligible in comparison with R . The signs in the formula assume that the capacity of condenser 1 is greater than that of condenser 2.

Method 10, Self-Inductance Measured in Henrys.—This is a deflection method of comparing self-inductance and resistance, analogous to Maxwell's method as modified and employed by Lord

Rayleigh and Dr. Schuster in their work on the determination of the ohm in absolute measure.¹ The connections of Fig. 1 are used. (a) Balance the resistances for zero deflection when steady current is flowing. (b) Reverse the commutator K and measure the resulting ballistic deflection d_1 . This deflection is to be corrected in the usual way to the value which would have been found, if there had been no damping. (c) Change the resistance on one side of the circuit between A and B by a small amount ΔR , and measure the resulting deflection d_2 when steady current is flowing. If R is the total resistance on each side before the change, and T is the measured period of vibration of the needle of the galvanometer, the inductance is

$$L = \frac{\Delta R \cdot R}{R + \Delta R} \cdot \frac{T}{4\pi} \cdot \frac{d_1}{d_2}. \quad (14)$$

It is of interest to note that the dimensions of inductance (a length) appear in this equation as resistance (velocity) multiplied by time.

*Method 11, Self-Inductance Measured in Henrys, Kohlrausch's Deflection Method, Using Induced Currents.*²—The coil whose self-inductance is to be determined is connected in one branch of the differential galvanometer circuit as shown in Fig. 1, and an equal, non-inductive resistance is included in the other branch. Points A and B are connected to a Weber magnet-inductor, or to the secondary of an induction coil. (a) By this means an induced current of short duration is divided between the branches of the galvanometer circuit. The quantity of electricity passing through each branch is the same, but since current grows more slowly on the side having the larger inductance, there is a sharp movement of the galvanometer needle from rest in its position of equilibrium, to another, momentary position of rest. This measured deflection is d_1 . (b) By sending induced current from the same source as before through half of the windings of the differential galvanometer, connected in series with a large resistance, a ballistic deflection is obtained and measured. This is to be corrected for damping. Its corrected value is d_2 . Let R_1 be the resistance of the coil whose

¹ Proc. R. S. London, Vol. 32, p. 116, 1881; Phil. Trans. R. S. London, 1882, Part II., p. 677.

² Ann. d. Phys. u. Chem., Vol. 31, p. 594, 1887.

inductance is desired, or of the equal, non-inductive ballast; R_2 the resistance of each half of the galvanometer; R_3 the resistance of the source of induced current; R_4 the large resistance included in part (b); T the period of vibration of the needle of the galvanometer. The required inductance is

$$L = \frac{d_1}{d_2} \cdot \frac{T}{2\pi} \cdot \frac{(R_1 + R_2)(2R_3 + R_1 + R_2)}{R_4 + R_3 + R_2}. \quad (15)$$

Method 12, Mutual Inductance Measured in Henrys, Deflection Method. — Two coils are connected in series, first with their magnetic fields coinciding in direction when current flows in a given direction, second with their fields opposed. If L_1 , L_2 and M are the self-inductances and mutual inductance of the two coils, the total inductance is $L_1 + L_2 + 2M$ in the first case and $L_1 + L_2 - 2M$ in the second case. Measure these total inductances by method 10, each in henrys. The required mutual inductance is one fourth of the difference between the measured values.

Method 13, Capacity Measured in Farads. — This is a deflection method of comparing capacity and resistance. The condenser, whose capacity C is to be determined, is connected in parallel with non-inductive resistance and introduced into one branch of the galvanometer circuit, in place of capacity C_1 as shown in Fig. 4, C_2 being omitted. (a) Balance the resistances for zero deflection, using steady current. (b) Reverse the commutator K and measure the ballistic deflection resulting. Let this when corrected for damping be d_1 . (c) Change one of the resistances by a small amount ΔR and measure the permanent deflection d_2 , using steady current. Let R be the total resistance from A to B by either branch before the change; R_1 the resistance in parallel with the condenser; and T the period of vibration of the needle of the galvanometer. The required capacity is

$$C = \frac{\Delta R \cdot R}{R_1^2(R + \Delta R)} \cdot \frac{T}{4\pi} \cdot \frac{d_1}{d_2}. \quad (16)$$

Method 14, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of One of Them, Null Method. — Coils 1 and 2, whose self-inductances are L_1 and L_2 and whose mutual in-

ductance is M , are connected as indicated in Fig. 6. For given direction of current from A to B , the magnetic fields of the coils are to be opposed in direction.

