AN IMPROVED METHOD OF MEASURING THE INFRA-
RED DISPERSION OF MAGNETIC ROTATION;
AND THE MAGNETIC ROTATORY
DISPERSION OF WATER.

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I N a previous article¹ the writer described a bolometric method of measuring the electromagnetic rotation of the plane of polarization of infra-red radiations, if of not too long wave-length. This enabled the determination of the variation of magnetic rotation with wave-length — or the magnetic rotatory dispersion — for a spectral region several times as large in extent as that which could be explored by the customary visual methods. But measurements were difficult, and the several sources of error and rather large corrections to be applied, necessarily detracted somewhat from the certainty of the result.

During the past year, however, a number of radical changes have been introduced, which not only remove in great part the above defects, but also lend to the bolometric method possibilities for sensitiveness in the detection and measurement of minute rotations, considerably exceeding those of visual methods of observation. Hence, as it is believed that the advantage appeals to investigators of making a more thorough study of the magnetic rotatory dispersion of a large class of substances, this improved bolometric method of measurement is described. Results are also given of measurements by its use, of the magnetic rotatory dispersion of water.

The fundamental principle is as already described,² and is essentially that due to De la Provostaye and Desains.³ Light passes in succession through a polarizing agent, through the rotating substance in a magnetic field, and through an analyzer, whose principal

¹ Phil. Mag. (6), 61, p. 41, January, 1906.

² Loc. cit.

³ Ann. Chim. Phys. (3), XXVII., p. 32, 1849.

plane makes an angle of 45° with that of the polarizer. Any rotation of the plane of polarization will cause an increase or diminution of intensity of the light transmitted by the analyzer, which may be measured bolometrically, and for any desired wave-length if the beam is dispersed.

The practical difficulty which must be encountered is that small and unavoidable variations in the intensity of the light source may be falsely interpreted as rotations, and may indeed mask entirely the true effect if it is small. It appeared that it might be possible to overcome this difficulty by utilizing two equal beams from the same source of light, and having them fall eventually on two bolometers, so connected that their currents opposed each other at the galvanometer. For then any variation of intensity of the source would affect both beams equally and produce no resultant deflection, while if only one of the beams was polarized and made to pass through the magnetic field, it alone would suffer changes of intensity due to rotation, and the angle of rotation could be measured as readily as before.

A large number of trials have demonstrated the possibility of compensating in this way for variations of the source, but it has been found that for best results, the two beams should be as nearly identically similar as possible, and that instead of two bolometers, the two strips of a single bolometer may be used to advantage. To fulfill these conditions the following arrangement has been adopted, the general disposition of magnet and mirror system being as before, and as indicated in the diagram of the previous paper.

Light from a Nernst glower, after reflection from a concave mirror and passage through a double image prism and down the axis of the electromagnet, is brought to a focus between the pole-pieces. Only the direct, and therefore achromatic image from the prism is used, hence this prism might be replaced by a Nicol, although the latter, being longer, would cause more absorption. A second mirror converges the beam, after passing through a second double image prism, on a Vierordt double slit. The two images of the source falling vertically above one another on the two halves of the slit, form, after passing through the spectrometer, two slightly separated, and, with the exception of the azimuths of polarization,

entirely similar spectra. These are received on the two strips of a bolometer, so constructed that the strips instead of being side by side, lie vertically over one another. The intensity of either spectrum for any given wave-length may thus be measured by shuttering either half of the slit, — or any relative change of intensity. This last is due to rotation, for, since the plane of polarization of the radiation as it passes between the poles of the magnet makes an angle of 45° with each of the two planes of transmission of the analyzing double image prism, any rotation will cause a diminution of intensity of one image on the slit and a corresponding increase in the other. This change being measured by the bolometer for any given wave-length and divided by the actual intensity of either spectrum for that wave-length, readily gives by the simple computation which will follow shortly, the magnitude of the rotation.

Of the two evident advantages resulting from this arrangement, one has already been mentioned. That is, the differential basis on which the measurement of rotations is placed, whereby the effect of small variations of the light source may be made negligible, and consequently far greater steadiness and certainty in the determination of small rotations secured. The second advantage results from the fact that the effect of any rotation is clearly doubled, since the increase of energy on one bolometer strip is accompanied by an equal decrease on the other, resulting in a doubled galvanometer deflection. Practically the only disadvantage results from the absorption of the calcite double image prisms, which prevents measurement beyond about $\lambda = 2.5 \mu$. But in very many cases the absorption of the rotating substance itself precludes work beyond this wave-length. In certain other cases, too, particularly that of the magnetic metals, the interesting part of the curve is obtained well within this region of the spectrum, and indeed barely outside the limits of the visible part.

There are, moreover, a number of good features arising from the fact that many of the errors of adjustment, which are almost impossible to avoid, are rendered of negligibly small effect by this arrangement. The reason for this will appear after a brief discussion of the theory of the method.

