II.—*A Further Inquiry into the Limits of Microscopic Vision and the delusive application of Fraunhofer's Optical Law of Vision.*

No. II.

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(Read 13th November, 1878.)

PLATE III.

THE writer has been more particularly led into the present subject by the wide-spread belief that the limit of microscopic vision has been reached by the resolution of Nobert's lines drawn at the rate of 112,000 per inch, which probably gives the 1-200,000th for the diameter of the smallest line supposed to be visible. It will not be uninteresting to relate the history of this belief.

The celebrated Fraunhofer (as stated in his Memoir to the Bavarian Academy of Sciences, June 14, 1823) succeeded in ruling lines as close as 30,000 to the Paris inch, which he found totally invisible with the Microscope. He also announced that if λ be the wave-length, and the light fell perpendicularly to the surface of the ruled glass, sine $\theta^{(n)}$ would become imaginary, and therefore the lines would produce no coloured spectra; and he concluded, says Sir John Herschel, "that an object of less linear magnitude than λ can in consequence never be discerned by Microscopes as consisting of parts."*

The skilful optician Nobert, believing in this result obtained by Fraunhofer, utterly despaired that anyone would ever succeed in describing his finest lines on glass.†

Now with regard to this very conclusion of Fraunhofer, Sir John Herschel regards it as "one which would put a natural limit to the magnifying power of Microscopes, but "which," says he, "we cannot regard as following from the premises" (*sic*).‡

Well, Dr. Woodward first achieved the honour of resolving these lines with a Powell and Lealand $\frac{1}{8}$ immersion in 1869;§ and in consequence of the grave doubts expressed by their maker, he wrote to Dr. Barnard, a distinguished mathematician (Pres. Columbia College), who replied that "with an objective that takes

* Art. "Light," 'Encyc. Met.,' p. 490.

† Nobert thus wrote to Dr. Colonel Woodward, U.S., dated Barth, Feb. 26, 1869. He expressed his belief that the resolution of the higher bands is an impossibility when light is permitted to fall on closely ruled lines. "The formula," says he, " $\sin. x = \frac{\lambda}{b}$ (Fraunhofer's), if by λ we designate the length of the undulations, by b the distance between two lines of the grating, and by x the angle of the refracted rays, gives for $\sin. x$ an impossible value when b becomes less than λ ;" that is to say, when the distance between two lines is less than a wave-length, the lines will become invisible.

‡ 'Encyc. Met.,' art. "Light," p. 490.

§ See 'Month. Mic. Jour.,' Dec. 1869, quoted by Dr. Woodward.

in a cone of an angle of from 140° to 175° it is nonsense to talk of this question as settled by theory. We shall continue to see closer lines just in proportion as Microscopes and modes of illumination are improved.*

That has long been the firm opinion of the writer. In the first paper on the subject of the limits of vision, he stated, "I believe this limit has not yet been reached;"† and farther on, p. 181, "With special adaptations to subdue or destroy the brilliant diffractions of too bright an illumination, many minute details before completely effaced may be brought into distinct revelation." When these remarks were made, the microscopical world had been recently favoured with the beautiful formula introduced independently, I believe, by Professors Helmholtz and Abbe, in which further elucidation of the principle was given by a new formula including the semi-angle of aperture of the objective used. Applying these similar results, I obtained, for mean rays of wave-length, $\frac{1}{46182}$ of an inch (46,000th nearly) the following results:—

A TABLE OF PROPORTIONATE RESOLVING POWERS ‡ (some of the details of which were as follows):—

Full Aperture of Object-glass.	Proportionate Resolving Power.	Semi-aperture.
179°	$\sin. \alpha = 99996$ per inch	$89\frac{1}{2}^\circ$
175°	$\sin. \alpha = 99905$ "	$87\frac{1}{2}^\circ$
150°	$\sin. \alpha = 96590$ "	"
120°	$\sin. \alpha = 86600$ "	60°
$12^\circ 38'$	$\sin. \alpha = 11000$ "	$6^\circ 19'$

I hope to show in the following paper that however truly this optical law may be deduced from the premisses, it utterly fails for minute dark lines.

