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“Experiments on the Relative Strength of Cast-Iron Beams.”¹

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It is evident that, after the exhaustive labours of Messrs. Barlow, Hodgkinson, and others, the effect of any experiments designed only to measure the strength of cast-iron beams of well-known sections would be merely to multiply examples and not to introduce anything new. It is hoped by the Authors that they are guilty neither of presumption nor of ignorance of existing authorities in instituting these experiments, inasmuch as their object is not so much to determine the absolute strength of the pieces, as to compare the behaviour of the beams under similar conditions; and to determine, among other things, if theory is right in asserting that certain changes of form of the section can be made without affecting the strength or general behaviour of the beam. For example, according to theory² a beam of hollow circular section ought to give results identical with those of a beam having the same sectional area, the form of the section, however, being arrived at by massing up the metal of the hollow cylinder about a diameter parallel to the line of application of the load. Also, the double tee-section (C) ought to behave in the same way as the box-section (E).

The different sections experimented upon were the following:—The small double tee-section (C) is to compare with the box-section (E), the box-section being arrived at by dividing the web and placing one-half to form each of the vertical sides of the box.

¹ This Paper was read at a meeting of the Students on the 16th of April, 1886.

² The Authors wish it to be understood, that in speaking of theory, they mean theory as far as mathematicians have succeeded in employing it. For theory is to practice what algebra is to geometry, and there can be no doubt that, were the theory of any subject complete, it would be possible not only to account for, but to calculate and be ready for, every incident that could occur in the practical working of that subject.

The solid circular sections L and M (L being turned in order to remove the skin) compare with—

(1) The hollow circular sections N and P (P being turned in order to remove the skin).

(2) The hollow cylindrical section massed up about a diameter parallel to the line of application of the load (R).

FIG. 1.

Upwards of sixty pieces were cast of the various sections just alluded to, and also ten test-pieces, 15 inches long and $1\frac{1}{4}$ inch in diameter, for the determination of the specific extension, specific compression, and crushing load of the material. All the pieces were, of course, cast from the same ladle, and, with the exception of three 36 inches by 1 inch by 1 inch bars, all were cast on end.¹

¹ This cast-iron was composed of:—1 part Damelton, 5 parts best scrap.

All the beams, except those marked F and S, were designed to have a sectional area of 3 square inches.

Ten beams were rectangular in section (A and B), six were tested as they came from the foundry, and four were planed all over so as to remove the skin.

Three, four, or five of each of the other sections were cast, and one specimen of each section was kept in reserve until the Authors had satisfied themselves that any apparently abnormal results were really due to the form of the bar, and not to the quality of the casting.

A special allowance was made in the patterns of those beams which were to be machined (A, B, L, P), so that ultimately the sectional-area of these beams was the same as that of the un-machined ones.

Two beams were designed to have a constant strength along the whole span. This result was arrived at in two different ways:—

(1) By varying the depth (H).

(2) By varying the breadth (K).

The dimensions of the beam marked S are reduced from those of the beam found practically to be the strongest of the double tees with unequal flanges by Mr. Hodgkinson.

As regards the theoretical shape of the double tee-section, it might be interesting to point out one or two details.

If the assumptions upon which the beam theory is founded are admitted as correct, it can be shown mathematically that, in order to dispose of the metal in the best way, it must be so arranged that the neutral-axis divides the depth in the ratio of the breaking-stress in tension to the breaking-stress in compression, and if the area of the web is neglected, the areas of the flanges must also be in this proportion.

The actual form of this section, arrived at by purely theoretical investigations, is due to Professor Karl Pearson, M.A., LL.B., of University College. Mr. Pearson finds that section to be theoretically the strongest in which not only are the before-mentioned conditions fulfilled, but where also the difference in the thicknesses of the upper and the lower flanges is as great as possible. This means, in mathematical language, where the upper flange has a finite and the lower an infinitely small thickness, or where the one has a finite and the other an infinitely great thickness.

All the pieces to be subjected to transverse-stress were provided with small faces at suitable points. These faces were planed true, and afforded proper bearings for the knife-edges of the testing-machine.

