JOURNAL

OF THE

FRANKLIN INSTITUTE

OF THE STATE OF PENNSYLVANIA,
FOR THE PROMOTION OF THE MECHANIC ARTS.

VOL. CXXXVIII. AUGUST, 1894.

No. 2

THE Franklin Institute is not responsible for the statements and opinions advanced by contributors to the *Journal*.

THE GRAPHICS OF THE EFFICIENCIES OF THE STEAM ENGINE.

BY R. H. THURSTON,
Director of Sibley College, Cornell University.

Among the many fragments of Rankine's work on the economics of the steam engine which have been left us as nuclei of later detailed and more complete studies, none, perhaps, better illustrates the remarkable perspicacity of the man than his statement of the principle properly governing the adjustment of the best ratio of expansion, and his exemplification of that principle by the application of the touchstone of financial result. James Watt, a century ago, and more (1782), patented his invention of the use of the expansive action of steam in the engine, with the purpose of effecting the most economical possible conversion of thermal into dynamic energy; but he was never able to secure those gains in the practical operation of the machine

which his mathematical computations had indicated as scientifically possible and as practically obtainable. A hundred inventors since the time of Watt have wrecked their fortunes, and thousands of engine builders and users have met with disaster, striking on the same rock. The vast discrepancy between the seemingly exact and positive predictions of the thermo-dynamic and mathematical theory of the engine and its practical results in everyday work have seemed to all the early engineers too great to be attributed to other than defects of construction and to errors in restricting the expansion of steam or in so constructing the machine as to make it impracticable to secure at once considerable expansion and steady motion. For a century it thus happened that efforts were constantly being made by inventors and builders to find some way of grasping that immense profit which was apparently just within reach by the simultaneous rise of steam pressures and the expansion ratio. It is only recently that we have learned precisely where the difficulties lie, and what must be the direction of improvement to insure the remedy of the still existing defects and further approximation to the thermo-dynamic conditions and to those results which are the ultimate object of every great invention relating to this wonderful machine.

Watt discovered the facts which are now known to constitute the obstacle to success in the endeavor to carry out his principle and to gain full advantage in transformation of the heat-energy stored in steam; but he exhausted all his ingenuity in the endeavor to remove what he thought the main barrier to complete utilization of his idea—irregularity of action of a piston subjected to varying pressures—and probably never discovered that, with ever so smooth a motion, profitable expansion must be enormously restricted by that phenomenon, the investigation of which was the subject of his very first experiments with the old Newcomen model of the University of Glasgow. It was reserved for Daniel Kinnear Clark, in England (1852), for Hirn, in France (1855), and for Isherwood, in the United States (1860), to show that the primary source of this restriction of profit-

able expansion was to be found in the fact that the internal wastes by condensation of steam, consequent upon the heat-conducting character of the cylinder walls—to “cylinder condensation” as it is now generally, though, perhaps, inelegantly, termed—increase more rapidly than the gain by expansion, thus giving a low ratio of maximum economic value. Rankine and Clausius, and Thomson and Zeuner, built up the whole thermo-dynamic theory of the heat-engines; but it was only after a quarter of a century had passed that the differences between the ideal and the real heat-engines came to be fully understood, and that their effect upon the economical working of the machine was recognized. Rankine earned added fame by his declaration of the principles of finance bearing upon the case and by showing that here, as in every other task set the engineer, the results must be gauged by the final measure of all engineering—the work accomplished by the unit of coin paid out; but the effect of the Watt phenomenon did not enter into his computations, and its extraordinary effect upon the proportions of the steam engine remained to be revealed by the work of the last and the current decades. The essential principles affecting the proportions of the engine which will give the best results, all things considered, are probably all now familiar to every well-informed engineer and to many contemporary thermo-dynamists; and the exact methods of investigation of the subject will become equally familiar to all when the now numerous researches in the physics and the economics of the case, now in progress in the hands of numberless engineers and men of science, shall have fully revealed the data needed for their development.