Let α be the angle which the plane of polarization of the incident

radiation makes with the plane of transmission of the analyzer for the beam which goes to the upper half of the slit. Then if I' represents the intensity for any wave length of the incident beam, the intensities of the two transmitted beams will be :

$$I_1 = k_1 I' \cos^2 \alpha \text{ to the upper half of the slit}$$

and

$$I_2 = k_2 I' \sin^2 \alpha \text{ to the lower half,}$$

where k_1 and k_2 are factors which take account of all losses which may occur to the two beams, due to any cause whatever, *e. g.*, narrowing of either half of the slit, absorption, etc.

Any small rotation of the plane of polarization δ will cause a relative change of intensity for the two beams of

$$dI = 2I'(k_1 + k_2) \sin \alpha \cos \alpha \cdot \delta.$$

Dividing by I , the intensity on either strip of the two are equal, or the mean if they differ slightly,

$$\frac{dI}{I} = 4\delta \frac{(k_1 + k_2) \sin \alpha \cos \alpha}{k_1 \cos^2 \alpha + k_2 \sin^2 \alpha}$$

or,

$$\text{if } \alpha = 45^\circ \delta = \frac{1}{4} \frac{dI}{I} \text{ radians or } \frac{45}{\pi} \frac{dI}{I} \text{ degrees.}$$

This shows that if $\alpha = 45^\circ$ a given rotation δ will be indicated by the same value of dI/I whatever the values of k_1 and k_2 . Furthermore, $\sin \alpha \cos \alpha$ being a maximum at 45° , a variation even as great as 4° either way changes it by less than 1 per cent, which means a correspondingly small change in the above factor if $k_1 = k_2$. If the two intensities are made equal, as is in general done to secure the compensating action explained above, $k_1 \cos^2 \alpha = k_2 \sin^2 \alpha$, but the change in the factor for a variation in α of several degrees from 45° , is still barely a per cent.

The practical importance of these several facts is, that the factor by which the measured quantity dI/I must be multiplied to give the angle of rotation, is practically a constant unless the variation of α from 45° is unreasonably large. This means that neither small errors

in the adjustment of the polarizing arrangement, nor unknown absorptions or losses of radiation in any part of the apparatus, can have any appreciable effect on the accuracy of the results.

It is of interest to compare the possibilities for accuracy and sensitiveness of this bolometric method of measuring angles of magnetic rotation, with those of the ordinary visual methods. It is evident at once that the relative accuracies of the two will depend very largely on the magnitude of the rotation to be measured. For in ordinary polarimetry, rotations being measured as the difference between two settings of the analyzer, the probable error of the result is practically the same for small as for large rotations. On the other hand, in the present bolometric method, the probable error is by no means constant but is rather a fraction of the whole which is not many times greater for small than for large rotations. It is quite possible then, that, while the accuracy of visual observations may be very considerably greater for relatively large angles of rotation, *i. e.*, of several degrees, the bolometric method may excel in accuracy when the rotation is a hundred times as small.

A few figures will illustrate to what extent this is true. The accuracy of single settings with the most improved form of polarimeter is of the order of $.01^\circ$. With the ordinary half shade arrangement the writer must confess to frequent discrepancies in successive measurements of several times this amount. On the other hand, in the bolometric method, a rotation of $.01^\circ$ would mean a ratio dI/I of 1:1433 or a galvanometer deflection of one centimeter if the total energy incident on either strip corresponded on the same basis to 1433 cm. This, measured of course with reduced sensibility, is only one-half or one-third of what has readily been secured in practice in parts of the infra-red in the spectrum used, and a much smaller fraction of what would be available if the beam was undispersed. Hence it is evident that rotations several times smaller than could be detected visually, may, under proper conditions, be measured bolometrically. The possibilities for sensibility appear to be limited — supposing a rotating substance of not too great absorption — only by the intensity of the available source of light, and by the success with which this, as well as the bolometer and galvanometer are protected from various disturbing elements.

Of course the two methods are in no sense competitive, applying, as they necessarily do, to entirely different parts of the spectrum but it would seem that the bolometric method might perhaps find a special field of usefulness, outside of its natural application to rotatory dispersion determinations, in the detection of minute rotations. Unfortunately it is not applicable in general to the measurement of rotations due to naturally active substances, save in such a case as Brace's experiment on ether drift, where the effect to be measured is differential. In this case, however, it would probably be difficult to secure a naturally active substance sufficiently transparent in the infra-red to transmit enough energy for the measurements, through the great thickness required.

It is, perhaps, worth noting that the spectrometer and bolometer of this arrangement, taken alone, constitute a type of spectro-photometer applicable to infra-red comparisons of various light sources.

THE MAGNETIC ROTATORY DISPERSION OF WATER IN THE INFRA-RED SPECTRUM.

Verdet¹ measured the magnetic rotation of water in a constant field, for the solar lines, *C* to *G*. Van Schaik² extended the determinations into the ultra-violet, finding that the rotation dispersion might be well represented by the expression $a\lambda^{-2} + b\lambda^{-4}$.³ It was thought desirable in the present case to measure this dispersion for water in the infra-red, both in order to secure determinations for a larger spectral region over which formulæ might be tested, and also to find if the numerous infra-red absorption bands have any appreciable effect in modifying the form of the curve.