The whole of the experiments were made in the Engineering Laboratory of University College, Gower Street, with one of Greenwood and Batley's 100,000 lbs. horizontal testing-machines.¹ It is needless to go into any detail concerning this machine, as it has now become well known, although the particular machine referred to here was the first of its type, and was especially constructed for Professor Alex. B. W. Kennedy, M. Inst. C.E., by the above-mentioned firm. The load is applied by a hydraulic ram, and balanced by a dead weight. The machine has been frequently calibrated and found to be correct.

The tensile- and the compressive-strains, as well as a large number of deflections, were measured by an ingenious mechanism, the design of Professor Kennedy, which has been fully described and illustrated by Mr. P. V. Appleby, Stud. Inst. C.E., in a Paper on "Experiments on iron and steel."² As this gear is of rather delicate construction, and could not be left on the piece until fracture occurred, and as it was desirable to measure the deflection at the instant of fracture, another gear was contrived by the Authors for the purpose. It consists of a pointer A B (Fig. 2), one end A of which is kept lightly pressing on the beam at the centre of the span; the other end B holds a pencil, which rests on the surface of a piece of prepared paper stretched on a drum, the axis of which is at right angles to the mean position of the pointer. It has been mentioned before that the pull of the hydraulic ram is balanced by a dead weight. This weight is attached to a carriage, which runs along a steel-yard; thus the position of the carriage at any moment is proportional to the load, and therefore to the stress at that moment. A fine strong silk cord was attached to the carriage, was next passed round the two pulleys E and F, and was then allowed to hang freely, a small weight being attached to the end by an elastic band, in order to secure as nearly uniform tension as possible throughout the whole length of the cord. Both the spindles G and S, as well as the pivot of the pointer, moved on centres, which were rigidly connected to the cross-head of the machine. Thus, as the load on the beam increased, the end B of the pointer moved nearly parallel to the axis of the drum, while the motion of the carriage on the steel-yard caused the spindle G to rotate, which rotation was communicated to the drum by means of the silk band K. In this way a

¹ For description of this machine see "Engineering," of the 26th of September, 1879.

² Minutes of Proceedings Inst. C.E., vol. lxxiv. p. 258.

between two limits until the set was practically eliminated, and then a series of readings was taken from which calculations were made. The beams were all tested on a span of 20 inches, and the load was applied at the centre.

The results are fully recorded in the Tables which accompany this Paper, and it will only be needful to touch upon one or two details.

The results of the machined beams of rectangular section (A and B) are confirmatory of the law that the breaking loads of beams vary as the square of the depth and as the breadth. The four planed-up specimens were 2.93 inches by 0.9 inch, therefore A was 3.26 times as deep as broad, and B was 3.26 times as broad as deep. Hence the breaking-load of A ought to be equal to the breaking-load of B $\times \frac{(3.26)^2}{3.26}$, or A's breaking-load ought to be 3.26 times as large as B's. The mean breaking-load of the machined A's was 4.82 tons, and of the machined B's 1.50 ton, and $\frac{4.82}{1.50} = 3.21$, which practically agrees with 3.26.

The reason of this law is readily understood on considering the resisting efficiency of these sections as determined graphically. (For an explanation of the graphic method see Appendix.)

If a beam be twice as deep, but of the same breadth, as another, not only is the area of the equivalent figure doubled, but the arm of the resisting couple is also doubled; hence the beam becomes $2 \times 2 = 4$ times as strong when the depth is doubled without altering the breadth. Now suppose the breadth to be doubled while the depth remains unaltered, then the area of the equivalent figure is doubled; but the arm of the resisting couple is not altered, hence it becomes only twice as strong when the breadth is doubled.