Graphical methods of representation are usually more intelligible to the engineer than are the algebraic processes; and this is especially true of the engineer whose genius lies in the direction of invention and of construction, rather than of theoretical investigation, and where the conditions to be studied and examined with reference to their relative interactions are numerous and where those mutual influences are obscure. An algebraic equation gives, in its solu-

tion, a single isolated datum; a curve in a diagram, the equation of which curve is known and permits its construction, exhibits an infinite number of successive values simultaneously, and illustrates, at the same time, the method of variation of those data and the presence or absence of maxima and minima. The diagram itself may present to the mind at a single glance the whole succession of phenomena in a series, or a complete cycle of operations involving the solutions of various equations, and for numerous different values if algebraic methods are adopted for the case. A half-dozen computations and a few strokes of the pencil, on the other hand, reveal, often, all the details of operation of a complicated system, and tell just where and how the engineer may look for results of maximum possible value. For the cases to be studied in the present instance, we shall probably find graphical methods the only practically available system of revelation of the information and the facts sought. It is for this reason, and because they are also vastly more satisfactory in their legibility and comprehensiveness, that they are adopted in the work which follows.

The Distribution of Energy supplied the steam engine is easily stated in a general way; but the exact distribution into utilized power and wasted energies is not always readily ascertainable, even with modern facilities for their measurement. Assuming this analysis to have been effected in any given case, as for a modern "high-speed" engine of moderate power, and working with high steam-pressure, some such balance-sheet as the following would be obtainable. Should the columns fail to balance, it would indicate either that a false measurement had been made, that the errors of careful observation, even, were sensible and perhaps cumulative, or that some source of waste had escaped observation altogether, as often did, in fact, the internal thermal wastes, during the greater part of the history of the machine, and up to within a few years.

BALANCE SHEET OF HIGH-SPEED ENGINE.

RECEIVED.		EXPENDED.	
	<i>Per cent.</i>		<i>Per cent.</i>
Energy stored in steam produced in the boiler and transferred to the engine	100	Utilized by conversion into dynamic energy at the engine .	9
	—	Wasted by friction	1
Total	100	Indicated Power	10
		External thermal waste	2
		Thermo-dynamic wastes	60
		Internal thermal waste	28
		—	—
		Total	100

Of the several quantities in the right-hand column, the first is easily and exactly calculable, as it is a purely thermo-dynamic problem, and capable of precise solution. The second item may be approximately computed, or deduced by reference to the experimental determinations of friction losses exhibited in the operation of similar engines. The first is the dynamometric or "brake" power of the engine, and the second is measured by the difference between the "indicated" and the "brake" powers. The fourth is heat lost by conduction and radiation, and may be computed or observed in any case with sufficient accuracy for all practical purposes of the engineer, and for the present case. The fifth is the necessarily rejected heat of the thermo-dynamic operation, is dependent upon the cycle adopted, the range of temperature worked through in the conversion of energy, and is precisely computable. The last, the internal wastes of heat by "cylinder condensation," may be accurately measured in the case of the engine constructed and in operation, and can be approximately computed or may be deduced by reference to the performance of other engines, where the results sought are for the case of an engine proposed and to be designed. We make these measurements or computations, or both, for any number of sets of conditions, and may establish a curve for each of these elements of the problem, the ordinates of which may be made the costs of the energy represented, and the abscissæ may be taken as the measures of the steam used or any convenient quantity defining the conditions of the case, as, in

the cases to be here used in illustration, the ratio of expansion adopted, unity weight or volume of steam being always taken into the engine at the point of cut-off.

The measure of cost here to be taken is the number of pounds of steam demanded per horse-power per hour to supply the energy utilized or wasted, as the case may be. With steam at a pressure of 120 pounds absolute, 105 by gauge, nearly, and with a back pressure of three pounds above vacuum, clearance being negligible and the engine well jacketed, the quantity of steam demanded for efficiency unity would be about two and one-half pounds per hour per horse-power, and, for any other efficiency, 2.5 divided by that efficiency, nearly. This quantity, as computed for the ideal case, the ratio of expansion being fixed, is invariable and incapable of modification by any human power. The wastes, other than thermo-dynamic, which latter waste is also inalterable, are capable of amelioration by various expedients, some of which are easily applicable, others difficult and as yet impracticable.