An announcement that it is possible to descry with microscopic apparatus the millionth of an inch would be almost too startling to believe. The human eye can distinguish a hair under favourable conditions of light and background subtending an angle of even less than a second. The black line dividing close double stars, such as χ Ursæ Majoris, which are both of the same (fourth) magnitude, does not subtend in the telescope with a power of 300 diameters many seconds of arc. Besides this, the Microscope differs only from the telescope in the length of its focus and smaller aperture, which, according to received dogma, gives great advantages of vision to the instrument with so great an angular aperture. We cannot doubt, either, from the tales of travellers, that birds of prey possess exceedingly acute vision, by which they can descry small objects at a

* See 'Month. Mic. Jour.,' Dec. 1869, quoted by Dr. Woodward.

† P. 175, 'M. M. Jour.,' Oct. 1876.

‡ P. 181, *ibid.*

great distance. I myself knew a friend who could see with the naked eye Jupiter's satellites, and dot down their position though ignorant of astronomy. If a simple organ of sight can distinguish such objects as subtend only a second or two, it would seem strange that modern glasses can only show objects presenting many seconds to the eye at the last visual image formed in the eye-piece.

In the following observations I shall endeavour to substantiate a fact apparently irreconcilable with the results of the now famous formula.*

In point of fact, the opinion has now become established both in Europe and America, that Nobert's lines 112,000 per inch (or lines of that size) are the closest that can be seen; and that the law enunciated in the footnote forbids the hope of farther advance in minute definition.

Now, considering the readiness with which a fine horsehair can be distinguished against a light cloudy sky, as also spider lines at several feet distance, I determined to mount upon glass several spider threads and measure their diameter by means of Browning's spider-line recording micrometer. After many trials, I found the smallest of these measured 1-35000th in diameter (Fig. 4).

I measured them by means of Powell and Lealand's magnificent $\frac{1}{8}$ dry lens. On this spider thread I could perceive irregularities, nodules, and marks; but the general thickness was remarkably true. Some others measured $\frac{1}{10000}$, $\frac{1}{18000}$, and coarse agglomerations, cord-like, were as thick as fine spun glass $\frac{1}{3000}$ th.

It then occurred to me to make a novel use of the "Aerial Micro-meter" formerly described by me, consisting of the "Browning" inverted beneath the sub-stage, as also placed in a reversed position (see Fig. 3).

The law established contains two remarkable elements: the kind of light, i.e. the length of the wave, and the aperture of the objective. For blue light (wave $\lambda = 53000$ per inch) intermediate between blue and indigo, this, with an aperture of 150° , would give

$$\text{Extreme limit of visibility } \epsilon = \frac{1}{2 \sin. 75^\circ} = \frac{1}{53000 \times 2 \times .960}, \text{ nearly} = \frac{1}{102000}.$$

* This is thus stated:—

If ϵ represent the smallest interspace recognizable between two bright lines or disks, on the condition that the diffraction fringe of one does not overlap that of its neighbour; and

If λ be the length of the wave of light under consideration, which for mean rays equals $\frac{1}{48182}$ of an inch; and

If α be the semi-aperture of the objective,

$$\epsilon = \frac{\lambda}{2 \sin. \alpha} = \frac{\lambda}{2} \text{ (when aperture} = 180^\circ),$$

and

$$= 1-96590\text{th (when aperture} = 150^\circ).$$

So that with the more favourable blue ray the smallest interval visible among contiguous bright disks or lines is about one hundred thousandth of an inch, and that only with the largest aperture. Such is the belief disseminated.

About ten years ago I requested Messrs. Beck to make for me an "iris diaphragm" with "adapters" on each side. By this ingenious contrivance, screwed between an objective and the body, the angular aperture could be instantly reduced at will.

It seems, on the face of it, not a little surprising, considering this famous optical law, that the visibility of lines of great minuteness is very little affected by great reduction of objective aperture, by means of this instrument, or by using low-angled objectives of sufficient power and excellence of manufacture. Apparently this is another failure of the celebrated law, as roundly stated and generally received.

