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XI. *On the Theory of Ionization by Collision.*

By P. O. PEDERSEN*.

1. **I**N an article in this journal Norman R. Campbell † treats of the mathematical theory of ionization by collision, making the simplest possible assumptions with regard to the nature of this ionization. Only ionization due to collisions between electrons and molecules is considered, and his assumptions may briefly be stated as follows:—

- (1) An electron will produce a fresh pair of ions (electron + positive ion) when it collides with a neutral molecule, if previously to that collision its velocity v satisfies the following condition:

$$v \geq v_0 = \sqrt{\frac{2e}{m} V_0}, \quad (1)$$

where m is the mass of the electron, $-e$ its charge, and V_0 a certain constant voltage depending on the nature of the molecule.

- (2) Collisions between electrons and molecules take place unelastically, so that after collision the electron is free again but has the velocity zero.
- (3) The electrons set free by ionization have also the velocity zero.
- (4) The velocities of the electrons due to the electric field are so great that the thermal velocities of the electrons and the molecules may be disregarded ‡.

2. Norman R. Campbell does not give the exact solution. He says §: “Now it is possible to calculate accurately according to the theory what should be the current between electrodes a distance a apart (fig. 1) in a gas at pressure p ,

* Communicated by the Author.

† *Phil. Mag.* (6) vol. xxiii. p. 400 (1912).

‡ With regard to the question of the validity of these assumptions, see J. S. Townsend, ‘Electricity in Gases’ (Oxford, 1915), and the discussion between Townsend and Norman R. Campbell, *Phil. Mag.* (6) vol. xxiii. pp. 856 & 986 (1912).

§ *Loc. cit.* p. 404.

when (the electric field) X has a given value, but the calculations are extremely complex, and the resulting

Fig. 1.

formula for the relation between i , a , p , X is not of a form to which Townsend's measurements can be applied." He therefore assumes that the number of electrons n_x varies continuously with the distance x from the cathode and thus obtains an approximate solution.

The exact solution is, however, quite simple and may be given a form to which Townsend's measurements may easily be applied. The deduction of this solution may therefore, perhaps, be of sufficient interest to be given here.

3. Let L be the mean free path of the electron; then the probability s that a new collision results in ionization is determined by

$$s = e^{-\frac{l_0}{L}}, \quad \dots \quad (3)$$

where

$$l_0 = \frac{V_0}{X} \quad \dots \quad (4)$$

The electrons coming from A (see fig. 1) fall into two groups 1 and 2. In the first we shall reckon the electrons whose initial free path is greater than l_0 and which will thus ionize at the first collision. The second group includes the electrons whose initial free path is less than l_0 : of these the first collisions which occur between $x=0$ and $x=l_0$ will not result in ionization.

The number of ionizations which take place in the space element between the planes x and $x+dx$, and which is

due to the first group of electrons, we shall call $N_0(x) \cdot dx$, where the suffix 0 denotes that the electrons come directly from A without intermediate collisions.

Fig. 2.

We have then

$$\left. \begin{array}{l} \text{for } x < l_0 : N_0(x) = 0, \\ \text{and for } x \geq l_0 : N_0(x) = \frac{n_0}{L} \cdot \epsilon^{-\frac{x}{L}}. \end{array} \right\} \text{(See fig. 2, I.).} \quad (5)$$

We shall then proceed to the determination of the first ionizations which are due to group 2.

In the element of space $(y, y + dy)$, where y in the same way as x signifies the distance from A and where $0 < y < l_0$, the number of collisions per second between electrons and molecules is $\frac{n_0}{L} \cdot dy$, but none of these collisions give rise to ionization. The next collision where these $\frac{n_0}{L} \cdot dy$ electrons take part will in the element of space $(x, x + dx)$ give $N_y(x) \cdot dx \cdot dy$ ionizations where

$$\left. \begin{array}{l} \text{for } x < y + l_0 : N_y(x) = 0, \\ \text{and for } x \geq y + l_0 : N_y(x) = \frac{n_0}{L^2} \cdot \epsilon^{-\frac{x-y}{L}}. \end{array} \right\} \quad \cdot \quad (6)$$