The results of the double tee (C) tested upright and on its side (D) show clearly the greater resisting efficiency of the material when placed at as great a distance as possible from the neutral axis. The double tee (C) broke under 2.63 times as great a load as D, and was considerably stiffer. It is difficult to state what is the depth, and what the breadth of such sections as these; and it seems not unreasonable to consider the mean breadth of the equivalent figure (see Appendix), and the arm of the couple of internal resistance, as the equivalents of the breadth and the depth of the section. Unfortunately, it is impossible to test the accuracy of this suggestion by a comparison of the results given by these particular pieces, as several of them exhibited very unsound fractures. But in former experiments on the transverse strength

of various materials the Authors tested some beams of similar sections, and an examination of the results then obtained will serve to determine the point.

On a span of 20 inches the double tee broke at 9,500 lbs. on the centre of the span, and the double tee tested on its side broke with a load of 2,190 lbs., or about $\frac{1}{4\frac{2}{3}}$ of the breaking-load of the first. On a span of 9 inches the respective breaking-loads were 28,440 lbs. and 5,760 lbs., the double tee when on its side breaking with about one-fifth the load it could bear when upright. These results show the double tee upright to be nearly five times as strong as the double tee on its side.

Now the mean breadth of the equivalent figure of the double tee upright was 0.5 inch, and the arm of the couple of internal resistance was 1.88 inch. The respective values of these quantities in the other beam were 0.5 inch and 0.84 inch. Hence the breaking-loads ought to be in the ratio $\left(\frac{1.88}{0.84}\right)^2 \times \frac{0.5}{0.5} = 5.00$, which is practically correct.

The behaviour of the double tee (C) and the box-section (E) shows that, as far as this modification of the form of the section is concerned, the theory is correct that the strength of the beam remains unaltered, of course within the limits of reasonable design. They are equally stiff, equally strong, yield the same transverse modulus, and behave generally in the same way under stress.

The results given by the beams designed to have constant strength throughout the whole span (H and K), (1) by varying the breadth only (K), (2) by varying the depth only (H), show equal stiffness in both sections, and equal values of the ratio of the breaking-load to the weight of the beam.

Owing to practical considerations, the ends of the parabolic girder became much stiffer than they ought to have been; thus these two beams cannot be said to compare satisfactorily one with the other.

The behaviour of those beams of circular section, and its modifications (L, M, N, P, R), may be compared with considerable accuracy, inasmuch as their sectional-areas varied only to the extent of about 3 per cent. from the area for which they were designed, namely, 3 square inches. For convenience in reference, the solid circular section will be denoted by M; solid circular turned, in order to remove the skin, L; hollow circular section, N; hollow circular section turned, P; hollow circular massed up about a diameter parallel to the line of application of the load, R.

Of these five, M shows the lowest mean breaking-load, but M

and L are very nearly equal as regards strength. L shows a higher ultimate deflection, due possibly to the fact of the skin having been removed, which circumstance would naturally render the outer fibre more free to extend. The same thing is noticed in the behaviour of P, but it is not so marked as in the case of L.

The stiffness of M and N are about the same, but the breaking-load of N is higher than that of M, which is only to be expected, the metal being more advantageously placed in N than in M.

L and P show a similar difference, but not quite so marked, possibly because the ratio of the depth of the skin to the total depth of the beam is not so large in P as in L.

R has a lower mean breaking-load than either P or N, the progression being $N > P > R$, but, on the other hand, R is decidedly stiffer. This may be due to the fact of the shape of the section approaching somewhat to the form of a double tee.

Theoretically, that is so far as theory is employed, N, P, and R, ought to be identical in their behaviour. Practically, however, they differ in strength and in stiffness.

Calling the breaking-load of N unity,

$$N = 1.000$$

$$P = 0.908$$

$$R = 0.872$$

the breaking-loads being in each case the mean of the results given by three beams exhibiting sound fractures.

The order of stiffness (calculated from the ultimate deflection) is:—

$$R = 1.000$$

$$N = 0.793$$

$$M = 0.763$$

$$P = 0.730$$

$$L = 0.574$$

The order of values of ratio $\frac{\text{breaking-load}}{\text{weight of beam}}$ is:—

$$N = 1.000$$

$$P = 0.920$$

$$R = 0.850$$

$$L = 0.695$$

$$M = 0.654$$

This shows that of these five sections N behaves on the whole in the most satisfactory manner, exhibiting the highest breaking-

load, and the highest value of the ratio of the breaking-load to the weight of beam, and being second in order of stiffness.

