V.—DISCUSSIONS.

THE UNIVERSAL AND THE *A FORTIORI*.¹

MR. PICKARD-CAMBRIDGE formulates the following arguments:—

- (1) A is greater than B, B is greater than C, $\therefore$ A is greater than C;
- (2) A is next to B, B is next to C, $\therefore$ A is next to C;
- (3) A cheats B, B cheats C, $\therefore$ A cheats C;

and he asks what the principle is that discriminates (1) that is valid from (2) and (3) which are invalid. We see directly, he says, that relations of the kind specified between A and B, and between B and C necessarily involve, in the case of (1) a corresponding relation between A and C, and that in the cases of (2) and (3) they do not. 'This apparently does not satisfy Dr. Mercier.' I beg Mr. Pickard-Cambridge's pardon: it does satisfy me. It satisfies me completely and entirely. It is, in fact, my case. It is what I am contending for. We see *directly* that the conclusion in (1) is valid, and that those in (2) and (3) are not. We see it directly, without any reference to a universal outside of the argument. We see it directly, not indirectly through a universal. That is my point. That is the point at issue. I hold, as Mr. Pickard-Cambridge says, that the conclusion can be reached, and is in fact reached, from the postulate alone, that is, from the premisses as they are stated, without praying in aid a *principium* from outside.

The criticism raises, says my cross-examiner, two points: (a) that the major or universal premiss (the *principium*) is not derived from the postulate (A is greater than B, and B than C); and (b) that it is unnecessary to the inference.

On the first point, he says, it is perhaps enough to say that the major premiss of a syllogism is not usually 'warranted' by the minor, or derived from it. I agree that the major of a syllogism is

¹ *Note by the Editor of MIND.*—Dr. Mercier originally included in the above article an extended reply to Mr. Shelton's charge of plagiarism and other personal accusations. I have requested him to omit this reply on the ground that a continuance of such personalities in the pages of *MIND* is intolerable. In fairness to Dr. Mercier it is necessary to add that in the opinion of the Editor, Mr. Shelton's charge of plagiarism is utterly baseless and that his abusive language was quite unjustifiable and in every way regrettable. The Editor apologises to readers of *MIND* and to Dr. Mercier for having allowed such language to appear at all.

not derived from the minor, but I do not agree that this is enough to say. I have a good deal more to say about it. In the first place, I have to say that the *a fortiori* is not a syllogism. In my opinion the major and minor premisses of a syllogism are both of them postulated, and the major is not derived from the minor, but is part of the original postulate. But the *a fortiori*, as it stands in (1), is not a syllogism; and when we try to convert it into a pseudo-syllogism by concocting a universal to fit it, as Mr. Shelton tries to do, then, I say, so far from deriving our conclusion from the universal, we derive our universal from the conclusion, or rather, from the argument as a whole, after it is completed.

If A is greater than B, and B is greater than C, then we conclude, *directly* as Mr. Pickard-Cambridge asserts, that A is greater than C. But what do we mean by A, B, and C? We may, if we please, use A, B, and C as proper names of individual things, and then, say the logicians, we require a universal outside and beyond the argument to validate and legitimatise our conclusion. But it has escaped their notice that A, B, and C may not be proper names of individual magnitudes. They may be symbols standing for any magnitudes whatever, and in that case the *a fortiori*, as stated in (1), is already a comprehensive universal, and no universal constructed on the same lines can be more comprehensive or more universal. For if A, B, and C stand for any magnitudes, they stand for all magnitudes, for 'any' and 'every' alike mean all. 'Any' means one taken at random as a sample of all that for the purpose in hand are alike. 'Every' means all taken one by one. The difficulty of concocting a universal to include this example of the *a fortiori* is now explained. It is an endeavour to formulate a universal more comprehensive than an equivalent universal expressed in other terms, that is to say, virtually more comprehensive than itself. The concocted universal of Jevons, Mr. Sidgwick and Mr. Shelton means exactly the same as the *a fortiori* in (1), neither more nor less. Now I aver that whether we take the A, B, and C of this argument to be proper names of individual things, or whether we take them to be symbols representing any magnitudes whatever, in other words, whether we take the propositions as particular or as universal, does not matter a straw to the validity of the argument, or to our ability to arrive at the conclusion *directly* from the premisses. If the conclusion is valid in the universal, it is valid in the particular, and if it is valid in the particular it is valid in the universal; and it does not matter in which order we take them. Moreover, if we need a universal to validate the particular (which is, or may be, already a universal) then we need a higher and more comprehensive universal to validate the J.-S.-S. universal; and a still higher and more comprehensive to validate that, and so on *ad infinitum*.

