means of the Fourier theorem, were therefore four times as large as the curves here shown.

I subjoin herewith a few specimen analyses. In the table, column I. contains the vibration number of the fundamental, *i.e.* the pitch at which the vowel was sung, and its amplitude percentage; column II. contains the vibration number of the first overtone, *i.e.* the octave of the fundamental, and its amplitude percentage, and so on.

I	II	III	IV	V	VI	VII	VIII	IX	X	XI
113	226	339	452	565	678	791	904	1017	1130	1243
9'9	15'4	2'5	0'8	11'5	16'1	3'3	8'1	21'2	7'2	3'2
144	288	432	576	720	864	1008	1152	1296	1440	1584
32'9	4'3	1'9	0'3	14'8	2'1	8'8	13'5	7'2	1'3	4'7
171	342	513	684	855	1026	1197	1368	1539	1710	1881
5'6	4'8	2'9	17'5	5'0	33'1	19'9	2'1	6'1	1'9	1'1
226	452	678	904	1130	1356	1582	1808	2034	2260	2486
9'2	3'6	34'6	8'0	31'J	2'6	6'2	1'4	0'8	1'2	1'4

It will be sufficient here to state as briefly as possible the conclusions which I believe to be warranted. The vowels as produced by the human organs of speech are composed in the first place of two elements, that due to the vibration of the vocal chords and that due to the resonance of the mouth, throat and nose cavities. It is not always possible to separate clearly these two elements, but at the pitches shown in the above table the problem is quite simple for the vowel *a*. The fundamental is due to the vocal chords, and the overtones that are strongly reinforced are due to the mouth and throat resonance. The vowel *a*, at any pitch, and pronounced by any clear voice, contains the following partial tones:—

(1) The fundamental, with the first two or three overtones. The fundamental varies greatly in relative amplitude for reasons which I do not as yet attempt to formulate. The overtones are all weak, unless reinforced by the mouth resonance as set forth below.

(2) The overtone or overtones whose frequencies of vibration chance to fall between 1000 and 1300 vibrations to the second, the maximum seeming to lie at about 1150. This is the main characteristic of *a*, which serves to identify it to the ear, and remains remarkably constant, no matter what the fundamental may be. If the fundamental has 144 vibrations to the second, overtones VII., VIII. and IX., with frequencies of 1008, 1152 and 1296 respectively, will all be present, but VIII., with 1152 vibrations to the second, in much the largest amplitude.

(3) The overtone or overtones whose frequencies of vibration chance to fall between 575 and 800 vibrations to the second for men's voices, with a maximum at about 675; or between 675 and 900 with a maximum at about 800, for the voices of women and children. This is presumably the resonance of mouth and throat cavities resounding as one vessel, and is not as constant as the main resonance described above. If the fundamental has 144 vibrations to the second, we shall therefore find overtones IV. and V. present, with frequencies of 576 and 720 respectively, but V., being nearer the point of maximum resonance, will be the stronger.

The two regions of resonance are indicated in the table by printing in larger figures the amplitude percentages of those overtones which fall at or near the points of maximum resonance, and are therefore present with large amplitudes. My analyses reveal for the vowel *a* no other region of resonance that is constant or important except these two. On the basis of many observations it would be possible to plot a curve which would represent the mouth resonance for the *a*-position. A tentative chart of this sort will be found in the articles already cited, and in general for a fuller discussion of the whole subject the reader is referred to those articles. LOUIS BEVIER.

Rutgers College, New Brunswick, N.J.

Illustrations of Lepidoptera.

CAN any of your readers assist me in finding some artist who is really competent to produce, by any process which combines extreme accuracy with a reasonable cost, a large series of illustrations of variation in butterflies? The difficulty of getting such work done in the country under my own eye is very great, and I am unwilling to do as some of my friends advise, and get them made in Germany. Where the variation is a question of pattern only, photography would probably be the most satisfactory process; but where colour is the leading feature of the variation, chromo-lithography seems the most likely to succeed.

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The number of works on natural history which are privately published in England seems to point to a want of enterprise among English publishers which I cannot understand, as the market for really well illustrated works on ornithology, botany and entomology is certainly increasing. Any suggestions addressed to me will be gratefully received. H. J. ELWES.

Colesborne, Andoversford R.S.O., Gloucestershire.

"Billiards Mathematically Treated."

THE review, in your issue of March 1, of Mr. Hemming's book on billiards reminds me that I have never yet seen a satisfactory explanation of the following question:—

Why does a billiard ball when struck to the right or left of its vertical axis nevertheless travel in the direction in which the cue travels?

It only does so when the cue tip is chalked, otherwise it travels in a direction of a line through the centre of the ball and the point of contact with the top of the cue, as may be expected from the laws of dynamics. The chalking of the cue of course enables the striker to put on what is known as side, *i.e.* to make the ball revolve on its vertical axis; but why does it also allow the ball to travel in a straight line instead of going off at an angle? ENQUIRER.