In small structures, where the depth of the skin bears an appreciable ratio to the total thickness of the piece, the skin materially enhances the ultimate strength. The results of the pieces tested with and without the skin are given below.

The modulus of elasticity in tension is denoted by E_T , and that in compression by E_C :—

$$\begin{aligned} E_C \text{ \{ obtained from test-pieces \} } &= 15570000. \\ E_T \text{ \{ with the skin . . . \} } &= 13950000. \\ E_C &= 1.116 E_T. \end{aligned}$$

$$\begin{aligned} E_C \text{ \{ obtained from turned \} } &= 13900000. \\ E_T \text{ \{ test-pieces . . . \} } &= 12470000. \\ E_C &= 1.114 E_T. \end{aligned}$$

$$\begin{aligned} E_C \text{ with skin} &= 1.12 E_C \text{ without.} \\ E_T \text{ with skin} &= 1.1187 E_T \text{ without.} \end{aligned}$$

It appears from this that the modulus in compression is about 12 per cent. higher than the modulus in tension under both conditions, that is, with skin and without; but that the respective values of E_C and E_T are smaller when obtained from test-pieces with the skin removed.

It appears also that the skin influences the ultimate transverse strength.

The mean breaking-load of A_{14} and A_{15} reduced to the mean area of A_{11} and A_{12} is 5.83 tons.

The mean breaking-load of B_{14} and B_{15} reduced to the mean area of B_{11} and B_{12} is 1.77 ton.

The mean breaking-load of A_{11} and A_{12} = 6.6 tons.

do. do. B_{11} and B_{12} = 2.11 tons.

Ratio of non-machined to machined in first case = $\frac{6.6}{5.83} = 1.13$.

Ratio of non-machined to machined in second case = $\frac{2.11}{1.77} = 1.19$.

It might perhaps be not uninteresting to consider the tendency of the results of these experiments with regard to the ordinarily accepted theory, by means of which the stresses in a beam are commonly calculated. The following remarks, it must be understood, apply solely to cast-iron.

This theory is based upon the assumption—

(1) That the stress varies directly as the distance from the neutral-axis.

(2) That the tensile-modulus is equal to the compressive-modulus.

(3) That every plane section of a beam remains a plane section after flexure.

(4) That the shear is inappreciable.

Now formulae based on these assumptions give values which agree very fairly with values obtained by experiment, as long as the load on the beam is well within the maximum load that the piece will bear. When, however, the load approaches the breaking-load, values of the deflection obtained from the formulae are no longer reliable; they are smaller than those actually measured. The ultimate tensile-stress on the outer fibre calculated from the formula: bending moment = stress $\times \frac{I}{y}$ (where I = moment of inertia of section, and y = distance of the outer fibre from the neutral axis) is invariably greater than the ultimate breaking-stress obtained from a bar pulled asunder in direct tension. (For convenience in future reference this ratio will be called $\frac{\text{breaking-stress from beam}}{\text{breaking-stress from test-piece}} = \theta$.)

Now, of course, the question arises, does this abnormal stress really exist in the outer fibre, or is it only an inaccurate result brought about by mathematical reasoning based on insufficient premises?

If the assumptions are not of themselves sufficiently incorrect to account for so large a discrepancy as is sometimes exhibited—and it is probable that the Authors are justified in taking this view, the cause of this discrepancy must be sought in the existence of other sources of strength, left out of account in the ordinary calculations, and in the fact that the tensile-modulus of cast-iron decreases as the stress increases. The tendency of these results is to show that this discrepancy decreases as the stiffness increases, that is, those sections which were relatively the stiffest show the least ratio θ . Individual pieces of the same section show also a variation of θ in the same direction as the stiffness. In Fig. 3 the values of ratio θ are plotted to a horizontal scale of maximum deflections, and a line drawn through these points so as to take an average, as it were, is nearly straight and rises gradually. Of course it is now impossible to estimate how much of these ultimate deflections was set, but still from the general behaviour of the pieces, and from the results of other experiments, it appears highly probable that the ratio θ varies in the same direction as the relative stiffness. Thus it might be concluded that, collectively, these

sources of strength, and the relative stiffness or the deflection, are functionally connected.