Nor must it be forgotten, though it usually is forgotten, that the argument: If A is greater than B, and B is greater than C, then A

is, greater than C, is not the only instance of the *a fortiori*. Far from it—there are many others. Here is one:—

- (4) If A is less than B,
and B is less than C;
then, *a fortiori*, A is less than C.

And here is another:—

- (5) If I answer an antagonist who is confused and discourteous,
Then, *a fortiori*, I shall answer one who is clear and courteous.

These are arguments *a fortiori* as much as (1), but the universal so painfully and laboriously concocted for (1) will not fit these. Now, either the mental process employed in reaching the conclusion is different with every specimen of the *a fortiori*, or it is the same in all. I do not think that any one who interrogates his own mind, and watches the march of the reasoning therein, will contend that it is different. Clearly, all these arguments are on the same model. Are they then governed by the same universal? Logically they should be. If they are, what is this universal? and who will formulate it? It is a sum in proportion. If it takes three logicians to concoct a universal to cover (1), how many will it take to concoct a universal to cover (1), (4), and (5)? Nine, or the square of nine? I suggest that the last figure is the nearer the mark. If it is said that though the principle, that of subsumption under a universal, is the same in every case, yet the universal need not be the same in every case, then I ask on what ground it is asserted that the principle underlying the *a fortiori* is subsumption under a universal. To this question I can get no answer. As far as I know there is no answer except that Aristotle said so, and this does not satisfy me. If there is any other answer, perhaps Mr. Pickard-Cambridge can succeed where Mr. Shelton has failed, and can give that answer. Some principle there must be that validates every instance of the *a fortiori*. If it is, as the logicians say it is, subsumption under a universal, let us have that universal. If it is not subsumption under a universal, then it must be some other principle. I maintain that it is another principle: that it is the principle of substituting one term for another which implies it. As the explanation of this principle given in my *New Logic* is not clear to Mr. Pickard-Cambridge, I will try to explain it more clearly:—

- If A—is greater than—B,
and B—is greater than—C;
then, *a fortiori*, A—is greater than—C.

If the conclusion is compared with the first premiss, it is seen at once that the conclusion differs in nothing but in the substitution of C for B. Whatever the process by which the result is reached, it cannot be denied that this is the result. The result is the substitution of C for B in the first premiss, or of A for B in the second. Of this there cannot be a shadow of doubt. The