It will be convenient to explain here two practical points:—

A.—*The method used in finding the diameter of the spider lines enclosed within the micrometer.*

B.—*The method employed in measuring the absolute reduction of the object in miniature.*

A.—The Rev. Mr. Dallinger has given us his beautiful measurements of the flagella in monads, by drawing an equivalent line with a very hard fine pencil on white paper, by means of the *camera lucida*. By this process he, after a great many observations, determined its diameter to be less than the two hundred thousandth of an inch.

The plan I adopted was by finding divisions on glass placed in the focus of the eye-piece which appeared perfectly coincident in diameter with the observed spider line; and then substituting a scale of a hundred thousandth of a metre, a most careful measurement was made of the apparent size of the diamond cut. The process was much facilitated by altering the length of the draw tube, and changing the objective until the most acceptable result was arrived at. I am indebted to Mr. Beck for the use of an exquisite scale of this kind, as also for the loan of $\frac{1}{20}$ th objectives, dry and immersion, which latter has reduced the miniature to the extraordinary minuteness and precision of definition, at seven inches, of one hundred and forty times less than the object.

On examining spider threads, gathered after recent spinning, with Powell and Lealand's best $\frac{1}{8}$ dry, and measuring them with the spider-line micrometer inserted in the body, I was charmed with perceiving the characteristic brilliant central band, due to a minute cylindrical lens of great beauty, and perfection of definition: and searching for threads lying flat and in close contact, I found some consisted of four cylinders in contact, showing four bright

bands running longitudinally. Taking a pair of these, the cross wires of the micrometer were accurately adjusted in the centre of each bright space, the result for this order of spider was (making the power 1000) with the micrometer

$$\frac{6.7}{10000} = \frac{1}{15000} \text{th very nearly (see Fig. 1).}$$

Different spiders spin much thinner webs, and seem to unite several according to the tension required. Another fibre measured $\frac{1}{17600}$, and some are discoverable $\frac{1}{35000}$ th. See Plate III.

B.—The reduction by miniature will be readily understood from diagrams, shown Fig. 3. There are two ways of deciding the ratio of reduction: the one by examining the size of the miniature itself, the other by finding the magnifying power of the apparatus used as a Microscope.

For this purpose it was especially mounted on the arm of the Microscope used to carry the body (exhibited to the meeting).

In these ways it was found that:—

Immersion	$\frac{1}{8}$	Powell and Lealand,	miniatured 36.7 times at $4\frac{1}{4}$ inches.
„	$\frac{1}{16}$	b. Gundlach	„ 50.17 „ $4\frac{1}{2}$ „
„	$\frac{1}{20}$	R. and J. Beck	„ 140 „ 7 „

The distance between object and spider lines in the focus of the positive eye-pieces varied accidentally with the length of the objective mount itself. But the “Beck glass” required seven inches to do it justice, and also to get the miniature sufficiently reduced. It was easy to form the image at any desirable distance, but then the mirror could not be used very well beyond seven inches, nor the micrometer held sufficiently steady without complex arrangements. The one shown is simple and adequate.

The miniature can, it is evident, be carried to any extent; which, however, is limited to certain dimensions depending upon two important conditions—brilliance or darkness. A very brilliant line or disk is enlarged considerably, whilst a dark line is little changed.

If you miniature the sun’s disk by viewing an aerial image of it formed by a 3-inch lens (100” distant), employing a magnificent $\frac{1}{10}$ immersion, you will get a disk reduced 1000 times theoretically; and since $\frac{1}{38200}$ of an inch is the diameter of the image of the sun formed by the 3-inch lens, its diameter miniaturized on the stage is 1000 times less, or

$$\frac{1}{382000} = \frac{4}{100000} \text{ * nearly.}$$

* See ‘Proc. Roy. Soc.’ No. 146, p. 428.

But in the microscope it appears quite the ten thousandth of an inch, or nearly four times larger than it ought to be, if light had no undulatory waves. And this too, whilst using the most exquisite glasses obtainable. This tremendous fact shows how hopeless it is to expect brilliant disks to appear of the proper or natural size, if I may so speak, in the microscope.