It is certain that the tensile-modulus of cast-iron decreases as the stress increases. This is shown by the results of Mr. Hodgkinson's experiments on the tensile-strength of cast-iron, and is corroborated in some very careful determinations of the extension of cast-iron under increasing loads by Professor Kennedy.

It is also not unreasonable to suppose that a fibre, by virtue of its being molecularly connected to less strained material, becomes capable of enduring a greater stress without breaking than if it were pulled in direct tension.

FIG. 3.

Then again the results of experiments, carried out by Professor Kennedy, with a view to measure the change of angle of the corners of a beam when bent, point to the fact that originally square corners do not remain square after flexure. The change of angle is, however, not sufficiently great to account for the discrepancy.

It must not be forgotten that the ordinary beam theory is a greatly simplified form of a complex problem, for it only takes account of the horizontal and the vertical components of the many stresses existing in a beam when loaded.

Taking these things into consideration, it may be concluded

that, at all events, it is known in what direction to look for a means of clearing up this difficulty, but, at the same time, the means may be of so complex a character, involving the use of so high mathematical reasoning, as to render it doubtful whether the end justifies the means; and whether it would not be more advisable, at least for practical purposes, to introduce an empirical coefficient in the formulae, which would allow for the stresses acting along the principal axes, the change of modulus, &c., and so give approximately correct values.

The Authors wish to express their great indebtedness to Professor Kennedy for the interest manifested by him in these experiments, and for many valuable suggestions as regards the method of testing the specimens.

The Paper is accompanied by several diagrams, from which the Figs. in the text have been prepared.

APPENDIX.

RESULTS OF EXPERIMENTS ON TEST-PIECES.

No.	Description.	Area in Square Inches.	Specific Extension in 1,000ths of an Inch.	Specific Com- pression in 1,000ths of an Inch.	E_T lbs. per Square Inch.	E_C lbs. per Square Inch.	Breaking- Stress. Tons per Square Inch.
10	With skin .	1.112	0.730	0.650	13,700,000	15,400,000	..
10	Turned . .	0.967	0.788	0.687	12,700,000	14,500,000	11.82
11	With skin .	1.090	..	..	..	..	..
12	„ .	1.090	0.625	..	16,000,000	..	11.50
13	„ .	1.100	0.633	..	15,800,000	..	..
13	Turned . .	0.862	0.644	..	15,530,000	..	11.85
14	With skin .	1.149	..	..	..	..	11.45
15	„ .	1.150	0.687	0.625	14,550,000	16,000,000	..
15	Turned . .	0.790	0.850	0.834	11,700,000	12,000,000	10.20
16	With skin .	1.120	very unsound			..	..
17	„ .	1.160	..	..	..	..	10.60
18	„ .	1.149	0.746	0.632	13,400,000	15,800,000	..
18	Turned . .	1.000	0.760	0.655	13,180,000	15,200,000	10.04
19	„ . .	0.862	0.770	..	13,000,000	..	11.40

Mean breaking-stress with skin = 11.18 tons per sq. in.

„ „ without „ = 11.06 „ „

Mean modulus in tension = E_T . . . = 14,680,000 lbs. per sq. inch.

„ in compression = E_C . . = 15,000,000 „ „

Crushing-load of struts, 2 inches long and 0.86-inch diameter =
68,600 lbs. = 53 tons per square inch.

Crushing-load of a similar piece = 66,500 lbs. = 50 tons per sq. in.

Both these pieces were cut from test-pieces which had been broken
in tension.

RESULTS OF EXPERIMENTS ON BEAMS.