whole question at issue is how this substitution is effected. The contention of the old Logic is—I am putting it in my own way, and not as it is taught in the books—that the substitution is effected by means of an analogy, using the term analogy in its strict and proper sense of a comparison of relations. They say that we have already in our minds a universal, in which the terms, instead of being singular, individual, and particular, as they may be in the instance before us, are plural, general, and universal; that we can recognise the validity of the argument when the terms are plural and universal, though we cannot recognise it when they are singular and particular; and that, the relations being the same, we argue by analogy that if it is true in the universal it must be true in the particular also. (The former contention of the old Logic was that the universal term of the universal proposition includes the very particular about which we are arguing, but I do not think that this position, evidently false as it is, is held now.) All these contentions I deny. I deny that we have the universal in our minds, and in proof of my denial I point to the fact that for ages logicians have been trying to discover and formulate this universal, and that no logician has succeeded to the satisfaction of any other logician. Moreover, I take the latest effort, the J.-S.-S. formula, and I am able to show that it means exactly the same as the *a fortiori* which it is put forward to validate. *Secundo*, I deny that it is any easier to recognise the truth or the validity of the *a fortiori* when stated generally than when stated particularly. We see its validity just as soon, just as plainly, just as directly, when we take the terms A, B, and C to be proper names of individual things as when we take them to be symbols of any magnitudes we please. This being so, it follows of course that I deny that the argument is an analogy, and I deny this on another ground also. Analogy is the comparison of relations, and 'the nature of an analogy is not altered, and its truth is not altered, by altering the nature of its terms, as long as the proportion (or relation) between them is maintained' (*New Logic*, p. 346). For this reason I deny that the truth of the *a fortiori* is any more evident in the universal than in the particular, and I invite any candid person to read the J.-S.-S. monstrosity (*MIND*, 93, p. 77) and say whether its truth is any more manifest than that of the *a fortiori* as stated in (1), taking the terms as proper names.

So far from this being the course of the argument, I aver that it is the very reverse. We see the validity of the *a fortiori* as soon as it is stated, and not until we have seen its validity in the individual case before us do we recognise by analogy that it must be true in any and every case in which the relations are similar. The universal is not anterior but posterior to the argument. No one ever thought of finding a general formula to include the *a fortiori* until he had seen an individual case of the *a fortiori*; and when we try to formulate a universal to fit that or any other

argument, we do so with an eye constantly on the individual instance, and construct our universal to fit the particular. I do not say this is so in the ordinary syllogism, in which a universal is given, but I say that when we are fishing for a universal, as we have to fish to find one for the *a fortiori*, we have the particular case first in our mind, and from this case, and in adaptation to it, we construct our universal. Is it plausible, is it reasonable, to contend that we reach our conclusion through a universal that we never formulate until after the conclusion is reached, that we cannot formulate without difficulty, and that, when formulated, is only fit to be laughed at? So hard put to it are those that uphold this view, that Mr. Shelton now says that we have the universal unconsciously in our minds (MIND, 96, p. 527). This is delicious. An unconscious universal in the mind may take rank with an immaterial dinner in the stomach, an unresisting support beneath the feet, or an impalpable weight in the hand. It is one of the garments that make up the Emperor's clothes.

It is Mr. Pickard-Cambridge's second query that he says is the crux. He quotes the rule I have laid down, that any term in a proposition may be replaced by any other term that *for the purpose of the argument* is implied in it, and my comment that the words italicised make all the difference, and he goes on to say that (presumably) the difference between the valid argument (1) and the invalid arguments (2) and (3) is that in the former the third term may for the purpose of the argument be substituted for the second, while in the latter it may not. So far I agree. But then he goes on to say that this difference is made by me to depend on some difference in the purpose of the argument in the two cases. Naturally therefore we ask 'What is the difference in the "purposes" of the two arguments?' This is the point at which the misunderstanding occurs. The passage that Mr. Pickard-Cambridge quotes should be read with its context, which shows that, in the case I was then considering, attention to the purpose of the argument would save us from a particular fallacy or invalidity; but does not show that it will save us from every fallacy or invalidity. It is true that in every case we must pay regard to the purpose of our arguments in order to avoid invalidity, but it is not true, and I have never said, that attention to the purpose of the argument will of itself ensure validity, or enable us to avoid every source of invalidity.

