to give the proper engagement with it, so that both dimensions become much smaller than in a winding-barrel, and the total space occupied is but a small fraction of that required for the latter device. The *chain-wheel* is fixed in direct line with the chain, and all lateral strains are avoided, while the flat bearings afforded for the chain by the pockets preserve the shape of the links and protect them from bending strains. The slack chain, after passing over the wheel, falls into a proper receptacle below.

From this analysis of the facts is deducted the proposition that chains, if well made, constitute the best form of flexible cord for sustaining the load in a crane, and that a well-constructed *chain-wheel* (as contra-distinguished from a winding barrel) is the best form of device for hauling in and paying out the chain; and therefore, the best method of crane construction involves the use of these two elements.

A REMARKABLE ERROR IN THE COMMON THEORY OF THE TURBINE WATER WHEEL.

By J. P. FRIZELL.

All writers who have attempted a theoretical discussion of the turbine water wheel, have agreed in representing the centrifugal force operating to increase or diminish the discharge of the wheel, by a certain head. The difference, namely, between the head due the rotatory velocity of the wheel at the point where the water enters the wheel, and the head due the rotatory velocity of the wheel at the point where the water leaves the wheel. Among those who have done so to my certain knowledge are Weisbach, Redtenbacher, Rankine, Morin, and Bresse.

To all these philosophers the very careful and complete experiments of Mr. Francis on turbine wheels have been accessible for the last thirty years; giving them, had they so desired, abundant means of comparing their theories with facts. They appear to have ignored these experiments entirely, and to have contented themselves with exhibiting the results of mathematical reasoning. Even in the late valuable and carefully considered treatise of Weisbach, no attempt is made to reconcile his conclusions with the results of these experiments or to explain the discrepancies that must have been known to exist.

To the engineer who attempts to compare theoretical results with those of experiment, nothing becomes apparent sooner than that the above-mentioned principle wholly fails to represent the action of centrifugal force. Mr. Francis, in his "Hydraulic Experiments," expresses the opinion that centrifugal force plays a less important part in the turbine wheel than is assigned to it by mathematicians, though he does not attempt any inquiry into the foundation of the formula by which that force is represented; being probably, like many others, content to regard it as one of those scientific truths which are only true on paper but not true in practice; as a principle which water ought, in deference to eminent authority, to observe, but which its habitual wilfulness and perversity lead it to disregard.

It is always worth while when we meet with a principle, so called, which does not represent facts, to inquire carefully whether it be well founded in theory.

The principle in question is demonstrated by Weisbach in "Mechanics of Engineering, etc." Cox's translation, p. 610, and more concisely in the introduction to his later work "Hydraulics and Hydraulic Motors," by Prof. DuBois. I reproduce the latter, though both are essentially the same, and this may be regarded as a type of the methods by which the principle is supposed to be demonstrated.

"If the path CAB , in which a body moves, is not at rest, but turning about an axis C , it imparts to the body a centrifugal force P , by reason of which the body either performs work or work is performed upon it, according as it departs from or approaches the axis of rotation C . Let M be the mass of the body, ω the constant angular velocity with which the path turns about C , and let r denote the variable distance CM of the body which is moving in the path CAB . We have the centrifugal force of the body (Art. 13) $P = \omega^2 Mr$, and for the work done by this force while the body describes an element

$M O$ of its path, and hence, while the radius $C M = r$ is increased by an amount $N O = d r$,

$$d W = \omega^2 M r d r.$$

Hence for the whole work

$$W = \frac{\omega^2 M r^2}{2}.$$

Now the velocity of rotation at the distance r is $v = \omega r$, and therefore

$$W = \frac{M v^2}{2} = \frac{v^2}{2g} G.$$

When we substitute instead of the mass M , the weight $G = M g$.

If the body begins its motion, not at C , but at any other point, A , at a distance r_1 from C , where the velocity of rotation is $v_1 = \omega r_1$, we have for the work done, while passing from A to M ,

$$\begin{aligned} W &= \frac{1}{2} \omega^2 M r^2 - \frac{1}{2} \omega^2 M r_1^2 = \frac{1}{2} \omega^2 M (r^2 - r_1^2) = \frac{1}{2} M (v^2 - v_1^2) \\ &= \frac{(v^2 - v_1^2)}{2g} G. \end{aligned}$$

If, then, a body moves in a rigid path or groove which revolves about a fixed axis, the vis viva of the body is increased or diminished by the product of the mass (M) and the difference of the squares of the velocities (v^2 and v_1^2) at the two ends (A and M) of the path, or by the product of the weight (G) and the heights

$$\left(\frac{v^2}{2g} \text{ and } \frac{v_1^2}{2g} \right)$$

due to these velocities. The increase takes place when the motion is from within outward, and the decrease when the motion is from without inward."

