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A Method of Determining the Numerical Value of Elasticities of Demand

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on rent, column four the percentage of column three spending *over 30 per cent.* on rent.

Weekly wage.	Percentage of workers spending		
	Up to 25 per cent. on rent.	Over 25 per cent. on rent.	(Of col. 3) over 30 per cent. on rent.
Up to 18 m. ...	62	38	20
18 „ 21 ...	66	34	13
21 „ 25 ...	77	23	10
25 „ 33 ...	89	11	4
30 „ 35 ...	94	6	1
Over 35 ..	97	3	—

In other words, of those earning under 21 m. a week more than a third pay away *over 25 per cent.* for rent; of those earning up to 18 m. a week one in five pays away *over 30 per cent.* for rent.

It is true that in a large number of cases the payment of these relatively excessive rents is facilitated by underletting or by the earnings of other members of the family being available to supplement the father's wages. But there can be no doubt that in the majority of the cases where the rent paid amounts to more than 25 per cent. of the wage received by the father the family suffers in its internal economy. The amount available for wholesome food and clothing is reduced below the minimum essential for physical efficiency. Last but not least, high rents among the working classes invariably spell overcrowding, with all its attendant evils.

ERNEST LESSER

A METHOD OF DETERMINING THE NUMERICAL VALUE OF ELASTICITIES OF DEMAND.

§ 1. An increasing amount of attention is being paid by State Departments and private persons to the study of workmen's budgets, and a number of tables have already been printed to show the proportion of their income which families in different income grades expend upon various principal groups of commodities. I wish to suggest that the statistical material thus made available affords opportunity to patient arithmeticians to investigate by a new method the magnitude of the elasticities of demand of some important consumable commodities.

§ 2. To exhibit the nature of this method I shall suppose that our data are better than they are, and that we possess tables showing the expenditure of the group of workpeople whose wages lie between 20s. and 21s., of the group whose wages lie between 21s. and 22s., and so on continuously for all wage levels. With this close grouping we may fairly assume that the tastes and temperament of the people in any two adjacent groups are approximately the same. That is to say, the quantity of "utility" obtained from the x th unit of any commodity, say tea, is equal for typical men in the 20s. to 21s. group, and in the 21s. to 22s. group. Let this quantity for all values of x be fx ; or, in other words, let the utility curve in respect of successive increments of the commodity tea be represented by $u = fx$. We are entitled to assume further, in the absence of special evidence of correlation of demand, that the utility curve of both groups for tea is approximately independent of the quantity of other commodities consumed, and, therefore, of the marginal utility of money. Let this marginal utility to the lower and higher wage-groups respectively be μ_1 and μ_2 , and the quantity of tea consumed by these groups x_1 and x_2 . Then, since the price paid for the commodity must be the same for both groups, we know that this price p is equal both to $\frac{1}{\mu_1} fx_1$ and to $\frac{1}{\mu_2} fx_2$. These two expressions are, therefore, equal to one another. But if, as is reasonable to suppose when the wages of the two groups are close together, x_2 differs only slightly from x_1 , fx_2 may in general be written $fx_1 + (x_2 - x_1) f'x_1$.

$$\therefore f'x_1 = \frac{1}{x_2 - x_1} \cdot \frac{\mu_2 - \mu_1}{\mu_1} fx_1.$$

But the elasticity of the utility curve in respect of any consumption x_1 is known to be equal to $\frac{fx_1}{x_1 f'x_1}$. Therefore the elasticity of the utility curve for tea in respect of both our groups

$$= \frac{x_2 - x_1}{x_1} \cdot \frac{\mu_1}{\mu_2 - \mu_1}$$

But, since a small change in the consumption of any ordinary commodity on which a small proportion of a man's total income is spent cannot involve any appreciable change in the marginal utility of money to him,¹ the elasticity of the utility curve in

¹ Strictly, of course, such a change must involve some alteration in the marginal utility of money unless the demand of the commodity in question has an elasticity equal to unity. If the elasticity is anything other than this, a change in its consumption will be accompanied by a transference of money from expenditure upon it to expenditure on other things, or *vice versa*. This must effect the marginal utility of money in these fields, and its marginal utility, if affected in one field is, since it must be equal in all, affected in all.

respect of any consumption x_1 is equal to the elasticity of the demand curve in respect of that consumption. Therefore, the elasticity of demand for tea of the lowest wage-group, in respect of its consumption of x_1 units of tea, is given by the equation :

$$\eta = \frac{x_2 - x_1}{x_1} \cdot \frac{\mu_1}{\mu_2 - \mu_1}$$

If we knew the relative values of μ_1 and μ_2 , this equation would enable us to determine the elasticity of demand of the lowest wage-group for all commodities in respect of such quantities of the commodities as they are consuming. Similar equations would enable us in like manner to determine the corresponding elasticities of each of the other wage-groups. If it is objected that our result would in practice be impaired by the fact that the higher wage-groups are apt to consume a better quality of commodity, as well as a greater quantity than the lower wage-groups, the difficulty is easily overcome by substituting in our formula for the quantities consumed (x_1 and x_2) of the commodity in question figures representing *aggregate expenditures upon it*. This device escapes the suggested objection by treating improved quality as another form of increased quantity.