The total number of ionizations $N_{(0-l_0)}(x) \cdot dx$ in the element of space $(x, x+dx)$, which are due to electrons the direct collisions of which occurred between $x=0$ and $x=l_0$, is determined by :

$$\left. \begin{aligned} \text{for } x < l_0 : \quad N_{(0-l_0)}(x) &= 0, \\ \text{for } l_0 \leq x \leq 2l_0 : \quad N_{(0-l_0)}(x) &= \int_{y=0}^{y=x-l_0} N_y(x) \cdot dy = \frac{n_0}{L} (\epsilon^{-\frac{l_0}{L}} - \epsilon^{-\frac{x}{L}}), \\ \text{and for } x > 2l_0 : \quad N_{(0-l_0)}(x) &= \int_{y=0}^{y=l_0} N_y(x) \cdot dy = \frac{n_0}{L} (\epsilon^{-\frac{x-l_0}{L}} - \epsilon^{-\frac{x}{L}}). \end{aligned} \right\} (7)$$

The total number of ionizations $N(x) \cdot dx$ in the element of space $(x, x+dx)$, which are due to electrons which either come directly from A or the direct collisions of which took place within the distance l_0 from A, is thus determined by :

$$\left. \begin{aligned} \text{for } x < l_0 : \quad N(x) &= 0, \\ \text{for } l_0 \leq x \leq 2l_0 : \quad N(x) &= \frac{n_0}{L} \cdot \epsilon^{-\frac{l_0}{L}}, \\ \text{and for } x > 2l_0 : \quad N(x) &= \frac{n_0}{L} \cdot \epsilon^{-\frac{x-l_0}{L}}. \end{aligned} \right\} (\text{See fig. 2, II.}) \cdot (8)$$

It is obvious that the number $N_1(x)dx$ in the element of space $(x, x+dx)$, which are due to collisions with electrons originally coming from A, is determined by :

$$\left. \begin{aligned} \text{for } x < l_0 : \quad N_1(x) &= 0, \\ \text{and for } x \geq l_0 : \quad N_1(x) &= \frac{n_0}{L} \cdot \epsilon^{-\frac{l_0}{L}}. \end{aligned} \right\} (\text{Fig. 2, III.}) \cdot (9)$$

The total number of liberated electrons of that class which we called Class 1, of which the collisions are due to impacts between molecules and electrons originally coming from A, is therefore

$$n_1 = (a-l_0) \frac{n_0}{L} \epsilon^{-\frac{l_0}{L}}. \quad . \quad . \quad . \quad (10)$$

It is also possible to realize the correctness of this result directly, but the procedure followed above has the advantage that it gives some information about the "geometry" of ionization by collision.

These n_1 electrons cause directly by collision with molecules $N_2(x)dx$ ionizations in the element of space $(x, x+dx)$, here

$$\left. \begin{array}{l} \text{for } x < 2l_0: N_2(x) = 0, \\ \text{and for } x \geq 2l_0: N_2(x) = n_0(x-2l_0) \left(\frac{\epsilon^{-\frac{l_0}{L}}}{L} \right)^2. \end{array} \right\} \quad (11)$$

The total number of the liberated electrons of the class high we will denote by Class 2 is thus

$$n_2 = n_0 \frac{(x-2l_0)^2}{1 \cdot 2} \cdot \left(\frac{\epsilon^{-\frac{l_0}{L}}}{L} \right)^2. \quad (12)$$

Similarly, for Class ν ,

$$\left. \begin{array}{l} \text{for } x < \nu l_0: N_\nu(x) = 0, \\ \text{and for } x \geq \nu l_0: N_\nu(x) = n_0 \frac{(x-\nu l_0)^{\nu-1}}{(\nu-1)!} \cdot \left(\frac{\epsilon^{-\frac{l_0}{L}}}{L} \right)^\nu. \end{array} \right\} \quad (13)$$

and the total number

$$n_\nu = n_0 \frac{(x-\nu l_0)^\nu}{\nu!} \left(\frac{\epsilon^{-\frac{l_0}{L}}}{L} \right). \quad (14)$$

Fig. 3.

Fig. 3 shows an example of the distribution of the first four ionization classes.

The "centre of gravity" for the ν th class—by which we

understand the centre of gravity for the area drawn in accordance with fig. 3 representing $N_\nu(x)$ —lies at the distance r_ν from A given by

$$r_\nu + \frac{\nu}{\nu+1} (a + l_0). \quad (15)$$

Fig. 4 shows for a definite case the position of the centre of gravity for the first ten classes and their relative magnitudes.

Fig. 4.