Fig. 6.

(a) Balance the resistances for zero deflection, using steady currents.

(b) Having connected non-inductive resistance in series with coil 2, adjust this resistance for zero ballistic deflection when the commutator K is reversed. If R_1 and R_2 are the total resistances from A to B by way of the branches including coils 1 and 2 respectively, the ratio of the self-inductance of

coil 1 to the mutual inductance of both coils is

$$\frac{L_1}{M} = \frac{R_1}{R_2}. \quad (17)$$

Method 15, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of One of Them, Deflection Method. — The coils, whose self-inductances are L_1 and L_2 and whose mutual inductance is M , are connected in series and included in one branch of the galvanometer circuit, in place of L_1 in Fig. 1. (a) Balance the resistances for zero deflection when steady current is used. (b) Connect the coils so that their total inductance is $L_1 + L_2 + 2M$. Reverse the commutator K and measure the ballistic deflection d_1 . (c) Connect the coils so that their total inductance is $L_1 + L_2 - 2M$. Reverse the commutator K and measure the ballistic deflection d_2 . (d) Rearrange the connections according to the scheme indicated in Fig. 2, one coil in each branch of the galvanometer circuit. R_2 is omitted and the total resistance R , from A to B , is made the same as in parts (a), (b) and (c). After balancing the resistances for zero deflection with steady current, reverse the commutator K and measure the ballistic deflection d_3 . The required ratios are

$$\frac{M}{L_1} = \frac{d_1 - d_2}{d_1 + d_2 + 2d_3}, \quad \frac{M}{L_2} = \frac{d_1 - d_2}{d_1 + d_2 - 2d_3}. \quad (18)$$

L_1 is assumed greater than L_2 in the derivation of these equations.

Method 16, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of a Third Coil, Null Method.—Coils 1 and 2, whose self-inductances are L_1 and L_2 and whose mutual inductance is M_{12} , are connected according to the scheme indicated in Fig. 6, coil 1 in one branch of the galvanometer circuit. Coil 3, whose self-inductance is L_3 , is included in the other branch of the galvanometer circuit. Non-inductive resistance is connected in series with coil 2 and with either coil 1 or 3 as may be necessary. (a) Balance the resistances in the branches of the galvanometer circuit for zero deflection, using steady current. (b) Adjust the resistance in series with coil 2 to obtain zero ballistic deflection when the commutator K is reversed. If L_1 is greater than L_3 the magnetic fields of coils 1 and 2 must be opposed; if L_1 is less than L_3 the fields must coincide in direction. (c) Transfer coil 3 to a position in series with coil 1, leaving coil 2 in its former place, and again balance the resistances for steady currents. (d) Repeat adjustment (b) with the new arrangement of coils. The magnetic fields of coils 1 and 2 are now opposed in direction. If R_1 and R_2 are the total resistances from A to B by way of the branch circuits containing coils 1 and 2 respectively, for adjustment (b); and R_1' and R_2' are the corresponding resistances for adjustment (d); the required ratio of inductances is

$$\frac{L_3}{M_{12}} = \frac{R_1' R_2 \mp R_1 R_2'}{2 R_2 R_2'}. \quad (19)$$

The choice of sign depends upon the relative magnitudes of L_1 and L_3 .

Method 17, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of a Third Coil.—The mutual inductance of two coils, M_{12} , is to be compared with the self-inductance of a third coil L_3 . Connect coils 1 and 2 in series in one branch of the galvanometer circuit; and coil 3 in the other branch. By preliminary trial test the relative magnitudes of L_3 and the total inductance $L_1 + L_2 + 2M_{12}$. Connect non-inductive resistance in parallel with the larger of these inductances, and determine according to method 3 the ratio $(L_1 + L_2 + 2M_{12})/L_3$. This assumes that coils 1 and 2 are connected so that their magnetic fields coincide in direction. Keeping coils 1 and 2 in series, interchange the terminals

of one of them so that their fields are opposed, and determine by the same method as before the ratio $(L_1 + L_2 - 2M_{12})/L_3$. The difference between the ratios thus determined is four times the required ratio of M_{12} to L_3 .