THE answer to "Enquirer's" puzzle is very simple. If you let a perfectly hard and smooth cue tip impinge at any angle on a perfectly hard and smooth ball, the only force exerted will be normal, and the ball will start in the line from the point of impact to the centre of the ball. At the same time, during the brief duration of the impact the cue tip will slide a minute distance along the surface of the ball, but in consequence of the perfect smoothness will exert no tangential force on the ball.

Now take the extreme case in the opposite direction. Let the cue tip be such that it bites on the ball without any sliding whatever during the impact. In this case the point of the ball struck moves exactly in the direction of the motion of the cue—the centre of gravity of the ball moves in precisely the same direction with a rotation added which is familiar to us as side—and this will be so (if the bite is perfect), whatever may be the angle between the direction of the cue and the surface of the ball. With the ivory tips in use more than a century ago, you had something approaching, though not quite reaching, perfect smoothness—and the ball flew off not quite, but very nearly, in the normal direction. With modern tips in good order and well chalked the bite is sufficient to prevent any slipping and drive the ball in the line of the cue, provided the angle between the cue and the surface of the ball is not too small; and if this condition is fulfilled the ball will start in a direction exactly parallel to the line of the cue.

If the angle is too small, as by playing side too near the edge of the ball, there will be some slip and the ball will go wrong (not quite normally, but a long way from the desired direction). It will be a miss-cue. The nearness to the edge which can be played without miss-cueing is different for different players, and depends upon some nicety in the handling of the cue which does not admit of definition.

Roberts can play side vastly nearer to the edge of the ball than I should dream of attempting, and I daresay a good deal nearer than "Enquirer" could try with success. But the broad fact is, that with a well chalked cue you can insure (within certain limits of angle) that there shall be no slip.

If the cue is not chalked you can still do the same thing within very much narrower limits, that is to say, you can put on a very little side.

The ivory tip, the chalked tip, and the unchalked tip, are three different cases intermediate between the extreme theoretical cases of perfect smoothness with unresisted slip and perfect bite with no slip at any angle however small.

I do not think that any one could have foreseen that a cue tip could be made to bite to the extent to which it does. But somebody in America or France found it out by experiment, and for the first time made billiards the scientific game which it has now become. G. W. HEMMING.

The Micro-Organism of Faulty Rum.

IN the course of our investigations upon this organism, first alluded to by us (NATURE, vol. lvi. p. 197, June 1897), and subsequently by others (vol. lix. p. 339, February 1899), we have found that from spirit of 70 per cent. of alcohol, which has

been in our possession for three years, we can still obtain successful cultures in gelatine; the various forms, which we have previously described, have been observed as before.

Our object in sending this note is to call attention to the extraordinary vitality of this organism under such untoward circumstances, owing doubtless to its carefully entrenched position.

V. H. VELEY,
LILIAN J. VELEY.

Oxford, March 10.

Drunkenness and the Weather.

ON reading the letter of Prof. Dexter on "Assaults and Drunkenness" (p. 365), I notice that there is one great fallacy in the argument.

When a man is intoxicated and commits an assault, the result is entered in police reports as "assault," the more serious offence overshadowing the less. So that, in all probability, many of the cases of assault referred to in the statement were also cases of drunkenness, but were not tabulated as such.

The temperature is an important element; for its variations are probably the cause of the change of character of the offences recorded. The same quantity of alcohol will, as has often been noticed, have very different effects in the summer and in the winter. In hot weather alcohol has a stimulating influence; this is much less marked in the winter, and during this season the sedative effect is certainly more noticeable.

Studying Prof. Dexter's curves in this light, and assuming the absence of any other fallacies, we may reasonably conclude that the number of those arrested for drunkenness or its results varies but little throughout the year. Probably the same people supply the cases of drunkenness in winter and of assaults in summer.

R. C. T. EVANS.

9 Heathcote Street, Gray's Inn Road, W.C., March 3.

Mechanical Methods of Calculating Logarithms.

THE following mechanical method of finding logarithms seems to be as simple as any that have been proposed, and has the advantage that it gives the logarithms of all numbers without interpolation, and at the same time affords a proof of the fundamental property of the function.