It is thus evident that it is by the amelioration of wastes in the ordinary steam engine that we are to advance in its improvement, if at all, aside from the gain to be effected by that increase of temperature range in expansion, which is the method of thermo-dynamic improvement available. It is in the reduction of extrathermo-dynamic wastes that we are to seek the most fruitful ways of promoting the economy of operation of the heat-engines of whatever kind, while the increase of thermo-dynamic efficiency and the reduction of thermo-dynamic losses will probably occur continually, but slowly, up to some as yet undetermined limit, which limit is, nevertheless, probably not very far distant. Both the thermo-dynamic improvement and the gain by reduction of extra thermo-dynamic wastes must be looked upon as legitimate ways of further improvement of the steam engine as a heat-engine, as a transformer of thermal energy into dynamic form; but the inaccessibility of the zero of the absolute thermometric scale renders the former comparatively unpromising of result, while the interception of external heat losses, the abolition of internal heat wastes

due to the heat-conducting character of the cylinder walls, and the elimination of friction, are at present more simple and promising directions of advance.

As will be presently seen, each of the extra thermodynamic wastes becomes a larger and larger tax upon the thermo-dynamic machine as the latter becomes more efficient through extended expansion, and each, acting by itself, would, if carried sufficiently far, place a limit to economic heat-conversion by such expansion; which limit is the earlier reached as the waste is greater. Thus, with prolonged expansion, a time would always come when further expansion of the steam would develop less power than would be, at that instant, required to overcome the resistance of friction; and, from that point on, the power exerted by the machine upon the working machinery would be a negative quantity. Similarly, external wastes of heat might, beyond a given ratio of expansion, amount to more than the quantity usefully converted, and efficiency would thus again be made negative. Internal heat wastes in this manner render the total ultimate efficiency of the engine a negative quantity at a comparatively early stage in expansion, as these wastes are great at low ratios even, and are a rapidly increasing fraction of the energy supplied with increasing expansion. Thus the magnitude of the ratio of expansion at which final total efficiency becomes a maximum is a gauge of the extent to which avoidable forms of waste exist in the engine, and of the success attained in the design, construction and operation of the engine. That engine which gives its best effect at the highest ratio of expansion is, other things equal, the best engine, viewed from the standpoint of physical perfection. This is perhaps best shown by the diagram herewith presented.

In figure 1, let the ordinates of the various curves be made proportional to the weights of steam employed per horse-power and per hour, under the conditions assumed in the construction of each of the several curves, and let the abscissæ measure the simultaneous values of the ratios of expansion in the given engine, and with the initial and back pressures, as here assumed—120 and three pounds,

respectively, above vacuum. Compute, first, the quantity of steam required for the "ideal case" at each of these ratios of expansion, and construct the lower curve of the series by passing it through the several computed points. Similarly, compute, or find by reference to results of experiment, the additional and total quantities demanded for the engine when the wastes due friction are taken into account, and thus obtain the second curve. Next add the weights of steam required to supply the heat wasted externally by conduction and radiation, and, as a final determination, find the internal wastes by the action of the cylinder walls, thus ascertaining the locus of the upper limiting curve of the series.

The lower line is obtained by the method of Rankine, and accurately represents the efficiencies and the costs of power for the ideal representative case taken. It is seen that the steam expended varies from about 40 pounds per horse-power and per hour at full stroke, to 20 at 3.5 expansions, to 16 at a ratio of expansion of 5, to 12 at 12 expansions, and about 10 pounds at a ratio of 20. The gain continues indefinitely, but at a decreasing rate, until a ratio of about 40 brings us to the limit at which the expansion line begins to fall below the back pressure line. Were there no wastes of extra thermo-dynamic character, this would be the method of operation of the engine which would insure highest efficiency and highest "duty." The friction loss in the case here taken, costs a quantity of steam which is decreased with the enhanced efficiencies of higher expansions, is but little affected by changing loads and power, and which, as a percentage of ideal costs, increases constantly with increasing expansion and decreasing power. It is thus found to place a limit to gain, as here taken, at about twenty-five expansions, and adds, throughout the whole range, not far from three pounds of steam per hour and per horse-power to the costs of useful work. External heat wastes still further exaggerate costs and restrict the profitable expansive action of the engine and a ratio of twenty is found on the curve to be as much as can be employed to advantage when all these wastes are taken into