Method 18, Comparison of the Mutual Inductance of Two Coils with the Self-Inductance of a Third Coil, Deflection Method.—Connect the three coils as in the last method, omitting the resistance in parallel with the larger total inductance. (a) Balance the resistances for zero deflection, using steady current. (b) Connect coils 1 and 2 so that their magnetic fields coincide in direction and measure the ballistic deflection d_1 , obtained when the commutator K is reversed. (c) Connect coils 1 and 2 so that their magnetic fields are opposed in direction and measure the ballistic deflection d_2 , obtained when the commutator is reversed. (d) Transfer coil 3 to a position in series with coils 1 and 2, altering resistances in series so that the total resistances from A to B by either branch of the circuit may be the same as they were in measurements (b) and (c). With the magnetic fields of coils 1 and 2 coinciding in direction, measure the ballistic deflection d_3 , obtained when the commutator is reversed. (e) With the coils arranged as in (d), except that the fields of coils 1 and 2 are now opposed in direction, measure the ballistic deflection d_4 . The required ratio is

$$\frac{M_{12}}{L_3} = \frac{(d_2 - d_1 - d_3 + d_4)}{2(d_1 + d_2 - d_3 - d_4)}. \quad (20)$$

The signs in this equation are arranged in accordance with the supposition that L_3 is greater than $L_1 + L_2 + 2M_{12}$. Other relative magnitudes will require other signs.

Method 19, Comparison of Self-Inductance with Capacity, Null Method.—The coil whose self-inductance is L is connected in one branch of the galvanometer circuit, according to the scheme indicated in Fig. 1. Non-inductive resistance is included in series with the coil. The condenser whose capacity is C is connected in parallel with any required portion of this resistance, by means of traveling plugs. (a) Balance the resistances for zero deflection, using steady current. (b) Shift the traveling plugs to obtain zero ballistic deflection when the commutator K is reversed. If R is

the resistance then bridging the terminals of the condenser, the ratio of self-inductance to capacity is

$$\frac{L}{C} = R^2. \quad (21)$$

*Method 20, Comparison of Self-Inductance with Capacity, Niven's Null Method.*¹—Branch circuits are made up between points *A* and *B*, including on one side a non-inductive resistance, one coil of the galvanometer and the inductance *L*, in order; on the other side a non-inductive resistance, the other coil of the galvanometer and a second non-inductive resistance. (*a*) Balance the resistances for zero deflection, using steady current. (*b*) Connect the condenser whose capacity is *C*, one terminal to a traveling plug which is placed at a point in the non-inductive resistance, in the branch in which *L* is found, near the galvanometer; the other terminal to a traveling plug which is joined to the opposing resistance at a point further from the galvanometer. Adjust the traveling plugs to obtain zero ballistic deflection of the needle of the galvanometer when the commutator *K* is reversed. If *R*₁ is the resistance from point *A* to the first plug, and *R*₂ is the resistance from *A* to the second plug, the required ratio is

$$\frac{L}{C} = R_1^2 - R_2^2. \quad (22)$$

Method 21, Comparison of Self-Inductance with Capacity, Deflection Method.—Connect the coil whose self-inductance is *L* according to the scheme indicated in Fig.

7. Capacity *C* is to be joined in parallel with either one of two equal resistances 1 and 2, each *R* ohms. (*a*) Balance the resistances for zero deflection, using steady current. (*b*) Measure the ballistic deflection *d*₁, resulting from reversal of the commutator *K* when the condenser is in the position

1. (*c*) Measure the ballistic deflection *d*₂, when the condenser is in

Fig. 7.

