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The Dynamical Deflection and Strain of Railway Girders.

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There is no subject in practical science which has been more elaborately investigated than the theory of the statical transverse strength of beams. It has fortunately happened that two different classes of investigators—mathematicians and experimentalists—have co-operated in the research; and the result of their united labors has been a valuable and comprehensive system of knowledge.

But the DYNAMICAL strength of beams, or their capability of sustaining weights moving rapidly over them, has never been satisfactorily discussed. There does not appear to be extant a single theoretical investigation of this subject—and the deficiency is due to two causes: it occurs partly because the subject has but comparatively recently grown into importance; partly because of its excessive and insuperable difficulties when investigated by the exact methods of theoretical mechanics. The following paper is a contribution to a more accurate knowledge of this important question, which has at length attracted the attention due to its influence on the security of railway traffic. The necessity of further inquiry seems to be generally acknowledged among engineers; and by the recommendation of the Commissioners of Railways, in a published minute of the 29th of June, 1847, a government commission has been appointed for the very purpose. The minute expresses a doubt "whether the experimental data and the theoretical principles at present known are adequate" for the "designing iron bridges, when these are to be traversed by loads of extraordinary weight at great velocities."

There seems to exist great discrepancy of opinion as to the effect of the velocity of transit. Some have imagined that it may become a source of safety, by causing the railway train to pass over before the girder has had time to yield. Others, again, have estimated the effect of the moving load as highly as six or seven times that of the same load at rest. In the following investigation, both these opinions will be shown to be incorrect: they are here cited merely as indications of the extreme uncertainty prevalent on the subject.

The method of inquiry about to be explained consists, not in determining the dynamical strain absolutely, but by comparison with the corresponding statical strain. The results will consequently be much simpler than they would be if the dynamical strain were estimated independently. The deflection which a given load *at rest* upon a girder produces, will be always taken as one of the known data of the problem. The determination of this statical deflexion, as it forms the basis of all the remaining calculations, is the first point of inquiry.

When a beam is not affected by a permanent set or defect of elasticity, it appears, both from theory and actual experiment, that the deflexion by a weight resting at its centre, is very nearly proportional to that weight—that is, if a given number of tons deflect it one inch, double the number of tons will deflect it two inches. This result is arrived at by Professor Mosely in his "Mechanical Principles of Engineering," and M. Navier in his "Resumè de Leçons de Construction," by independent methods. Its near accordance with practical truth has been abundantly confirmed by experiment, as may be verified by reference to numerous published accounts of actual observations on the subject, and especially to Mr. Hodgkinson's invaluable "Experimental Researches on the Strength of Cast-Iron." This work gives the results of an exceedingly large number of experiments, made by the author and others, on the transverse strength of beams loaded at their centres; and although these beams were of very different forms and dimensions, the law indicated is nearly observed in all of them. Whether the section of the beam be rectangular, triangular, or T-shaped, with the vertical rib either upwards or downwards, the constant ratio, in each beam, of each deflexion to the corresponding load is nearly maintained: and the same remark applies to beams of the form most useful for railway purposes—that of an upper and lower flange connected by a vertical rib.

It will be found, however, by reference to the tables of Mr. Hodgkinson's work, that the actual deflexions are somewhat more than the theoretical law would make them. This discrepancy may be accounted for by attributing it to the defect of elasticity, which the ordinary theory of beams does not consider. As this defect is not generally very great, it will here in the first instance be neglected: the deflexions will primarily be estimated as if the elasticity were perfect; and subsequently the modifications due to defect of elasticity will be taken into consideration.

Work Done on the Deflexion of a Beam.

The "work done" by a moving force may be defined to be the product of that force into the distance through which it acts. A familiar instance of the use of this measure is the Steam-Engine; where the work done receives the particular name of Horse-Power. If the pressure on the piston were uniform, that pressure (in pounds) multiplied by the distance through which it is exerted (in feet) would, if divided by 33,000, give the horse-power. But in the steam-engine, and all other practical instances, where the pressure is not uniform, but varying, it is impossible to calculate the work done by this direct multiplication. Where the value of the moving force is constantly altering, we may resort to either of the following methods of ascertaining the work done by it,—we may multiply its *average* value by the distance through which it acts; or, when that average cannot be ascertained, we may consider the whole distance divided into elementary portions, so small that it may be supposed without sensible error—that the pressure is at least uniform while it acts through each portion in succession. The aggregate work done, is the sum of the work done on each of these portions—that is, it is the sum of the products of each portion of the distance and the corresponding pressure.

This process of summation when carried out with the greatest possible accuracy, is equivalent to that of mathematical integration; in which case, the work done by a varying pressure may be defined, in mathematical language, to be the integral of the product of the pressure, and its "virtual velocity." The work done in deflecting a beam by pressure at its centre is easily ascertained, if that pressure be assumed proportional to the deflexion. Calling the deflexion x, and therefore the pressure $\alpha\, x$, (where α is a constant depending on the dimensions, &c. of the beam,) we have—

$$\text{work done} = \int_0^x \alpha x\, dx = \frac{\alpha\, x^2}{2} = \frac{\alpha\, x}{2} \cdot x.$$

Now $\frac{\alpha\, x}{2}$ is the pressure or weight which would statically maintain half the deflexion x. Hence *the work done in producing a given deflexion is equal to the weight which would statically maintain half the deflexion, multiplied by the whole distance of deflexion.*

The value of this rule will appear hereafter.