No.	Description.	Area in Square Inches.	Z.	Deflection at calculated stress of 10,000 lbs. per Square Inch on Outer Fibre.		Breaking Load.		Breaking Stress in Square Inch.	Ultimate Deflection in Inches.	Transverse Modulus in lbs. per Square Inch.	Br'king Load Weight.	θ .	Fracture.
				Deflection in Inches.	Load at Centre in lbs.	Lbs.	Tons.						
A ₁₁	Rect.	3.2	1.627	0.015	3,250	14,780	6.6	20.3	0.074	14,660,000	869.4	1.815	Sound.
A ₁₂	"	3.05	1.55	"	"	10,400	4.64	14.99	0.0804	"	630.0	1.33	{ Small blow-holes on tension side.
A ₁₃	"	2.63	1.287	"	"	10,960	4.89	19.00	0.104	11,800,000	776.0	1.719	Perfectly sound.
A ₁₄	" planed	2.63	1.287	"	"	10,660	4.76	18.50	0.0834	13,200,000	754.0	1.67	Perfectly sound.
B ₁₁	Rect.	3.121	0.5304	0.0436	1,060	4,680	2.09	19.7	0.204	14,520,000	279.0	1.76	Sound.
B ₁₂	"	3.255	0.5693	"	"	5,580	2.49	21.89	0.2107	"	318.0	1.96	Sound.
B ₁₃	"	3.055	0.5092	"	"	3,920	1.75	17.14	0.188	15,500,000	230.5	1.53	Sound.
B ₁₄	" planed	2.63	0.395	"	"	3,415	1.52	19.3	0.269	16,100,000	239.0	1.74	Sound.
B ₁₅	" planed	2.63	0.395	"	"	3,335	1.49	18.8	0.2906	13,830,000	235.0	1.70	Sound.
C ₁₁	"	3.407	2.59	0.0172	5,186	16,680	7.44	14.36	0.064	12,300,000	953.3	1.285	Unsound.
C ₁₂	"	3.389	2.68	"	"	16,350	7.3	13.6	0.0521	12,000,000	921.0	1.217	{ Cinder-flaw and blow-hole.
D ₁₃	"	3.266	0.995	"	"	5,160	2.30	11.57	0.0792	12,400,000	341.0	1.035	{ Blow-hole in compression flange.
D ₁₄	"	3.506	1.12	0.034	2,135	7,380	3.29	14.70	0.099	11,200,000	410.0	1.315	Perfectly sound.
E ₁₁	"	3.193	2.265	"	"	16,300	7.27	16.05	0.066	"	1,015.0	1.43	Not very sound.
E ₁₂	"	3.01	2.214	0.018	4,430	20,056	8.95	20.20	0.101	12,830,000	1,234.0	1.805	{ Blow-hole in compression flange.
E ₁₃	"	2.94	2.176	"	"	13,620	6.08	14.00	0.0598	"	864.4	1.25	Sound.
E ₁₄	"	2.95	2.22	"	"	13,050	5.82	13.12	0.0557	"	803.3	1.17	Not very sound.
E ₁₅	"	3.119	"	"	"	"	5.00	"	"	"	"	"	"
F ₁₁	"	3.378	3.588	"	"	"	"	"	"	"	"	"	"
F ₁₂	"	3.338	3.635	0.0146	7,270	19,440	8.68	12.00	0.0479	"	1,143.0	1.074	Unsound.
F ₁₃	"	3.44	3.7	"	"	16,560	7.39	10.00	0.0406	11,150,000	946.0	0.895	Very unsound.