account. Finally, taking up the internal heat wastes by action of the metallic surfaces enclosing the fluid, and assuming, as here, that the engine is of such size and character as to waste, in spite of jacket-action, from twenty per cent. at full stroke, to fifty per cent. at seven expansions, and still higher proportions at greater ratios, the proportions found actually wasted in, for example, the Sandy Hook experiments reported by the writer to the A. S. M. E., some years ago, it is found that a maximum efficiency is attained, as shown on the upper line, at about seven expansions, or at the point experimentally found for a more efficient engine, operated at a lower pressure, by Hirn and Hallauer. This is probably as large a loss, and gives as radical an illustration of the facts of such cases, as can be fairly expected. No well-designed and properly constructed engine should give a higher curve of final efficiency than is here presented, unless of very small size or working under peculiarly disadvantageous circumstances.

Whatever the form or dimensions of the engine, such construction of its several elements of efficiency as is here exemplified will exhibit the facts in which the designer is most interested, and show the costs of power and the most economical adjustment for high duty. All the thermodynamic, and all the thermal, and all the dynamic elements are here brought into view, and all measures are taken by a common and intelligible scale. The scale actually to be adopted will depend upon circumstances. Where the steam is supplied under different conditions, and thus may have a variable measure of intrinsic or conventional value, the vertical scale should be made in thermal units, rather than weights of steam. In some cases it should be made a scale of coal-weights, while in the solution of some of the most important problems presenting themselves to the engineer, this scale should be one of money-costs.

The Comparison of Various Types of Engine may be made by these graphical methods with the greatest ease and with most satisfactory results. Every essential element of economical operation being thus capable of representation on a common scale, the construction of such a diagram

as is here illustrated and described for each permits the selection of the machine likely, on the whole, to prove best; and we are thus enabled to identify the best adjustment of each and to make the comparison of a pair or of a series of available designs under their several individually best conditions of working. Adhering to the case thus far studied, that of the simple engine, it may be required to ascertain which of a number of machines of stated and required power, but differing in proportions of stroke and diameter of cylinder, or in speed of piston and of rotation, or even simply in clearness, is most desirable. The construction, on a common scale, of a set of these diagrams will reveal to the eye and at a glance the probably best design. The loss by clearance is treated precisely like the wastes by external loss of heat or by internal condensation or leakage, and all may, as in fact illustrated above, be taken as a common internal waste, since it often happens as here that they cannot well be separated with exactness. The best practical method of making the comparison is perhaps that of making the diagrams on tracing paper or cloth and superposing them, one upon the other in pairs, eliminating, one after another, the least desirable cases.

Compound or "Multiple-Expansion" Engines afford the most interesting and probably the most useful applications of such methods of graphical representation of problems of this sort. In such comparisons as have been just referred to, when multiple-expansion, series-expansion—or cascade-expansion, as French writers sometimes call it—is to be considered, it becomes especially important to ascertain not only what relations of efficiency exist at the best adjustment of each, but often even more important to determine the method of variation of efficiencies with varying expansion, and the relative values of each type throughout their respective as well as their common ranges of working. This process of complete study which, ordinarily, is most laborious and troublesome—when, especially, their respective best conditions of operation are to be identified and compared—becomes easy, interesting and exact when the graphical process can be accurately carried into effect. It then

COST-VARIATION DIAGRAM FOR VARIOUS TYPES OF STEAM ENGINE.

becomes as easy to discover the best ratios of expansion, and the ranges of working throughout which the simple engine is inferior or the more complex structure is superior, as to find absolute values for specified single cases. But it is in the exhibition of the relative values of the several multiple-cylinder engines that this process is perhaps likely to prove of most service and to best illustrate the essential principles involved.