In the argument 'If A cheats B, and B cheats C, it is equally clear that C is not, for the purpose of the argument, implied in B'. Thus Mr. Pickard-Cambridge quotes me; and he goes on to say that it is indeed clear that the inference 'A cheats C' does not follow; but the problem is to know how that *non sequitur* is dependent on any difference in the purpose of the argument. He says I do not reveal this. Here again there is a misunderstanding, and I am glad of the opportunity of clearing it up. Again we

must take the passage with its context. I was insisting that the argument $A : B$, and $B : C$ does not always justify the conclusion $A : C$, and used the relation of cheating, among others, to illustrate the point. The argument is not valid, I asserted, unless C is, for the purpose of the argument, equivalent to B or included or implied in B ; and if cheating or sundry others be taken as the common relation, then this condition is not fulfilled. I could, of course, have said that if A cheats B , and B cheats C , then C is not for the purpose of *any* argument implied, etc., in B ; but in fact I was dealing solely with the argument $A : B, B : C, \therefore A : C$, and this is *the* argument to which I was referring. The *non sequitur* does not depend on any difference between the purpose of the argument about cheating and the purpose of the *a fortiori*, for if we endeavour to get from the cheating premisses a relation between A and C , our purpose is, *mutatis mutandis*, the same as the purpose of the *a fortiori*. It is to obtain a relation between A and C . The *non sequitur* nevertheless does depend on the purpose of the argument, for it results from taking for the purpose of the argument premisses that are not adapted to that purpose, or, to put it otherwise, applying the premisses to a purpose that they will not serve. The importance of the purpose of the argument is not only internal to the argument, in preventing such treatment of the premisses as is inconsistent with the purpose, but also is external or preliminary to the argument, in guiding us to the choice of premisses suitable to our purpose, or, in logical exercises, finding a purpose that the premisses will suit.

'Let us suppose,' says Mr. Pickard-Cambridge, 'that the question "valid or invalid" does depend on the purpose of the argument, and not on some essential difference between the kinds of relation referred to.' But he is not entitled, from anything I have said, to suppose any such thing until appropriate premisses are stated for some purpose. *The validity or invalidity of an argument depends (inter alia) upon the adaptation of the relation in the premisses to the purpose of the argument.* That is my point. That is my position. That is one of the most important matters in which the *New Logic* differs from the old. The old Logic never mentions the purpose of an argument, and never recognises that an argument has any purpose except to be crammed into a syllogism. And the old Logic assumes and declares that in argument or reasoning there is only one relation that can possibly be used—the relation of subsumption. The *New Logic* asserts not only that every argument must have a purpose, and that its purpose is vital to its validity, but also that any relation whatever, even subsumption, can enter into argument or reasoning as freely as any other, provided always that the relation is adapted to the purpose of the argument, or, if the relation is given, that a purpose adapted to it can be found.

The argument (3) about cheating illustrates both assertions. If

A cheats B, and B cheats C, then from these premisses we cannot conclude that A cheats C, for to this purpose the premisses are not adapted. They give no warrant for substituting C for B in the first premiss, or A for B in the second, and therefore they reveal no relation between A and C. But it would be a very gross mistake to suppose that these premisses yield no conclusion, and an equally gross mistake to suppose that there are no premisses from which we can draw the conclusion that A cheats C. If A cheats B, and B cheats C, nothing is easier than to find a purpose that will fit these premisses, such that a valid conclusion may be drawn from them by paying regard to this purpose. Let the purpose be to find a similarity between A and B: then these premisses will justify us in concluding that both are cheats. Let the purpose be to find a similarity in the fortunes of B and C: then the premisses will allow us to conclude that both are cheated. Or, if our purpose is to discover the maximum of honest men among them, we can conclude from the same premisses that it is not more than one. But these are mere academical exercises. Their artificiality is stamped upon them. In real life we do not take up arguments and then seek a purpose that will fit them. We start with a purpose, and seek premisses that will serve that purpose. There is no argument without a purpose, except in Logic; and except in Logic, the purpose dominates the argument.

If we want to discover a relation of cheating between A and C, the premisses in (3) will not serve our purpose, but it is not difficult to discover premisses that will. The first Canon of Inference is that we may postulate what we please (saving self-contradictions) and we may, therefore, state the following premisses:—

If A—obtains money by false pretences from—C,
and Obtaining money by false pretences—is—cheating;
then A—cheats—C.