Distinction of Gradual and Instantaneous Loading.

When experiments are made on the strength and deflexion of beams, they are generally loaded very gradually at their centres. Each addition to the load is allowed to produce its full effect before more be imposed. Consequently, at every stage of the experiment, the beam is in a state of statical equilibrium: the pressure of the load on the beam is always just equal to its weight, and is never increased by any momentum arising from downward velocity.

But if the whole load be suddenly and at once placed on the beam, while it is as yet undeflected, the effects are entirely altered. The deflexion is greater than the same load would produce if gradually applied; for when the beam has reached the point of statical deflexion, the momentum acquired by the downward motion urges it further; and the descent of load continues till it be *brought up* (so to speak) by the increased resistance of the beam. Afterwards, the beam and load rise again, as the deflexion has been carried beyond the degree at which it can be statically maintained.

In the case here supposed of instantaneous loading, nothing like impact or sudden collision occurs. The pressure at the centre of the beam is finite and continuous. The load does not *fall upon* the beam—it is merely supposed to be placed originally in close contact with the beam, and then suffered to instantaneously rest upon it.

For the sake of elucidation, one or two instances of analogous action may be cited. If a common balance have its fulcrum above the points of suspension of the scales, and a weight suddenly rest in one of the scales, the lever will turn through a much greater angle than if the same weight were applied in small successive portions.

If an elastic string suspend vertically a weight from one end of it, the string will be more stretched if the whole weight be suffered to act at once, than if applied in small portions. It will be found, that if the extension of the string be proportional to the stretching force, the extension produced by the descending weight will be *twice* that due to the gradual effect of the same weight.

A light cylinder of wood, loaded at its lower end, and floating vertically in water, furnishes another illustration. If the cylinder be raised a little above its position of equilibrium and then let go, it will sink twice the distance it has been raised, if the motion be so small that the resistance is equal to the hydrostatical pressure.

In the same way, in a perfectly elastic horizontal beam, loaded at its centre, the effect of instantaneous loading is double that of gradual loading. For, by a known principle of mechanics, when a material system moves from one position of rest to another position of rest, the work done by the retarding forces is equal to the work done by the accelerating forces. For any small deflexion of a beam by instantaneous loading, its position of ultimate deflexion is one of instantaneous rest, for immediately before it arrives at that position, all the parts of the beam descend, and immediately after, ascend. Also, the work done by the accelerating force is the weight actually resting on the beam, multiplied by the space of deflexion: and the work done by the retarding forces is, by what has been said above, "equal to the weight which would statically maintain half the deflexion, multiplied by the whole distance of deflexion." Therefore, putting the two amounts of work done equal to one another, we see that the weight actually upon the beam is that which would statically maintain half the deflexion. In other words, *the deflexion is doubled by instantaneous loading.*

Transit of a Single Weight.

We now proceed to examine the effect of the transit of a single weight *along* the girder, and first of all to show that its effect cannot exceed that which it has just been estimated to produce, if stationed at the centre of the girder and allowed to descend freely from the undeflected position—in other words, it will be proved that at whatever rate the weight may travel over the girder, its ultimate strain and deflexion cannot be more than double the corresponding statical effects produced when it *rests* at the centre of the girder.

There is a general rule of constant use in engineering which, expressed in practical language, states that power is never gained, but only modified, by the intervention of machinery. This rule may be more scientifically expressed and extended by tracing it to its origin—it is a particular case of the principle known in theoretical mechanics, as the Conservation of *Vis Viva*. This principle may be very conveniently enunciated by employing the term "work done," as defined above: and it then assumes this form of enunciation—that the *vis viva* gained or lost by a system in moving from one position to another, is equal to twice the difference between the work done by the accelerating, and that done by the retarding, forces in the same interval.

From this it follows, that where there is no gain or loss of *vis viva*, there is *no* difference between the work done by the accelerating and retarding forces respectively. Hence, if the parts of the system be moving at the same velocity in the second position as in the first—or if both positions be positions of rest—the aggregate work done in the interval by the retarding forces is equal to that done by the accelerating forces.

A very simple case will illustrate this theorem. If a locomotive-engine travel a mile along a railway, and its velocity at the end of the mile be the same as at the beginning of the mile, the work done by all the forces which have resisted its motion is in the aggregate just equal to the horse-power developed in the steam-cylinders. And this equality holds good, however the engine have moved in the interval—whether on a straight level road, or on severe curves and gradients—whether the speed were uniform or very irregular—whether the steam were on the whole time, or the engine during large parts of the journey moved by its momentum only. The intermission of the moving force and all other irregularities disappear in the result. To establish equality between the work done in moving, and that done in retarding, the engine, all that is necessary is that the engine be moving neither faster nor slower at the end of the mile, than at the beginning of it.