Using the foregoing method, measurements were made of the magnetic rotation of water for various infra-red wave-lengths, in a field of approximately 13,000 C.G.S. units. Because of its low rotatory power and high infra-red absorption, it is a peculiarly difficult substance to test in this way, but it was found that by using a rather thin cell — .616 cm. — a fairly consistent series of determinations could be made over the range of spectrum from $\lambda = .7 \mu$ to $\lambda = 1.3 \mu$, or as far into the infra-red as the ordinary dispersion has been determined.

¹ Ann. Chim. Phys. (3), 69, p. 415, 1863.

² Arch. Néerl., 17, p. 373, 1882.

³ The careful experiments of Siertsema (Arch. Néerl. (2), 6, p. 825, 1901) give results in fairly good agreement with these.

A detailed description of the magnet and mirror system has already been given¹ and need not be repeated here, as this part of the apparatus was identical with that formerly used. The double image prisms of calcite and glass, used as polarizer and analyzer in the present case, were of 5 cm. and 4.5 cm. aperture, so that a rock salt prism with 8 cm. faces utilized practically the full beam transmitted. The differential bolometer had strips each 8 mm. by .5 mm. It proved in practice an extremely satisfactory instrument, for, as the balancing coils and slide adjustments were contained in the same case as the bolometer itself, "drift" was practically entirely eliminated.

The only correction necessary to the measurements was for the width in the spectrum of the bolometer strip and of the slit image, each .5 mm. wide. This for reasons previously explained,² was small, the calculated corrections varying from + 1.5 per cent. at $\lambda = .8 \mu$ to - 1 per cent. at $\lambda = 1.3 \mu$. Special tests with a wide slit, giving an exaggerated error, indicated corrections in general accordance with these. The effect of the thin cover glass ends of the cell was safely neglected, since preliminary trials indicated a rotatory dispersion curve for the glass very similar to that for the water, hence the effect of the ends would merely be to increase the equivalent thickness of the water cell 2 or 3 per cent.

In the following curve and table the results are summarized.

Magnetic Rotation of Water.

¹ Loc. cit.

² Loc. cit., p. 56.

The rotations plotted in the curve are half those actually measured — since the magnet current is reversed in the measurement — with the small correction above mentioned applied. The observed values given in the table are taken from the smooth curve and reduced to a scale which makes the rotation for sodium light unity. This point was determined by visual observations with a half shade polariscope.

TABLE I.

Wave-length.	Observed Rotations.	Calculated Rotations.	
		(2)	(1)
.589 μ	1.000	1.000	1.000
.70	.695	.676	.683
.80	.514	.496	.510
.90	.389	.389	.399
1.00	.312	.315	.322
1.10	.254	.259	.263
1.20	.220	.220	.220
1.30	.201	.193	.181

The agreement of observed with calculated results is only fair, and hardly as satisfactory as in the case of carbon bisulphide. Results calculated from two slightly different formulæ are given. (1) is with the use of the formula given by Drude.¹

$$\delta = \frac{1}{n} \left(\frac{a}{\lambda^2} + \frac{b\lambda^2}{(\lambda^2 - \lambda_1^2)^2} \right)$$

where $\lambda_1 = .127 \mu$ is the wave-length of an ultra-violet absorption band. The constants, being determined for wave-lengths .589 μ and 1.2 μ give $a = -.176$ and $b = +.582$, (2) uses the same formula modified by a term depending on the infra-red absorption band having its maximum at $\lambda = 3.02 \mu$.

$$\delta = \frac{1}{n} \left(\frac{a'}{\lambda^2} + \frac{b'\lambda^2}{(\lambda^2 - \lambda_1^2)^2} + \frac{c'\lambda^2}{(\lambda^2 - \lambda_2^2)^2} \right).$$

The constants, determined for wave-lengths .589 μ , .9 μ and 1.2 μ are $a' = -.517$; $b' = +.890$; $c' = +.804$.

It will be observed that there is only questionable advantage in

¹ Lehrbuch der Optik, p. 403.

favor of the second formula, which assumes the influence of an infra-red absorption band. This might perhaps be anticipated from the fact that the ordinary dispersion curve for water as determined by Rubens¹ as far as $\lambda = 1.256 \mu$, does not show the effect of such an absorption band—in the early infra-red at any rate.

It is of interest to note that the theoretical formula proposed by Becquerel² and further worked out by Larmor,³

$$\delta = \kappa \lambda \frac{dn}{d\lambda},$$

which fits very well in the case of carbon bisulphide,⁴ fails entirely here, giving values for the rotation several times too large at $\lambda = 1.3 \mu$.

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¹ Wied. Ann., 45, p. 253, 1892.

² Comptes Rendus, Nov., 1897.

³ Ether and Matter, p. 353.

⁴ A discussion of this formula in its application to carbon bisulphide was unfortunately omitted in the previous paper. However at the suggestion of Professor Larmor it has since been tried and found to apply very well to $\lambda = 2.0 \mu$ which is as far as the ordinary dispersion has been determined.