In the action of the multiple-expansion systems, the several lines of our diagram become altered in location, although retaining their distinctive forms. The ideal case only remains unaffected, the thermo-dynamic conditions being the same. * The friction-line is usually, but not always or necessarily, raised; the friction of the machine being in some cases substantially unaltered, and in others even reduced, by modifications of designs introduced by "compounding." In the "tandem-compound" engine, this change is often insensible or unimportant; in the three-crank compound this dynamic waste may be even reduced by the balance secured about the shaft-line. The external thermal wastes may be but slightly affected also, the reduction of the mean temperature of the external surfaces of the cylinders compensating roughly the extension of area. The internal heat wastes are most remarkably modified in good examples of such engines. The upper line of our diagram is brought down considerably and often very far below that location given it in the case of the companion simple engine. These wastes may be reduced from six or eight pounds, for example, to three or four in the compound, to two or three in the triple-expansion, or to even less weight of steam per horse-power and per hour, in the more complicated forms of engine, for similar total ratios of expansion. The restricted expansion in each of the cylinders and the reduced area of actively wasteful cylinder-wall, make the condensation of the entering steam much less than at the point of cut-off in the simple engine, and the steam thus condensed, being re-evaporated completely at its exit, enters the second cylinder in series as steam once more, is there again subjected to similar condensation, and thus goes on

through the series, however extensive, the waste from the one cylinder more or less completely meeting the demand for wastes in the next, and thus approximately reducing the total waste from this action to that comparatively small fraction of the steam supplied from the boiler, which is represented by the comparatively small condensation in a single cylinder of the series. Every step thus gained in the reduction of these internal thermal wastes permits corresponding advance to be made toward full utilization of the thermo-dynamic gain due expansion and raises the ratio of expansion for best effect.

In illustration: let the same thermo-dynamic case be assumed; the expenditures of steam being computed by the familiar equations for efficiency, and taking the consumption of steam to be 2.5 pounds for efficiency unity; giving, for any given ratio of expansion, not far from $w = 40 / \sqrt{r}$, as the minimum cost in steam expended, as before, for the ranges of expansion usual with such engines. Take the friction as unchanged and the external losses of heat as substantially constant, the one amounting to about ten per cent. of the rated power of the machine and the other as one-third that proportion, at that rating, and varying as in the case of the simple engine. Following experience with the most successful designs of mill-engines, and comparing cases in which the double-expansion engine reduces the internal wastes to one-half, the triple to one-third, and the quadruple to nearly one-fourth the magnitudes distinguishing our simple machine, we obtain the three curves in the figure lying between the upper line already obtained and the lower three lines of the diagram as first traced.

In each case we find the internal heat wastes constituting a heavy tax upon the operation of the engine; in each they become a larger proportion of total expenditures as the ratio of expansion becomes greater; in each case the point of minimum expenditure and the ordinate of maximum efficiency recede further from the best ordinate for the simple machine, as these losses become less; and, as already remarked, the ratio of expansion for highest total effect

becomes a gauge of the value of the type of engine studied. While the simple engine did its best work at a ratio of 7—all these machines are assumed to be jacketed—the compound raises the figure to 10; the triple-expansion to 12 or 13, and the quadruple-expansion engine does its best duty at a ratio not far from 18. With larger and faster engines, these values of the ratio of maximum effect will be found

FIG. 2. LOCOMOTIVE.

SIMPLE ENGINE UNJACKETED. COMPOUND ENGINE JACKETED.

Steam at $p_1 = 120$ lbs. absolute; $p_3 = 20$.

still more distributed, and will assume still higher values; but 7 or 8, 10 to 12, 13 to 15, and 18 to 20, are probable fair maxima, respectively, in modern good practice with the assumed pressure, for the several types of machine above taken.