	3-447	3-69	..	21,600	9-64	13-03	..	1,250-0	1-165	Unsound.
F ₁₄	3-08	..	..	13,260	8-6	17-2	0-105	1,130-0	..	Very unsound.
G ₁₁	3-179	..	..	21,190	9-46	20-04	0-115	1,240-0	1-79	Perfectly sound.
G ₁₂	3-12	2-50	0-022	21,750	9-71	19-42	..	1,280-0	1-73	Quite sound.
H ₁₁	3-41	2-527	..	21,950	9-8	19-6	0-103	1,510-0	1-75	Quite sound.
H ₁₂	3-43	2-568	..	19,380	8-65	16-85	0-0833	1,360-0	1-50	Fairly sound.
H ₁₃	3-48	2-592	..	21,150	9-44	18-22	0-089	1,385-0	1-63	Sound.
H ₁₄	3-34	2-595	..	16,410	7-34	14-13	0-0784	..	1-26	Sound.
K ₁₁	3-14	2-381	0-026	13,560	6-05	12-72	0-0636	1,154-0	1-138	Bad blow-hole.
K ₁₂	3-04	2-311	..	16,400	7-32	15-9	0-100	1,470-0	1-42	Quite sound.
K ₁₃	3-14	2-373	..	13,010	5-81	12-2	0-0625	1,100-0	1-09	Not very sound.
K ₁₄	3-00	2-330	..	14,600	6-55	14-05	0-0783	1,300-0	1-26	Sound.
L ₁₁	2-89	0-693	0-0235	7,500	3-34	24-15	0-1459	454-0	2-185	Sound.
L ₁₂	2-89	0-693	..	6,840	3-01	22-00	0-130	421-0	1-99	Sound.
L ₁₃	2-89	0-693	..	8,310	3-71	26-70	0-175	511-1	2-414	{Blow-hole on compression side.
M ₁₁	2-98	0-727	0-0214	6,460	2-88	19-83	0-102	410-0	1-77	{Blow-hole in centre.
M ₁₂	3-14	0-784	..	7,410	3-31	21-13	0-125	449-0	1-89	{Crystalline in centre.
M ₁₃	3-079	0-761	..	7,410	3-31	21-70	0-111	419-0	1-94	
N ₁₁	2-766	1-044	..	10,050	4-48	21-50	0-105	659-0	1-923	Sound.
N ₁₂	2-823	1-082	0-0196	10,560	4-71	21-80	0-1135	681-0	1-95	Sound.
N ₁₃	2-746	1-04	..	10,050	4-48	21-50	0-105	659-0	1-923	Cinder-flaw.
P ₁₁	2-82	1-025	..	10,230	4-56	22-30	0-128	660-0	2-00	Sound.
P ₁₂	2-73	1-007	0-022	8,730	3-89	19-36	0-109	572-0	1-75	Sound.
P ₁₃	2-82	1-025	..	9,300	4-15	20-20	0-117	610-0	1-82	Sound.
R ₁₁	..	1-03	0-019	8,850	3-95	19-1	0-0825	561-0	1-71	Sound.
R ₁₂	..	1-056	..	9,000	4-01	19-1	0-0907	580-0	1-71	Sound.
R ₁₃	..	1-093	..	8,910	3-98	18-20	0-0845	557-0	1-63	Sound.
R ₁₄	..	1-093	0-019	8,920	3-98	18-20	0-079	557-0	1-628	Sound.
S ₁₁	3-623	4-43	..	27,280	12-18	13-74	0-088	1,390-0	1-23	Unsound.
S ₁₂	3-834	4-62	0-036	31,560	14-10	15-20	0-0866	1,540-0	1-36	Sound.
S ₁₃	3-900	4-82	..	23,950	10-69	11-09	0-083	9,890,000	0-99	{Large blow-hole in tension flange.
S ₁₄	3-93	4-80	..	26,040	11-60	12-10	0-0928	1,270-0	1-082	{Large blow-hole in tension flange.

Turned

Turned

GRAPHIC METHOD of DETERMINING the MOMENT of INTERNAL RESISTANCE of
any CROSS-SECTION of a BEAM.

(See B. Baker "On the Strength of Beams, Columns, and Arches.")

Since one moment can only be completely balanced by another, the bending moments of the external forces acting on a beam can only be equilibrated by a moment of internal resistance. In the ordinary process of finding this internal resistance, only the tensile and the compressive couple are taken into account, other sources of resistance, such as shear, being practically of no consequence, and others such as values of principal stresses being very small and difficult to determine. From the fact that one side of a bent beam exhibits tension and the opposite side compression, it follows that there must exist one longitudinal section whose length remains unaltered during the process of bending. This section is known as the neutral plane. By Hooke's law the stress varies as the distance from the neutral plane; thus, considering a cross-section of a beam and a section of a very thin plane $A B C D$ (Fig. 4), so that the stress may be assumed to remain constant over the area $A B C D$, the intensity of stress over $A B C D$ will be equivalent to the intensity of stress in the extreme outer plane

FIG. 4.

