So the cave maintains itself, and the cliff recedes and all the wash of storms, and the pry and thrust of wedging roots and freezing water are not equal in their work of degradation in these special conditions, to this insidious, gnawing lichen. And the question is raised whether we shall not find on more careful study, that much of the real work of undercutting in cliffs generally, is to be properly attributed to these extremely efficient workers. Shall we not find their fine hand in evidence in the curious, concave faces of the "navy yard" in the Dales of the Wisconsin river and all along the cliffs of the Mississippi. And in the southwest, in the undercut precipices where the cliff-dwellers found a safe retreat from the wild tribes of the plains; and in the cliffs of the Vezere, where the prehistoric Frenchmen made their homes.

FORCE AS AN ENERGY FACTOR.

BY A. H. SAGE.

State Normal School, Oshkosh, Wis.

The discussions that have appeared in this journal recently by Professor Crew and Professor Stewart relative to the nature and peculiarities of the law of force, $f=ma$, have been of interest to me because I have found my students experiencing the difficulties mentioned. There is a considerable number of similar difficulties pertaining to the formulated law of physics, which makes a thorough appreciation of these laws difficult for the beginner.

The same obscurity or confusion, for example, as that referred to above is seen in the use of f and v in the law of momentum, $ft=mv$, in which v must be taken as the velocity acquired through the action of the unbalanced force f , and not as including any initial velocity which the mass may have possessed; that is, as Newton puts it, it is the "change of momentum" and not the total momentum of a body that measures the impulse.

Another case along the same line which my students meet is the difficulty of accounting for the error in the calculation of energy from the law, $E=fs$, when the mass on which the force acts possesses an initial velocity, i . For example, a 20 gram mass

having an initial velocity of 42 centimeters is acted on for 6 seconds by a force of 80 dynes. Then

$$F=ma=80=20a.$$

$$a=4.$$

$$v=at+i=4\times 6+42=66 \text{ cm.}$$

$$s=\frac{1}{2}at^2+it=\frac{1}{2}\times 4\times 6^2+42\times 6=324 \text{ cm.}$$

$$E=fs=80\times 324=25920 \text{ ergs.}$$

$$E=\frac{1}{2}mv^2=\frac{1}{2}\times 20\times 66^2=43560 \text{ ergs.}$$

Now, it is readily seen that the difference in the two results is due to the energy of the initial velocity which is included in the 43560 ergs, but not in the 25920 ergs calculated from the law $E=fs$. Nevertheless, the student who has just calculated the value of s from the law of acceleration, $s=\frac{1}{2}at^2+it$ usually replies that the initial velocity was included in the calculation of fs as well as in the other case since the space s contains the result of this initial velocity. Of course, the error of his reasoning lies in the fact that while the functioning capacity of a force is dependent on the space through which the force acts on the body, it is independent of the velocity of the body. Therefore, while the force really acted through a greater space on account of the initial velocity, its capacity for imparting energy was thereby increased in proportion to the increase in the space covered, but not in accordance with the extra velocity which caused that increase in space.

The appearance of this difficulty in the student's course affords a suitable setting for the elucidation of a further application of the law, $E=fs$, in such cases as that of the steam pressure against the piston of an engine. In such cases the body impressed (eg., the piston) is not a free body. Because of the resistance, the piston does not move through the space it would cover if free. Here we see the reverse application of the principle of conservation of energy exemplified above. In that case the increased space but not the increased velocity was a factor in the transference of energy; here, the decreased space covered, but not the subdued velocity, counts in the calculation of the energy imparted. From a physical point of view, it is such considerations as these that lead to the statement that $E=fs$ is the law of work, while $E=\frac{1}{2}mv^2$ is more strictly the true expression of the law of energy.

Every teacher in the course of his experience develops excellent methods of presenting some of these difficult matters to his classes. These pages afford a very suitable agency for the exchange of these acquisitions, and they are certainly needed by most of us. Perhaps, others like myself have hesitated to put in print their own petty experiences in these connections lest some technical inaccuracy of expression or detail make the endeavor seem crude or unscientific. But I am confident that any critical suggestions that may appear in these pages will have only helpfulness for its aim. It will be a great pleasure to me to see contributions along these lines appearing from time to time.

CORRELATION OF MATHEMATICS WITH BIOGRAPHY, HISTORY AND LITERATURE.

By JOS. V. COLLINS.

Stevens Point, Wis.

In the recitations of mathematics, especially geometry, the mind is usually given up to an exercise of pure thought. As a relief during the progress of the recitation teachers often interject extraneous topics. It is better, of course, to give something that relates to the subject and which is calculated to stimulate the class to their best endeavors. Perhaps the finest material available for use for such object is the experiences of men who figure prominently in the history of literature or science studies of the student.

If a boy who is not particularly interested in geometry but who is intensely interested in, say Washington, through his study of history, learns that Washington was once a student of the same branch, and its study early developed for him into a means of earning money to supply his mother's lack of funds, he may be led to take a greater interest in the subject. Or if another who has trouble with his geometrical demonstrations, but who is a great admirer of the poet Longfellow, learns that the latter found more difficulty in mastering the hard problems of mathematics than he did in any other of his studies, but who nevertheless never allowed himself to come to class without thorough preparation, he also may be stimulated to more faithful endeavor.¹

¹ Life of Longfellow edited by Samuel Longfellow, p. 28.