Now who can fail to perceive that this demonstration is entirely erroneous, assuming as it does that ω represents the angular velocity of the body M . In other words, that this body has the same angular velocity as the revolving disc. The body M has a motion along the path $C A B$ independent of its rotatory motion imparted by the disc. This movement is partly angular and partly radial. The angular element of this motion must be added to or subtracted from the angular motion of the disc to find the true angular velocity of the body M on which the centrifugal force depends. This would lead to a very com-

plicated expression for the centrifugal force, involving the equation of the path in which the body moves.

The above expressions would be true if the body had merely a radial movement on the revolving disc, but are entirely erroneous as soon as an angular element is added to that movement. How entirely a formula deduced from such assumptions must fail to represent the action of centrifugal force in a turbine wheel will appear from this consideration. All the above-mentioned writers prescribe as a condition of efficiency that the water at its exit from the wheel should have the same velocity as the latter, in the opposite direction. In other words, that the angular velocity of the water at that point must be null, while a moment's reflection will show that their formula for the centrifugal force can only be true upon the assumption that the water has, here as well as elsewhere, the full angular velocity of the wheel.

I append the following example of the application of this formula. In Mr. Francis's experiments of 1851 upon a Fourneyron Turbine, he measured the discharge and useful effect at all velocities, from the condition in which the wheel was held fast by the brake, to that in which it ran entirely free. In the table "Hyd. Exp.," p. 33, the discharges are all reduced to a uniform fall of 13 feet. It by no means follows, however, that the variations in the discharge, under these circumstances, are wholly due to changes in the centrifugal force.

As the velocity increases from zero, there is a progressive change in the resultant direction of the water entering the wheel, and it comes more and more into coincidence with the direction in which the water leaves the guides. There is a progressively diminishing loss of head due to abrupt change of direction. Presumably, these two directions coincide, and the loss of head ceases, at the velocity corresponding to the maximum efficiency of the wheel. Between these two limits of velocity, therefore, viz., 0, and the velocity of maximum efficiency, we should expect the discharge to increase faster than is called for by the formula for centrifugal force. The results of experiments show nothing of this kind.

From the point of maximum efficiency to the point where the wheel runs free, there is an increasing deviation between the direction in which the water enters the wheel and that in which it leaves the guides, and consequently, an increasing loss of head. Between these latter limits, therefore, we might expect the increase of discharge to be less than that called for by the formula for centrifugal force. Between

the limits O and wheel running free, we might expect these two causes of discrepancy to neutralize each other, and the formula for centrifugal force to represent the total increase of discharge between these limits better than between any other limits.

The velocity of the wheel, running free, was expt. 23. Internal circumference 37·766 feet per sec.

$$\text{Ext. circum. } 37\cdot766 \times \frac{8\cdot292}{6\cdot75} = 46\cdot393 \text{ ft. per sec.}$$

The difference between the heights due these velocities is

$$\frac{46\cdot393^2 - 37\cdot766^2}{2g} = 11\cdot288.$$

The discharge of the wheel, standing still, under a head of 13 feet was (Exp. 43) 136·725 cubic feet per sec. According to the theorem under consideration, the centrifugal force should have the same effect in increasing the discharge as an addition of 11·288 feet to the head. The discharge, therefore, should be

$$136\cdot725 \sqrt{\frac{24\cdot288}{13}} = 186\cdot88 \text{ cubic feet per sec.}$$

The actual discharge in Expt. 23 was 165·37 cubic feet per sec.

Induced Currents in Reciprocal Movements.—In an article published by du Moncel, on Dec. 16, 1882, upon the effects produced in the Griseom motor, he showed that when one of the poles of a permanent magnet is moved before a straight electro-magnet there are three successive induced currents: 1. The one resulting from the approach of the magnet, which is inverse and momentary. 2. Another, which is manifested during the whole passage of the magnet before the magnetic nucleus, and which is indirect. 3. A third, resulting from the removal of the magnet, which is momentary and which, although in the same direction as the first, corresponds to a direct current, because the inductive current is then exerted on an opposite extremity of the magnetic nucleus. Of these three currents only one has the same energy at the different points of the magnetic path. It is the one which corresponds to the passage of the magnet from one end to the other of the nucleus, and it belongs to the category of currents which du Moncel has called currents of polar interversions, and which he is now investigating.—*La Lum. Elec.*, Jan. 27, 1883. C.