In view of the extraordinary result of the measurement of solar spectra already alluded to, a very natural doubt will arise in the minds of those who have not had practice in this method of miniaturature, as to the correct effect of the glasses. Now the best process for solving the doubt is to watch the spider threads successively reduced from ten to fifty times. The operator will find it a slow process, as every possible adjustment of centricity and correction for aberration must be carefully attended to the whole time, as well as arranging the light. It cost me at first about six hours' work. But then the miniatures become so exquisitely smaller, the work in its very novelty becomes fascinating, and encourages one to persevere. The observer will have no chance of splendidly defining the millionth of an inch unless he is accustomed to high-power manipulation, and remembers that both upper and lower objectives must be both corrected by the screw collars for uncovered objects (dry or immersion), and change of distance of the focal images. Something too should be understood of the effect of change of "aperture" upon the appearance of a transparent cylinder of spider silk. It must be remembered that the aperture of the miniaturizing objective, as this is used in an inverted position, is greatly reduced as regards the incident pencils emanating from the spider lines.

A pencil of rays proceeding from the cross or intersection of the spider lines about six inches from the back glass, enters it at an aperture of a few degrees only, perhaps ten. Now if a cylinder of glass or spider gum be viewed with a low-aperture objective (say $1\frac{1}{2}$), it will present two black borders, and the breadth of these borders narrows as the aperture is increased, and *vice versa*. Also when the spider thread is diminished more and more, these black borders appear almost to coalesce until only a black line appears. The middle bright part vanishes with attenuation.

Then it may be further urged that a very fine glass forms miniatures of an object, theoretically, by merely optically reversing the rays as perfectly, indeed more so, than in the enlarged image of the same object. If therefore we can see the minute spider line very perfectly magnified one thousand times, we can, *a fortiori*, see the miniature, which is only fifty times smaller, with great precision. So much for the objection against the accuracy of miniatures formed by an excellently adjusted objective.

But a crucial test is supplied by observing sets of cross wires separated a small space. Fortunately I had requested Mr. Browning

to put a double set of cross wires, and also a set of parallel wires, in the micrometer. The head of the instrument is divided into one hundred parts, and a half or quarter part is readily seen with the naked eye. I may here observe, when the wires are reduced thirty-eight times by the $\frac{1}{2}$ (as one division of the micrometer is the $\frac{1}{100000}$ of an inch of motion in the wires), a single division for the miniature then reckons $\frac{1}{380000}$ th. But I found a quarter of a division made a perceptible difference in the apparent thickness of two coincident webs; *whilst three whole divisions separated the webs* so completely, that a narrow strip of light could be discerned between them (not much room here for swelling or enlargement of the lines!).

I then changed the glasses, putting the best glass in the body, and the older one (both newly formulated) in the micrometer: the definition was not so good. It required $3\frac{1}{2}$ divisions to separate the same lines.

This dividing of close lines by means of a very finely constructed micrometer is quite satisfactory to my mind, and I should hope conclusive as a *crucial* test to others who may witness it that the lines are very truly portrayed.

The following little circumstance has an interest of its own. Having conveyed my instruments home from the London Museum, S.K., I found the webs entirely covered with London dust. Upon getting them, however, into rapid vibration, I succeeded in shaking off nearly the whole before measuring them. A few minute particles adhere here and there; and though these webs are diminished fifty times—i. e. to the 300,000th of an inch—these particles of dust are visible on the web in this state of reduction. This result is the most surprising of all.

It was found that under this reduction (fifty times) it required five divisions to separate the spider lines, or a movement of $\frac{5}{100000}$ of the micrometer, i. e.

$$\frac{5}{50 \times 10000} = \frac{5}{500000} = \frac{1}{100000}.$$

Each division represented here on the micrometer head

$$\frac{1}{500000} \text{ of an inch}$$

in the field of view of the Microscope.

