While we must have uniformity and intelligent supervision to do the best work, still wherever unhampered by uninterested and ignorant authority, let us take up the work ourselves and do what we may toward unifying the course. If alternate days' work in algebra and geometry is not satisfactory, and we are not yet convinced that it is unsatisfactory, it is within the power of every teacher to arrange her daily program so that into each recitation aside from the daily progress made in the new subject under consideration may be introduced some review of arithmetic, algebra or geometry. Some drill in rapid multiplying, adding, and the practical arithmetical processes, in solving quadratic equations, factoring, some work in exponents, computing triangles, etc., etc., not in a haphazard way, but outlined to cover gradually the whole course so that there is always present this idea of preserving the factors that were in the product when it came to us, as well as adding new ones. Then with the extra time which the removal of useless work will give us, our mathematical product would hold at the time when we present it to the public or to the higher institution of learning, all the original factors raised to their highest power by continued multiplication, all the zero, fractional and imaginary factors replaced by real integral ones, all signs of operations gone, and the product reduced, simplified, ready to take its place in the great world's problem.

THE ANGLE SUM OF THE GENERAL POLYGON.

By J. S. BROWN,

San Marcos, Texas.

Definitions: A polygon is a broken line lying in a plane, whose end points coincide. The parts of the broken line are called the sides of the polygon. Polygons are either *simple* or *cross*. A simple polygon is one, no two of whose sides cross unless produced. A cross polygon is one, two or more of whose sides cross without being produced. Any part of a cross polygon which, taken alone, is a simple polygon is called a *segment*.

Angle sum: The angle sum of a polygon is the sum of the angles formed on one side of the perimeter of the polygon. Hence, there are two angle sums in a polygon, corresponding

The right angle is the unit of angular measurement in this paper.

to the two sides of the perimeter. Since the value of an angle does not depend on the length of the sides of the angle, and also since the angle sum of a simple polygon depends only on the number of sides and not on the shape of the polygon, it follows that a polygon may be deformed into another polygon whose angle sum is the same, either by moving the perimeter and lengthening or shortening the sides, as may be necessary or convenient, or by keeping the sides intact or parallel with themselves, and shortening or lengthening the sides as may be necessary or convenient. (The term *deformed* hereinafter used has the above meaning.) In deforming the polygon, no angle must change sides of perimeter, i. e., from interior to exterior or *vice versa*.

Any cross polygon may be deformed so that none of the segments overlap, that is so that the various segments are either exterior or interior to each of the other segments.

Cross polygons are of two types. One is illustrated by figure 1 in which the consecutive segments have their interior angles on opposite sides of the perimeter of the cross polygon. A polygon of this type can be deformed into a simple polygon, if it has an *odd* number of segments: into a cross polygon of two segments exterior to each other, if it has an *even* number of segments. In the first case, the angle sum is by ordinary proof $2(n \pm 2)$ right angles; in the second case $2n$ right angles.

The other type is one in which two or more consecutive segments have their interior angles on the same side of the perimeter of the polygon respectively. This type is the most general form of the polygon, i. e., a polygon drawn at random would likely fall in this type, and is illustrated in figure 2.

In this type there is at least one segment more than two of whose sides produced are sides of other segments. Evidently the segments of a cross polygon may be divided into two classes, one class having its interior angles on one side of the perimeter, and the other class having interior angles on the other side of the perimeter.

Problem: To find the angle sum of a polygon of the second type. Let $a+1$ be the number of segments whose interior angles are on that side of the perimeter occupied by the interior angles of a segment having more than two sides which produced are sides of other segments; or if there is no such segment then $a+1$ may be the number of segments whose interior angles are on one side of the perimeter and b be the number of segments whose interior angles are on the other side of the perimeter. Let $N', N'', \text{ etc.}$, be the number of sides of the $a+1$ segments respectively and let $N_1, N_2, \text{ etc.}$, be the number of sides of the b segments respectively.

Now referring to Fig. 3 it is evident that in counting the angles of the polygon, the angles at the cross points 1, 2, 3 are not angles of the cross polygon. So in estimating the angle sum these angles must be counted out, since they are counted in, in estimating the angle sum of the segments separately. In estimating the angle sum on both sides of the perimeter, the whole perigon about these cross points is counted in estimating the angle sum for the segments separately.

The interior angle sum of the $a+1$ segments is: $2[N' + N'' + N''' + \dots - 2(a+1)] - 4a$ since there are a cross points in $a+1$ segments. The exterior angle sum of the b segments is $2(N_1 + N_2 + N_3 + \dots)$. Adding these two expressions to get angle sum on one side of the perimeter, we have $2(N' + N'' + N''' + \dots + N_1 + N_2 + \dots - 2 - 2a) - 4a = 2(N' + N'' + \dots + N_1 + N_2 + \dots - 4a - 2) \dots \dots \dots (1)$

Let N be the number of the sides of the cross polygon, then

$$N = N' + N'' + \dots + N_1 + N_2 + \dots - 2(a+b) \dots (2)$$

$$\therefore N' + 2a + 2b = N' + N'' + \dots + N_1 + N_2 + \dots (3)$$

Substituting in (1) gives

$$2(N + 2a + 2b - 4a - 2) = 2(N - 2) - 4(a - b)$$

Since $4N$ is evidently the angle sum on *both* sides of the perimeter and $2(N - 2) - 4(a - b)$ is the angle sum on *one* side. Then $4N - [2(N - 2) - 4(a - b)] = 2(N + 2) + 4(a - b)$ the angle sum for the other side of the perimeter. These two may be written as one formula as follows: $2(N \pm 2) \pm 4(a - b)$, the double sign corresponding to the two sides of the perimeter.