§ 3. The result established in the preceding section is not, however, the one which prompted me to write this note. For, in order to obtain quantitative knowledge of the magnitude of any elasticity from that result, we need to evaluate μ_1 in terms of μ_2 ; and this we cannot do. The point to which I wish to draw attention may be displayed thus. By the process indicated above, the elasticities of demand in any wage-group can be determined as regards all the things consumed in that wage-group in expressions involving μ_1 and μ_2 . Let these elasticities be e_x, e_y, e_z , and so on. *It follows that any one of these elasticities can be determined in terms of any other without reference to μ_1 and μ_2 .* These unknowns are eliminated, and we obtain the formula

$$e_y = e_x \cdot \frac{x_1}{x_2 - x_1} \cdot \frac{y_2 - y_1}{y_1}$$

This result, it should be observed, only follows directly from the preceding argument provided that the commodities concerned are both such that only a small part of a typical man's income is normally spent upon them. In general, however, though the absolute formula for elasticities from which the result is derived is only valid on this assumption, the above comparative formula is approximately valid also in respect of two commodities on which a large part of a typical man's income is spent, provided that the part spent on the one does not differ greatly from that spent on the other. The reason for this is that the errors in the two

formulae for absolute elasticities which have to be combined will tend to balance one another. The only case, therefore, in which our comparative formula must be seriously suspected arises when it is used to obtain the relative elasticities of the demands of a group for two things on one of which it spends a large and on the other a small proportion of its income. Apart from this case, the formula, when applied to the statistics of quantities of, or expenditures upon, different commodities by neighbouring wage-groups, enables us to determine numerically the ratio of the elasticity of demand of any wage-group for any one commodity (in respect of the quantity of the commodity actually consumed by it), to the elasticity of demand of the group for any other commodity. This information will often be valuable in itself. It is important to know whether the demand of workers with 25s. a week for clothes is about twice or about ten times as elastic as their demand for food. But the information is also valuable indirectly. For by it, if we can in some other way—through the examination of shop-keepers' books or otherwise—determine the elasticity of demand of a group of workpeople for one thing, we have a bridge ready along which we may proceed to determine the elasticity of their demand for all other things.

§ 4. In explaining the above method I have, as indicated at the outset, tacitly assumed that our data are better than they are. This, I think, is legitimate, because there is no reason in the nature of the case why these data should not be improved; and, indeed, there is little doubt that they will be improved. Even then, of course, anyone attempting a detailed application of the method is certain to encounter serious difficulties, among which, perhaps, not the least will be that of deciding how far to treat different commodities separately and how far to group them together according to the purpose which they jointly serve. When put to the test these difficulties may, no doubt in some cases, prove insurmountable. From the results of an experiment made upon figures given in the second Fiscal Blue-book (pp. 215 and 217), I am, however, tempted to hope for better things. The figures refer to the expenditure upon "food" and "clothing" of groups of workpeople whose wages were respectively under 20s., between 20s. and 25s., between 25s. and 30s., between 30s. and 35s., between 35s. and 40s., 40s. and above. My method gave the ratio of the elasticity of demand for clothes to that for food for the several groups as follows:—

Workmen under 20s.	...	...	...	...	1.16
From 20s. to 25s.	...	...	...	...	1.31
From 25s. to 30s.	...	...	...	...	1.62
From 30s. to 35s.	...	...	...	...	1.25
From 35s. to 40s.	...	...	...	...	2.46

Apart from the drop in the ratio in the case of families earning from 30s. to 35s.—and it may be remarked in passing that the instances from which the average in this group are made up are only half as numerous as those in the two adjacent groups—these figures are continuous and in no wise incompatible with what we should expect from general observation. It is natural that among the very poor the demand for clothes should be nearly as inelastic as the demand for food, and that, as we proceed to groups of greater wealth, its relative elasticity should grow. My small experiment, therefore, is not discouraging. I sincerely hope that some economist will undertake a thorough exploration of this field of investigation. His labour would not, I think, be without result.

A. C. PIGOU

LETTER FROM JAPAN.

As stated in the last letter,¹ Japan did not escape the influence of the New York panic in 1907. The reason was, in the first place, that it came to her before she had quite recovered from the effects of the great war and from the financial disturbances of the extraordinary industrial buoyancy which followed the war. Her export trade had to suffer, because the demand for silk and “habutae,” the most important of the Japanese exports, had suddenly decreased, not only in the United States, but also in Europe, while the fall of silver prices caused the decline of her exports to China and other Oriental countries.

The depression increased towards the end of 1908 and the beginning of 1909. However, the revival of the world's trade and the higher price of silver caused Japan's export trade to increase, both in the Western and the Oriental countries. In the autumn of 1909 general recovery was in view. But there now arose a new cause for depression—the fall of the price of rice, on account of the presumed vast increase of the crop. In no other country does a single kind of agricultural product retain such an important position in the national economy as rice does in Japan. Nor in any other country does the agricultural class form such an important part of the community as in Japan. The fall of the price of rice caused a universal decrease of demand for every kind of article; and a general fall of prices was again experienced.

¹ See ECONOMIC JOURNAL, March, 1910.