It is interesting to inquire what effect separating the spider lines has upon the discriminating power of vision. The optical conditions of seeing a black line upon a white ground, and separating or clearly dividing between two close minute black lines, are totally different. The researches of Dr. Jurin, 150 years ago, and of Dr. Robinson, F.R.S., the astronomer, on the subject, are very interesting; but no observations have yet been made of the minute-

ness about to be related. The question arose, Is it possible to estimate a bright space between two spider lines when *total* separation is only the eight millionths of an inch, the lines themselves being the 8000th and the 7000th of an inch respectively, and reduced in the miniature thirty-eight times? Reducing the numbers to decimals, if S be the space reckoned between the centres of the spider lines, it is evident if t and t' be the spider lines in diameters, and x be the required interval (see Fig. 5),

$$x + \frac{1}{2}t + \frac{1}{2}t' = S;$$

$$\therefore x = S - \frac{1}{2}t - \frac{1}{2}t' = S - (\frac{1}{2}t + \frac{1}{2}t').$$

The value of S was found by carefully measuring the movement of the micrometer = $\frac{3}{100000}$, which just brought the bright separating interval into view. Therefore we have the required size of interval (considering it diminished thirty-eight times),

$$\begin{aligned} x &= \frac{3}{38 \times 10000} - \frac{1}{2} \cdot \frac{1}{38.8000} - \frac{1}{2} \cdot \frac{1}{38.7000} \\ &= 0.00000789 - 0.00000164 - 0.00000187 \\ &= \begin{array}{r} 0.00000789 \\ - 0.00000351 \\ \hline \end{array} \\ &= .00000438 \\ &= \frac{1}{230000} \text{ nearly,} \end{aligned}$$

or about half the interval between the centres of the wires.*

The astounding sight of wires or webs separated by an interval of light less than the two hundred thousandth of an inch can only be explained by the light being subdued. Indifferent glasses cause diffraction images, besides clouding over the view with residuary spherical aberration much more difficult of cure than the colour. Without this interval—I may say, this extraordinary interval—one might conclude the webs are in some mysterious manner enlarged in the miniature beyond the calculated value. And so they are in poor glasses; for the image appears blurred—swelled, as it were—or adumbrated. But now the lovely precision of definition witnessed in high-class glasses, not only of the webs, but of dust on them and specks on the lamp-glass, precludes any suspicion, in face of this interval, of the enlargement of the lines encroaching much upon its dimensions. Besides all this, as the webs pass and repass

* Putting the decimals into fractions,

$$S = \frac{1}{127000}; \quad \frac{1}{2}t = \frac{1}{610000}; \quad \frac{1}{2}t' = \frac{1}{532000}.$$

The above calculation, it must be remembered, refers to the effect of the micrometer screw diminished thirty-eight times by the Powell and Lealand $\frac{1}{2}$ best immersion.

each other, the smallest movement of the screw changes their apparent thickness before division or separation is seen.

The miniatures were measured as follows :—

At Distance. Inches.		Miniature reduced. Times.
6 $\frac{1}{4}$	 Very old $\frac{1}{8}$ Powell and Lealand	49
6 $\frac{1}{2}$	 1862 $\frac{1}{8}$ Powell and Lealand (immersion)	58
6 $\frac{3}{4}$	 1875 $\frac{1}{8}$	55
7	 1878 $\frac{1}{8}$ R. and "J. Beck" (immersion)	140
7	 1878 $\frac{1}{8}$ Beck (dry)	118·6
6 $\frac{1}{2}$	 1873 $\frac{1}{8}$ Gundlach (immersion)	91·3
5 $\frac{1}{2}$	 1877 $\frac{1}{8}$ Zeiss (oil immersion)	49
6 $\frac{1}{2}$	 1863 1-inch Powell and Lealand	6·07
6 $\frac{1}{4}$	 1851 $\frac{1}{4}$ Andrew Ross	27·6
6	 1870 $\frac{1}{2}$ Wray	13·4
6	 1870 $\frac{1}{2}$ "	29·5

To accurately adjust the observing and miniaturizing objectives in the same optical axis is easily done with low powers. If both are equal in power, the test of the quality is very severe, as I have shown elsewhere.* With a then excellent Powell and Lealand $\frac{1}{8}$ made for me in 1862, and improved by them after its return to the makers, a fog is still seen when observed by their brilliant newly-formulated $\frac{1}{8}$ immersion. But still the spider lines are visible. It is not till objectives of equal and I may say of surpassing beauty of definition are opposed to one another above and below, nose to nose, that their exquisite powers of displaying fine black details are exhibited.