In this case we substitute, not an implied term for one that implies it, but a ratio for one to which it is equivalent—to which it is given equivalent by the second premiss. If we compare the conclusion with the first premiss, it is incontestable that they do in fact differ solely in the substitution of one ratio for another. If we look at the second premiss it is incontestable that it does in fact declare the equivalence of the two ratios. If we consult our common sense, it will tell us that any term or ratio in an argument may be replaced by one that is, for the purpose of the argument, equivalent; and that is what is done in this argument. Where is the necessity for a universal?

I should therefore disagree on more than one ground with Mr. Pickard-Cambridge when he says 'the hinge of a valid argument is not its purpose but a nexus of relations which is necessary—therefore universal'. I say that the hinge of a valid argument is neither the purpose nor the relation, but the adaptation of the one to the other. An argument may contain a nexus of relations,

or it may contain but one, but it need not contain a universal. In nearly every case in which a universal is declared to exist in an argument, it is not in the argument, but is faked up, like the J.-S.-S. monstrosity, outside of the argument. It is never in the mind until after the argument is stated, the conclusion reached, and the fancied necessity of a universal remembered. Then it is easy to universalise one of the premisses, or, as in the J.-S.-S. monstrosity, the whole of the argument, and to make believe that it is in the argument, or is necessary to the argument. Then you have your universal. It is there, but it is not in the argument, and is not extracted out of the argument. It is faked up and added to the argument as an unnecessary and disfiguring excrescence.

As soon as it is recognised that any relation, even that of cheating, can enter into argument, not only does the necessity for a universal disappear, but also its practicability is in most cases abolished; for a universal, *quid* universal, can enter into only one relation, that of subsumption. It is curious that logicians should still insist upon the necessity of a universal in face of the admission into logic of arguments about individuals. In Scholastic days, when Logic was comparatively logical, the individual was shut out of logic, for an individual that, *quid* individual, is also universal, is a contradiction in terms. An individual and a universal are as antithetic as any antithesis can be, and if a universal is necessary in every argument, then there cannot possibly be any argument about individuals alone. However, in fact, arguments about individuals are constantly framed, constantly used, and constantly acted on; and at long length they have been admitted into logic—in a wooden horse.

If Wren designed St. Paul's
and St. Paul's is a cathedral church;
then Wren designed a cathedral church.

I ask any person with a reasonably open mind, which is the more likely, which is the more reasonable view to take,—That since we are told by the second premiss that a cathedral church is, for the purpose of the argument, equivalent to St. Paul's, therefore it may be substituted for St. Paul's in the first premiss, or that the conclusion is reached through some such universal as Every one who designs a church which is also a cathedral church designs a cathedral church? Besides, supposing the impossible, supposing this universal does enter into the argument, and is our warrant for the conclusion, what is our warrant for the truth or validity of the universal? Is it not that we see at once the equivalence of the terms, and recognise that one equivalent term may be substituted for another? And if we may do this in the universal, why may we not do it in the particular argument itself? why take two bites at a cherry—a cherry? a green pea! Yet this is what logicians do suppose, *mutatis mutandis*, with the *a fortiori*. In both the *a fortiori* and the argument about St.

Paul's is it not plain and manifest beyond all question and doubt that the universal is never used in the argument, but is faked up after the conclusion is reached? Can any one go through either argument and then assert that he had the universal in his mind while he was doing so? Is it not clear that the universal is never in the mind until the argument is complete, and then it occurs to the reasoner that, according to the rule he has been taught, there ought to be a universal knocking about somewhere, and that he had better find it or fake it? He cannot find it, and so he has to fake it, with what difficulty Mr. Shelton's efforts show. Will the most hide-bound logician assert seriously that there is a universal in the St. Paul's argument? And if there is none in that, what is the necessity for one in the *a fortiori*? How is it that there is all this pother and difficulty in framing the universal if it is in the mind all the time, and you cannot reach your conclusion without it? I put to Mr. Pickard-Cambridge the same question to which I can get no answer from Mr. Shelton. They say the argument *must* contain a universal, but why must it?