Comparing the range of competitive operation, it is seen that, in these particular cases, the compound competes with

the simple throughout the range from the ratio 4 to some point beyond our maximum limit on the diagram; that the triple-expansion competes with the compound from $r = 5.5$, and with the simple between 3.5 and the maximum; and that the quadruple similarly competes with the simple between 3 and the limit, with the triple between 5 and the extreme, and with the compound between 3.75 and 20 or more. It is assumed, in each comparison, that the basis of the competition is the duty, or the steam required per horsepower per hour. The simple engine competes, at its best ratio, with the compound and other engines only when the latter are working at abnormally low ratios of expansion, and each multiple-cylinder machine shows superiority over the preceding in our list at expansions which are always less than the best ratios of the simpler engine. These comparisons, however, obviously may result very differently when the complication introduced with the multiple-cylinder engine throws up the lines representing friction and external wastes to such height as to make the loss of energy in these directions exceed the gain by reduced internal wastes, and thus carries the upper line of the representative diagram for the latter above the line of total cost for the simpler machine with which it is compared. Very different and often very disappointing and unexpected results may thus follow where the engines are not suitably adapted to their working conditions, or when they are unskilfully designed and proportioned.

In all cases, whatever the type of engine, it is likely to be found that the best results are only to be secured when the extra thermo-dynamic wastes of the machine are intelligently treated and made as small as practicable. It is in this direction that the indications are that the greatest advances are to be made in the immediate future of the history of the steam engine.

Specific Applications of the principles illustrated in the general case just presented will be easily effected where the data are available. For example, let figure 2 represent the distribution of useful energies and wastes in the case of a locomotive. The thermo-dynamic case is com-

puted, as per Rankine and as in the earlier work of the author.* The steam consumption is found by experiment to be approximately measured by $w = 45 / \sqrt{r}$. This quantity is increased, when the friction of the engine is taken into account by about ten per cent. at ordinary expansions, and the external heat wastes add a very nearly equal amount; both these wastes, however, being found by experience to be variable slightly with varying power and to be somewhat less as an absolute amount and greater relatively with decreasing expansion. Friction is taken as six per cent. of the full power, at full stroke, heat waste as 0.5 B. T. U. per square foot per hour per degree difference of temperature between the internal and the external temperatures. Adding the internal wastes by condensation, leakage, clearance and wire-drawing, following Clark for the first-mentioned up to the middle of our range and later practice beyond, we obtain the upper curve and find our profitable expansion restricted to about $r = 2.5$; which accords with general experience from Clark's earliest work to the present time. But, should these last wastes be decreased by compounding or superheating, as is known to be practicable, to one-half their present amount, the limit-curve would fall to the second line, and the allowable ratio of expansion for highest duty and lowest consumption of fuel in regular work would fall correspondingly, giving a higher ratio $r = 4$ or 5, and a consumption of fuel or of steam more nearly three than four, and twenty-five than thirty-two, as the first set of figures would probably read. In other words, the gain would be not far from twenty-five per cent. Both the first and last results have been bettered by engines of each type; but the relative standing of the two should be very much as here indicated.

The two cases above studied are those of engines working under what is generally considered high steam-pressure. The following will illustrate the operation of the older type of machine, simple, unjacketed, condensing, with steam at sixty pounds absolute, forty-five by gauge, nearly. It is

* Rankine, p. 192, et seq.; Thurston's Manual, vol. i, arts. 116, 117, and especially 123, 137.

the engine employed for marine work a generation ago and of which numerous examples survive. In the figure, the lower curve, as before, represents the ideal thermo-dynamic

case. The consumption of steam, assuming the temperature of the hot well and feed such that 2.5 pounds would be demanded per hour per horse-power for efficiency unity, is

VOL. CXXXVIII.

again about $w = 40 / \sqrt{r}$, and varies between 40 at full stroke to 20 at $3\frac{1}{2}$ expansions and 15 at $r = 8$. Internal and external wastes are computed as before; and the internal wastes are taken as found by Isherwood for the average marine engine of his time. Here the dynamic wastes appear to be a heavier proportional tax on the machine than in the case of the locomotive or in that of the high-pressure condensing engine, and limit the gain by increasing expansion to about $r = 2\frac{1}{2}$ or $r = 2\frac{3}{4}$. By both compounding and jacketing, we convert the engine into the representative multiple-cylinder machine of Hirn's experiments and of Hallauer, and at once find it possible to gain by carrying expansion to a new maximum at $r = 6$ or $r = 7$, while the efficiency becomes a nearly constant quantity over a very wide range—from three to eight or more.