The Gundlach immersion is of very fine quality. On reference to the table, it diminished the spider lines 91·3 times when the distance between them and the miniature was 6 $\frac{1}{2}$ inches. This gave for the first and second lines ($\frac{1}{80000}$ and $\frac{1}{70000}$ th diameter respectively) miniature sizes of

$$\begin{aligned} \text{1st web} & \quad \frac{1}{730000} \text{ of an inch.} \\ \text{2nd web} & \quad \frac{1}{640000} \text{ "} \end{aligned}$$

The sizes of the web No. 1 with the different objectives may thus be tabulated :—

Dist.	Inches focus.	$\frac{1}{4}$ Ross	$\frac{1}{8}$ Powell	$\frac{1}{8}$ Powell and Lealand	$\frac{1}{80}$ Beck
6 $\frac{1}{2}$	1	1	1	1	1
	$\frac{1}{48000}$	$\frac{1}{220000}$	$\frac{1}{300000}$	$\frac{1}{460000}$	$\frac{1}{1120000}$

These were mostly at 6 $\frac{1}{2}$ or 6 $\frac{1}{4}$ inches. At a greater distance—10 inches—the diameters of the spider line of $\frac{1}{80000}$ with the two latter glasses would be,

$$\begin{array}{rcl} \frac{1}{8} \text{ Powell and Lealand} & & \frac{1}{80} \text{ Beck} \\ 1 & & 1 \\ \hline 640000 & & 1600000 \end{array}$$

* 'Phil. Transact.,' vol. ii., 1871.

These astonishing results, so contrary to what had been generally supposed, demand thorough investigation. And with a view to elucidate this unusually important subject, it will be interesting to inquire what is the visual angle of fine-line objects just visible by different observers.

	Diameter of Hair.	Distance visible.		Angle.
		feet.	seconds.	
Mr. Broun, F.R.S., 'Proc. Roy. Soc.' ..	·0026	36	$1\frac{1}{4}$	
Mr. Slack, P.R.M.S.	·003	$45\frac{1}{2}$	1	
With sun illumination and grey sky } background	·003	76	6-10ths	
Against white wall of house, sun still } shining	·003	113	4-10ths	
Lit up by sun glittering	·003	173*	..	

It is now requisite to determine what would be the visual angle of the spider line $\frac{1}{8000}$ of an inch miniaturd 140 times smaller with the Beck $\frac{1}{20}$, and then magnified up 1000 times by an eighth immersion with C eye-piece and about 10 inches of tube. Here

$$\text{Visual diameter of web } \delta = \frac{1}{8000} \div 140 \times 1000 \text{ at a distance of 10 inches.}$$

Hence

$$\sin. \theta = \frac{\text{Perp.}}{\text{Radius}} = \frac{\delta}{10} = \frac{1000}{140 \times 8000 \times 10} = \frac{1}{11200} = 18 \text{ seconds nearly.}$$

The most ready way of getting the value of the fraction in seconds is by recollecting that $60'' = \frac{1}{34\frac{1}{38}}$ nearly.

Referring now to the former table, it will be found by simple arithmetic that since the Beck $\frac{1}{20}$ immersion shows theoretically a visual angle of 18 seconds, miniaturd 140 times, a glass reducing only fifty-eight times ought to show at an angle of $7\frac{1}{2}$ seconds at a power of 1000, and at a power of 500 at about 4 seconds. I see the line plainly, most charmingly defined with 500, and can even see them when miniaturd only thirty times. A good deal might be written on this extraordinary fact. As the aperture of the objectives is diminished the spider lines look blacker, and therefore larger. I reserve this question for future treatment.

In inferior glasses the spider lines are thickened, and, besides this, garnished with secondary lines, true diffraction lines, and this you may see. I first detailed the method of miniatures in the 'Philosophical Transactions' eight years ago; but I have had nearly twelve years' experience of this method, and I have several times recommended it to the microscopical world with great cor-

* The glittering line here would afford a broad spurious line greatly enlarged.

diality. It is superior to all others for detecting residuary errors, and when these are nearly compensated the miniatures of spider lines of any size are portrayed with enchanting precision.