FIG. 5.

reduced in the ratio of the distance of $A B C D$ from the neutral plane to the distance of the outer plane from $N P$; or

$$\text{Stress in } A B C D = \text{stress in outer plane} \times \frac{O M}{O Q}.$$

Consider now the whole section divided into sections of planes, which, for the moment, may be considered to have first an appreciable thickness, and let $A B C D E$ (Fig. 5), be the mid points of these sections of the planes; also consider the planes to be so thin that the stress may be considered constant over the respective areas.

Let S = intensity of stress in the extreme outer rectangle.

Let n = area of extreme outer rectangle, then the stress in the—

1st rectangle		= $n S$
2nd	„	= $n S \frac{B O}{A O}$
3rd	„	= $n S \frac{C O}{A O}$
4th	„	= $n S \frac{D O}{A O}$
5th	„	= $n S \frac{E O}{A O}$

So the total stress in that section—

$$= n S + n S \frac{BO}{AO} + \&c.$$

$$= S \left(n + n \frac{BO}{AO} + n \frac{CO}{AO} + n \frac{DO}{AO} + n \frac{EO}{AO} \right).$$

The expression put in this form shows that instead of reducing the stress in the ratio of the distance of the rectangle from NP, the stress may be considered as constant, and the areas of these rectangles may be reduced in that ratio. If that be done a diagram is obtained such as the following shaded one (Fig. 6),

where $AB = MK \frac{OC}{OD}$.

Now where the planes become infinitely thin the diagram becomes a triangle (Fig. 7), and if the area ABO is then multiplied by the intensity of stress in the extreme outer fibre, the total effective stressed area of that half of the section is obtained.

If the modulus of elasticity in tension be equal to the modulus in compression, the compressed portion of the section must contribute as much to the internal

FIG. 6.

FIG. 7.

resistance as the extended portion; hence the effective stressed area on each side of the NP must be equal; hence the NP must pass through the centre of gravity of the whole section.

The resistance of the stressed areas may be considered as concentrated at the centre of gravity of this shaded area (called the equivalent figure); hence the area of either half of the equivalent figure multiplied by the distance between the centres of gravity of each half, and multiplied by the stress, constitutes the moment of internal resistance.

The quantity (area of half equivalent fig. $\times$ arm) is called Z. Thus $ABO \times CP = Z$.

Let b = breadth of rectangular section = AB.

d = depth = BK,

then $ABO = \frac{bd}{4}$ and $CP = \frac{2}{3}d$,

hence $Z = ABO \times CP = \frac{2bd^2}{12} = \frac{bd^2}{6}$.

SZ being the moment of internal resistance must be equal and opposite to the

bending moment, in order that the beam may be in equilibrium under the load.

Let M = bending moment ;

then

$$M - S Z \text{ must } = 0.$$

$$M = S Z$$

$$S = \frac{M}{Z},$$

hence, knowing M and Z it is possible to calculate S , the stress in the outer fibre.

Now the moment of inertia of this section about $N O P$ is $\frac{1}{12} b d^3$ which can be written $\frac{1}{6} b d^2 \times \frac{d}{2}$

$$= Z \frac{d}{2}.$$

So

$$M = S I \frac{d}{2}.$$

It can be shown that the equation $M = S I y$ holds for any section, y being the distance of the outer fibre of the half of the equivalent fig. considered from the neutral axis.

In symmetrical sections it does not matter whether the length of the outer fibre on the upper or lower side of the beam be taken to represent graphically the stress in that fibre: but in sections such as that of a tee-iron, the choice of the length of the fibre on the extended, or on the compressed side of the beam, materially alters the appearance of the equivalent figure, though not the value of the quantity Z . An explanation of this will be found in Mr. B. Baker's work "On the Strength of Beams, Columns, and Arches."