Let a flat ruler AB be provided at one end, A, with a hatchet edge (like that of the hatchet planimeter), so arranged that when the ruler is held horizontally, and the hatchet allowed to touch the paper, it touches at a point vertically below the edge of the ruler. The hatchet must lie in a vertical plane inclined at a convenient angle (say 45°) to the ruler. Let the ruler be held thus, with its edge touching a pin. On moving the ruler so that the hatchet does not slip sideways, the latter will trace a spiral curve on the paper. From its mode of generation the spiral clearly cuts all radii vectores at the same angle, and thus is the well-known equiangular spiral. Let OA be a radius vector of unit length, and OP one of length r . Let $\angle AOP = \theta$ where θ may be expressed in terms of any convenient unit, then we may define the logarithm by the equation $\theta = \log r$. Of course, θ depends on the angle of the spiral and on the unit of angle adopted as well as on r , and so is not yet completely defined. We can, however, immediately prove the fundamental property of the logarithmic function.

Imagine a copy O'A'P' of the diagram to be made on some extensible material, and to be extended equally in all directions in the ratio R:1. All angles remain unaltered, and the new curve is an equiangular spiral with the same angle as before. If, now, O' be placed on O, and the new diagram turned till A' lies on the old spiral, the two spirals, having the same angle, must coincide, and hence P' lies on the old spiral. Now $OA' = R$, $OP' = rR$, $\angle AOP' = \angle AOA' + \angle A'OP' = \angle AOA' + \angle AOP$, which gives $\log rR = \log r + \log R$, the fundamental property. If we further chose our unit angle so that $\log 10 = 1$, the spiral will give Briggian logarithms. It would, perhaps, be more convenient practically to adjust the angle of inclination of the hatchet so that $\log 10$ is represented by 100° , or perhaps by 360° if we divide the circle centesimally. It may seem that the logarithm, as defined above, still depends on the angle of the spiral, but this idea can be readily disproved by means of the equation $\log rR = \log r + \log R$. The logarithm, having been defined without reference to indices, may now be used to define the quantity x^n , where n is negative or fractional, and to give the index laws in a manner rather less artificial than that usually adopted (the fact that no indication is given of the many-valued character of a fractional power is, however, a drawback).

The hatchet planimeter may be used to obtain logarithms, but in a less simple manner. If the planimeter be placed with its point on a given straight line, and its length perpendicular to the line, and the point be moved through a distance x along this line, the inclination θ of the planimeter to the line is given by $x = a \log \cot \theta/2$, where a is the length of the planimeter. This gives an obvious mechanical construction for a logarithm.

Leeds, March 5.

H. C. POCKLINGTON.

THE CENTENARY OF THE BERLIN ACADEMY OF SCIENCES.¹

IT is with feelings of pleasure that we call the attention of our readers to the fact that rather more than one month ago the Academy of Sciences at Berlin, at its meeting on the 25th of January, commemorated with great rejoicing and some very pardonable pride the work which its members have done in the world during the last hundred years. The subjects which have been investigated by this distinguished body include almost every branch of human knowledge, and although at this date we are too near in point of time to be able to judge definitely and finally as to the value of the work which the German scholars and men of science, whose names are written on its books, have done, there is no room for doubting that they have enlarged the bounds of human knowledge in every direction, and have brought us many degrees nearer to the goal sought by all honest investigators.

The Berlin Academy has kept in mind what the true functions of an Academy of Sciences should be, for it has not sought to limit the number of subjects which its members desired to investigate, and it has not attempted to patronise or to foster the growth of one class of sciences, or of one branch of learning, to the exclusion of all others. It has encouraged knowledge of every kind, and has supported by its influence and money the workers in the most recondite branches of human learning, and its influence for good has been so far-reaching that it would need a volume if we attempted to describe the work which has been well and efficiently performed under its auspices. And the Academy of Sciences at Berlin has not only helped the world positively, as it may be termed, that is to say, by enabling its members to formulate and build up sciences, but negatively, by making it impossible for the faddist, and crank, and charlatan to press his views upon the non-expert, but well-educated, section of the German public. In this last capacity it has performed, very quietly and unobtrusively, but effectively, a most important duty, and it has succeeded in obtaining and holding a position of authority which cannot be gainsaid. It has proved to all the world that when it sets its seal of approval on a man's methods or works, those methods and works have permanent value. We may almost say that the work of German scholars and thinkers is so good because they possess in their country a high authority for the approval of which they are content to toil long and arduously, knowing well that its stamp is a hall-mark which the intellectual world will honour, and the full value of which will be duly credited to it. Of the universality of learning the Academy at Berlin has been a consistent and powerful patron, and the long list of great names which Herr Waldeyer, one of the secretaries of the Physical Section, brought to the notice of the members at its festival meeting is a splendid proof of this statement. Among historical investigators and jurists may be mentioned Fichte, Schleiermacher, Schelling and Trendelenburg; among students of linguistics and archaeologists, Boeckh, Bekker, Bopp, Curtius,

¹ *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin. 25 Januar. Öffentliche Sitzung zur Feier des Geburtsfestes Sr. Majestät des Kaisers und Königs und des Jahrestages König Friedrich's II.* In Commission bei Georg Reimer, Berlin.