To sum up:—The whole question of minute vision is the least visual angle first of naked vision, and secondly in instrumental vision.

It can hardly be expected that any Microscope, especially if connected with miniature apparatus, involving the total use of some twenty lenses arranged as nearly as possible with one continuous optical axis,—that any Microscope, I say, can ever equal the simplicity of human vision. But then, with the unassisted sight we can easily determine the limits of vision by receding from the object, and so making the visual angle smaller and smaller until the hair vanishes. This we may call the vanishing angle θ .

Now the art, if I may so speak, of making very minute objects visible, may be applied by my method to render them distinctly visible as they get smaller and smaller as miniatures, and at last reach the vanishing limit.

But to my eye, which is, I must confess, the worse for these experiments, lines can be formed under the Microscope which also by lowering the ocular power, or diminishing the miniature, resemble (I will not say absolutely identify themselves with) the vanishing phenomena of naked vision.

When I see spider lines sharply defined become beautifully less, and give one the same appearance as a hair upon a window-pane, vanishing as its visual angle reaches the limit, I am bound to believe, nay be assured, though against all modern belief and theory apparently, that I do see these exquisitely small lines just on the point of evanishment at a very small visual angle indeed. Anyone with ordinary sight can see a human hair on a window-pane against a moderately white sky at a distance of two feet and a quarter. This is an angle of 20 seconds.

At five feet it is	..	..	..	Nine seconds.
„ ten feet it is	..	..	..	Four and a half seconds.
„ twenty feet it is	..	..	..	Two and a quarter seconds.

Now, on comparison of the minute lines exhibited by me microscopically, the hair lines appear equally small in each mode, either by viewing them on a window-pane at a yard off, or in the microscope diminished fifty times, and then sufficiently enlarged. The irresistible conclusion from this comparison is that the eye can discover a minute hair line either on the window-pane or in the apparatus exhibited, at certainly a smaller angle than 20 seconds. In other words, the minute microscopic image appears as small as a hair several feet off, according to the acuteness of vision.

The highest experimental proof by comparison is thus strongly in favour of a line sharply and clearly defined, subtending an

angle of 20 seconds, and probably a good deal less, as 2 seconds is the visual limit that can be seen in the apparatus or by the eye alone.

Another very curious point is worth mentioning. Dr. Jurin 150 years ago stuck two pins on a window-pane, and found that when placed near each other he could not divide them except when the interval between them reached the wide visual angle of 30". But when only one pin was viewed, he could distinguish it at a visual angle of from 2 to 3 seconds!

This interesting fact explains what I have witnessed in separating the spider lines of the micrometer in these miniatures: the interval could only be seen when the lines were separated, centre to centre, three divisions (micrometer), each division representing $\frac{1}{400000}$ when a minuendo of *fifty times* was employed; yet one can see a most sensible thickening of the gossamers just beginning to separate by moving the micrometer half a division. From this, I presume, a similar phenomenon was produced, though very much less pronounced than Jurin's case. It is marvellous to me that a visible bright space between these lines can be seen at all when their centres are separated only three divisions, i.e. $\frac{3}{1000000}$ or $\frac{1}{300000}$ of an inch. Considering that there must be some residuary aberration, however small, and that the error of each set of glasses accumulates in the final image presented to the eye, it seems to me wonderful that, notwithstanding Jurin's fact, a division is visible between the gossamers at all with so light a movement as described.

In continuation of this subject, I propose to offer to the Society some researches on the effect of large and small apertures in object-glasses. I beg to commend this research to the earnest attention of the rising generation of microscopists. Unless I am very much mistaken, the idea propagated in reference to the limits of microscopic vision is totally erroneous; whilst for brilliant lines or minute disks of great brilliance, I have not the slightest hesitation in embracing the truths conveyed in the exquisite formula presented to the microscopical world by, I believe, independently, Professors Helmholtz and Abbe.

It is almost needless to remark that very firm supports and delicacy of the adjustments as regards spherical aberration and illumination are essential to the success of this refined kind of definition.