¹ Phil. Mag. (5), Vol. 24, p. 231, 1887.

the position 2. The required ratio of self-inductance to capacity is

$$\frac{L}{C} = R^2 \cdot \frac{d_2 + d_1}{d_2 - d_1}. \quad (23)$$

Method 22, Comparison of Mutual Inductance with Capacity, Null Method. — The two coils whose self-inductances are L_1 and L_2 , and whose mutual inductance is M , are connected in series, their magnetic fields either coinciding in direction or opposed. The total inductance of the coils, $L_1 + L_2 \mp 2M$, is then compared with the capacity C by method 19. If R_1 is the resistance in parallel with the condenser, required for zero ballistic deflection when the inductance is $L_1 + L_2 + 2M$; and R_2 is the resistance when the inductance is $L_1 + L_2 - 2M$; the required ratio is

$$\frac{M}{C} = \frac{R_1^2 - R_2^2}{4}. \quad (24)$$

Method 23, Comparison of Mutual Inductance with Capacity, Deflection Method. — The coils whose mutual inductance is M are connected in series and placed in the position occupied by the inductance L in Fig. 7. The capacity C is connected in parallel with either of the resistances 1 or 2, which need not be equal. (a) Balance the resistances for zero deflection of the needle of the galvanometer, using steady current. (b) Measure the ballistic deflection d_1 , resulting from reversal of the commutator K , when the coils are connected for total inductance $L_1 + L_2 + 2M$, the condenser, being disconnected. (c) Measure the ballistic deflection d_2 , resulting from reversal of the commutator when the total inductance is $L_1 + L_2 - 2M$, the condenser still disconnected. (d) Measure the ballistic deflection d_3 , when the condenser is connected in parallel with resistance 1 and the total inductance is $L_1 + L_2 + 2M$. (e) Measure the ballistic deflection d_4 , when the condenser is in parallel with resistance 2 and the total inductance is $L_1 + L_2 + 2M$. If R_1 and R_2 are the magnitudes of the resistances 1 and 2, the required ratio of mutual inductance to capacity is

$$\frac{M}{C} = \frac{R_1^2 + R_2^2}{4} \cdot \frac{d_1 - d_2}{d_4 - d_3}. \quad (25)$$

OTHER APPLICATIONS OF THE DIFFERENTIAL BALLISTIC
GALVANOMETER.

In addition to the methods of measurement described in this paper, many of the published methods of determining inductance and capacity which make use of the Wheatstone network, may be carried out with the aid of the differential ballistic galvanometer. The Rayleigh-Maxwell method of measuring self-inductance may be taken as an example. Four resistances, three non-inductive and one containing the inductance to be determined, form a quadrilateral, whose diagonals are occupied by a battery and a galvanometer respectively. If the galvanometer is replaced by a short wire, and the coils of a differential galvanometer are included with two of the resistances of the quadrilateral, one each side of a junction with the short wire, the measurement of inductance may be made as usual, in terms of resistance, deflection and period of vibration of the needle of the galvanometer. When the resistance of the galvanometer is small, there is no apparent gain in the use of the differential arrangement. When its resistance is large, then for given resistances and battery power, nearly twice the ballistic deflection may be obtained with the differential galvanometer, that would result from the use of the same galvanometer, its coils connected in series and employed as a ballistic galvanometer of the ordinary type. When the inductance to be measured is small and the resistance associated with it is large, relatively large current must be used to secure ballistic deflections which can be measured. Under these circumstances there is difficulty in maintaining balance of resistances when steady current is flowing, because of unequal heating effects in different parts of the circuit. Lessening the current required to produce a given deflection, decreases the heat liberated in the circuit in proportion to the square of the current, and tends to obviate the difficulty described.

Comparing the Rayleigh-Maxwell method with method 10 of the list now offered, the latter method has the advantage of greater sensitiveness. This may be established by comparison of the expression for the discharge through the galvanometer in the former method, with equation (4) of this paper. It is assumed for this comparison that the same amount of wire is wound in a single coil,

or differentially in two coils, and that the steady current flowing through the inductive resistance is the same in both methods. The advantage of greater sensitiveness, shared by other methods here described, increases in general with higher resistance of the galvanometer coils. The writer is convinced that partly because of greater simplicity of connections, leading to greater ease of manipulation, partly because of increased sensitiveness or smaller steady currents required for the same deflection of the galvanometer needle, the methods now presented will show some real gain in precision, over the methods which make use of the Wheatstone quadrilateral and a ballistic galvanometer of the ordinary type.

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