There is little to rejoin to in Mr. Shelton's reply.

I said that his assertion that I throw induction and deduction into a hotch-potch must be understood in a sense exactly the opposite of that in which the words are commonly used. Mr. Shelton now says what he meant was that I draw six differences between induction and deduction. On his own showing, therefore, my statement was correct.

He produced a universal that he said is—or is not, he is not clear which—the universal in the *a fortiori*. I answered that he had not found the universal. He now rejoins that he *has* found it, —and produces another one!

I asked him what reason he had for saying that every argument must contain a universal. He does not answer. The conclusive presumption is that he cannot.

He says the inference A is next to B, B is next to C, $\therefore$ A is next but one to C, is certainly wrong. It is certainly right in the ordinary and natural meaning of the words, that there is not more than one thing between A and C. He must be pretty hard put to it when he has to drag in the space of Reimann and the space of Lobatschewsky.

I said I doubted if Mr. Shelton knew, when he called my logic formal, what he meant by formal. He calls it formal again, but he does not explain what he means. The conclusive presumption is that he cannot.

I asked him how I had confused two spheres. He does not answer. The conclusive presumption is that he cannot.

He said that while professing to carry out the explication of what is implied in propositions, I had not really done so. I challenged him to find one implication that I had overlooked; and to encourage him I told him that since the publication of my *New*

Logic I had myself discovered some that are not described therein. He neither takes up the challenge nor apologises.

CHAS. A. MERCIER.

POSTSCRIPT.

One or two minor matters I relegate to a postscript.

For reasons given in my *New Logic*, I think that every deductive argument should be stated for what it is—a hypothetical argument. In my opinion we have no right to assert, and it is misleading, even if not actually untrue, to assert, that A is greater than B, or next to B, or that A cheats B, unless we have in our mind some specific A and B, refer to them as actually existing things or persons, and have reason to believe that what we say of them is true. If our argument is hypothetical, as in the cases in the text it is, then I think it should be stated hypothetically, and should run—*If A is greater than B, If A is next to B, and so on.* My reasons are many, but I will give only two, which I think are enough. In the first place, if the rule is adhered to it is impossible to mistake a deductive argument for an inductive argument, and the difference between them is made striking and manifest. In the second place, by stating the syllogism hypothetically we clear up completely the much-debated nature of the universal therein, and we solve with ease the excruciating problem of 'the *petitio principii* inherent in the syllogism'.

From several expressions in his paper it is evident that Mr. Pickard-Cambridge does not expect me to answer his questions, for he warns me repeatedly of the consequences of not doing so. I am afraid he will be disappointed to find that I do answer them, but I feel sure that he will not allow his disappointment to prejudice him in examining my answers. I can assure him that, accustomed as I am to be condemned without being read, or after being read unintelligently, it is a pleasure to meet a critic who is both honest and acute.

In my answer to Mr. Mayo (*MIND*, 95, p. 396) I said that for the last thirty years logicians have not claimed any utility for Logic. I might have extended the period considerably. Two hundred and fifty years ago the logicians of Port Royal wrote in the following terms, as translated by Mr. Ozell, just 200 years ago: 'This part [the syllogism] that we have now to handle, which comprehends the Rules of Reasoning, is esteemed the most important part of Logic, and is what is generally discussed with most care: But there is cause to doubt whether it is so useful as is imagined. . . . However, even though these Rules be considered only as speculative Truths, they will at least serve as an exercise to the Mind: And besides, it cannot be denied that they are of some use upon certain Occasions, and to certain

Persons, who, being of a lively penetrating Genius, do sometimes suffer themselves to be carried away by false Consequences, only for want of Attention ; which may in some measure be remedied, by the reflection they will make upon these Rules. Be it as it will, you have heard what is generally alleged in their Favour, and even something more.'

C. A. M.