In both these last two cases, the compound shows its superiority in range of profitable expansion, as compared with the simple engine, and it is seen more clearly than before how great are the losses in the simple engine with variable loads, on either side the best adjustment of cut-off and load for the individual machine, and the enhanced gain, for such cases, obtainable by the substitution of the condensing multiple-cylinder engine. The same superiority is obtainable by the adoption of the principle of "compounding" with the non-condensing engine, especially at high pressures, *provided* the limit is not passed at which the expansion line falls below the atmospheric, beyond which limit the machine rapidly loses efficiency.

Financial Conditions, as was shown by Rankine, as early as the middle of the nineteenth century, are the final limiting considerations in adjusting the power of an engine and its method of expansion to secure the best possible results in any given location and in stated conditions of the market for labor and material. Were it the purpose of this paper, it would be easy to introduce them into the diagrams which have been given to illustrate purely physical conditions; but these may perhaps be treated more conveniently by the methods earlier developed by the writer, and in which all

these obstructive elements and their varying effects are so treated as to permit both graphical and analytical deduction of the maximum commercial efficiencies—of which there are several—and the solution of many interesting as well as of the essential problems designated by the writer, respectively, as the “designer’s problems” and the “owner’s problems.”*

It is easy, however, to see that if the vertical scale of our diagrams were, as it might perfectly well be made, one of dollars instead of weight of steam, we might superpose upon the series of curves, constituting each, another, which should represent the varying costs of power external to the engine and the boiler plant, those costs which measure the annual value of capital invested and expenses incurred, apart from the internal costs of production of the power; such as, for example, in the former class, the interest on costs of engine, boiler, buildings, etc., and on the same scale which would measure the costs of the steam-supply already treated of. Referring to the first set of curves, assume the next to the upper line to represent these “internal” costs, and the upper curve to measure the “external” expenses, it is here seen that, as is the known fact, the latter being an increasing relative magnitude, the departure of the pair of curves from each other with increasing expansion, indicates that these new considerations dictate still further restriction of the ratio of expansion and still further narrow the range of nearly constant economy. As is elsewhere shown by the writer, this restriction is often more effective than even the enormous influence of internal wastes has been seen to prove. Where the designer seeks to ascertain what adjustment will give him the best work in proportioning an engine of either class to perform a specified amount of work, he often finds it necessary to adopt a ratio of expansion not exceeding two-thirds that dictated by the solution of the problem which it is the principal purpose of this paper to study; and when he again endeavors to ascertain to what power his client may drive

* Manual, vol. i, chap. vii; vol. ii, chap. viii.

up engine and boiler to obtain highest pecuniary returns from a "plant" already established and adapted to a specified original power—it being found that more power may be usefully employed or sold—he again finds that he may profitably restrict the ratio of expansion by perhaps another twenty-five per cent., and obtain large or at least fair profits on the new investment, notwithstanding the facts that the power added costs him more per unit than the original amount supplied by an engine precisely adapted to that rating, and that the new total power might be still more profitably produced by a new plant of proper rating and adjustment.

NEW CELLULOSE DERIVATIVES.*

BY CLAYTON BEADLE.

Nearly two years ago, my colleagues, Messrs. C. F. Cross and E. J. Bevan, and I, when working on the behavior of "mercerized" cellulose towards various reagents, discovered a new class of soluble cellulose derivatives. The ultimate products obtained from these promised to be of so much value and interest that we followed the work up with the greatest possible speed, one of us devoting his whole time to the prosecution of this work. It has resulted in the production of a large range of products, which we have every reason to believe are of great value. Mr. Arthur D. Little has been connected with us in working out the various developments over here. The results obtained by him, we believe, have largely added to the value and scope of our process. We published a brief account (*Chem. Soc. J.*, 1893, 837) of our early work on these compounds and indicated their probable line of developments. (*Soc. Chem. Ind. J.*, June 30, 1893.) After nine months' further work we are encouraged in the belief that the predictions we then made are on the eve of fulfilment.

*Read at the stated meeting of the Franklin Institute, April 17, 1894.