4. From the preceding it follows that the total number of electrons n_a which reaches the B electrode is

$$\left. \begin{aligned} n_a &= \sum_{\nu=0}^{\nu=\nu'} n_\nu = \sum_{\nu=0}^{\nu=\nu'} \frac{(a - \nu l_0)^\nu}{\nu!} \cdot \left(\epsilon^{-\frac{l_0}{L}} \right)^\nu \\ &= \sum_{\nu=0}^{\nu=\nu'} \frac{(h - \nu)^\nu}{\nu!} \cdot y^\nu = n_0 \sum_{\nu=0}^{\nu=\nu'} \frac{\left(1 - \frac{\nu}{h} \right)^\nu}{\nu!} (\alpha a)^\nu, \end{aligned} \right\} \quad (16)$$

where $h = \frac{a}{l_0}$, $y = \frac{l_0}{L} \cdot \epsilon^{-\frac{l_0}{L}}$, and $\alpha = \frac{1}{L} \epsilon^{-\frac{l_0}{L}}$. . . (16')

while ν' is the greatest integer which is less than or equal to h .

In figs. 5 *a* and 5 *b* the curves I. and II. represent $\log \frac{n_a}{n_0}$ as function of h for respectively $y=0.1$ and 0.2 . It is

Fig. 5 *a*. Fig. 5 *b*.

evident that I. and II. are very nearly straight lines, which intersect the abscissæ axis at the point $h=1$.

The same holds also for other values of y ; with sufficient approximation we can therefore put

$$\log \frac{n_a}{n_0} = \gamma y(h-1) \quad . \quad . \quad . \quad (17)$$

$$\text{or} \quad \frac{n_a}{n_0} = \epsilon^{\gamma y(h-1)}, \quad . \quad . \quad . \quad (18)$$

where γ is a coefficient which only depends on y .

In place of Townsend's relation

$$\frac{n_a}{n_0} = \epsilon^{a\alpha},$$

we thus obtain the equation

$$\frac{n_a}{n_0} = \epsilon^{\gamma a(a-l_0)}. \quad . \quad . \quad . \quad (19)$$

In the table below are given approximate values of γ calculated on the basis of equations (16) and (17) for values of y between 0 and $0.368 = \epsilon^{-1}$, which latter value is the highest y can obtain:—

y .	γ .	y .	γ .	y .	γ .	y .	γ .
0.00	1.00	0.10	0.92	0.20	0.85	0.30	0.79
.01	.98	.11	.91	.21	.84	.31	.79
.02	.98	.12	.90	.22	.84	.32	.79
.03	.97	.13	.89	.23	.83	.33	.78
.04	.96	.14	.89	.24	.83	.34	.78
.05	.96	.15	.88	.25	.82	.35	.77
.06	.95	.16	.87	.26	.82	.36	.77
.07	.95	.17	.87	.27	.81	.368 ...	.76
.08	.93	.18	.86	.28	.81		
.09	.92	.19	.86	.29	.80		

5. The determinations of l_0 , V_0 , γ , and α are made on the basis of measurements of corresponding values of $\frac{n_a}{n_0}$ and a , and by use of equation (17) as well as the table above. As example, we will treat the following series of measurements by Townsend (*loc. cit.* p. 277; Phil. Mag. (6) vi. p. 598, 1903):—

Atmospheric air; Pressure 1 mm. Hg.

$L = 3.2 \times 10^{-2}$ cm.; $X = 350$ volt/cm.

$a = 0$	0.2	0.4	0.6	0.8	1.0	1.1 cm.
$\frac{n_a}{n_0} = 1$	2.86	8.3	24.2	81.0	373	2250
$\log \frac{n_a}{n_0} = 0$	1.05	2.12	3.19	4.39	5.92	7.72

The values of $\log \frac{n_a}{n_0}$ are drawn with a as abscissæ (fig. 6).

One puts after a preliminary trial $l_0 = 0.05$ cm., and draws through the point c on the abscissæ axis determined by $0c = l_0$ a straight line cb which for small values of a

Fig. 6.

agrees as closely as possible with the found values of $\log \frac{n_a}{n_0}$. (For greater values of a , $\log \frac{n_a}{n_0}$ lies above the line cb on account of the ionization caused by the positive ions of which no account is taken.) From fig. 6 is obtained $\gamma a = 5.8$. To $l_0 = 0.05$ cm. corresponds $y = \frac{l_0}{L} \cdot e^{-\frac{l_0}{L}} = 0.328$, to which again according to the preceding table corresponds $\gamma = 0.78$. Consequently $a = \frac{5.8}{0.78} = 7.44$. From this is found by using equation (16') $l_0 = 0.046$ cm., and as this value is sufficiently near to the one previously assumed, namely 0.05 cm., there is no reason to make a new determination.

For the ionization voltage we now obtain,

$$V_0 = 0.046 \times 350 = 16.1 \text{ volt.}$$

A further discussion of this problem is to be found in a Danish paper*.

Copenhagen,
November 1919.

* *Det Kgl. Danske Videnskabernes Selskab. Mathematisk-fysiske Meddelelser*, i. part 7 (1918).