Another illustration will serve to show the extreme generality of the principle in question. If a certain quantity of water have to be raised a certain height, the amount of work actually requisite for effecting the object is in all cases equal to the weight of water multiplied by the vertical height. This amount of necessary power or work is incapable of being diminished by any mechanical or hydraulic contrivance. The water may be contained in a vessel which is drawn up perpendicularly, as from a well, or which is drawn up an inclined plane or by a spiral path; or the water may be raised by an Archimedian screw, or by buckets attached to the periphery of a revolving wheel, or by a hydraulic-ram, or by a force pump; or lastly, it may be thrown up in a jet, as from a fountain or fire-engine. But it is physically impossible, by these or any other methods, to diminish the requisite amount of labor. It is, of course, easy to increase the amount by a waste or unprofitable expenditure of labor, such as is caused by friction of the machinery, or the mutual action of the particles of water among themselves. But supposing no waste of force to occur—supposing all the power usefully employed in simply raising the water without doing anything else; then the amount of that power is in all cases just what has been stated—the weight of water multiplied by the vertical distance through which it is raised.

The rule is of universal application, and there is no other principle of dynamics of such great and constant utility in practical science; for it embraces all those cases of motion with which the engineer happens to be concerned—cases where the motion either ceases, or has the same values, at regularly-recurring intervals.

The case before us, of the transit of a weight along a girder, is a striking exemplification of this Principle of the Conservation of Work. For this principle enables us immediately to compare the effect of a weight moving along the girder, and that of the same weight stationed at its centre, and descending. If the deflexion be the same in both cases, the work done by the descent of the load in both cases is the same—namely, the weight multiplied by the vertical descent: and this is true, whatever be the path of descent. Now, it has already been shown, that in the case of instantaneous loading, the work done by the descent of the weight is equal to that necessary to produce in the beam the deflexion which twice the weight would statically maintain. Hence the traveling weight can do no more.

The value of this conclusion appears the greater; when it is considered that it avoids all hazardous hypotheses as to the forms assumed by the beam during the transit. However the beam may be bent—whatever may be the nature of its vibrations and internal action, this is certain,—that when its elasticity is unimpaired, a weight traveling along it cannot, under any circumstances whatever, more than double its corresponding statical deflexion. To suppose it capable of doing more, is to suppose the physical impossibility of a gain of power.

But though the traveling weight cannot, under any circumstances, produce *more* than double the statical deflexion, it is quite possible that it may do *less*. A large portion of the work done by the weight may be absorbed in producing lateral vibrations and other irregularities of motion in the beam. All these concomitant operations act by way of diminution, and tend to make the dynamical deflexion *less than* double the statical central deflexion.

In determining the actual amount of this diminution, the velocity of transit must be taken into account. For that there is some particular velocity for which the deflexion is a maximum, is obvious from this simple consideration—that when the weight travels exceedingly slowly along the beam, it always exerts a statical pressure, and does not tend to increase by momentum the deflexion beyond its statical amount;—and, on the other hand, when the weight travels with excessive rapidity, it may not have time during the transit, to sink even the distance of statical deflexion.—To take the limiting case, when the velocity is indefinitely great, the descent of the weight must be indefinitely small; for even if it fell freely, and there were no beam to support it, the distance of descent in an indefinitely short time is inappreciable.

Effects of the Inertia of the Beam.

There is, then, between the exceedingly high and the exceedingly low velocity, some particular intermediate speed which produces the greatest possible deflexion. Before, however, considering what that velocity is, or endeavoring to establish a direct relation between the velocity and the deflexion, it is necessary to examine more particularly the case just referred to—where the velocity of transit is so great, that the weight has not time to sink beyond a certain degree.

Now, there are two ways in which this consideration of time might be supposed to affect the amount of deflexion. The first is that already stated, where the period of transit is so short, that even if the weight descended freely, without support from the beam, its descent would be inconsiderable. This case may, however, be at once excluded, when it is considered that at all practicable railway velocities, the time of transit over a long girder (50 to 80 feet) could not be much less than one second, that a body would fall freely upwards of 16 feet in that time, and that its actual descent (equal to the deflexion of the girder) is only a few inches.

But there is another way in which the consideration of time might be supposed to affect the deflexion: there might not be time enough to overcome the inertia of the beam. This case requires more particular examination.

A person skating over a weak piece of ice may sometimes, by moving rapidly, glide over it safely before it have *time* to break—that is, before the pressure of his body have impressed on the ice the downward motion sufficient for it to attain the point of fracture while he is passing over it. Now, by the general principles of mechanics, the same pressure which, acting for a given time, would produce a great velocity in a small mass, will produce proportionably little velocity in a large mass. In order then that the inertia of the ice may, in the case supposed, be a cause of safety, it must be large in comparison with the pressure acting on it; that is, the mass of ice acted upon must greatly exceed the mass of the man's body.

In the same way, in order that the inertia of a girder might be a cause of security, the mass of the girder must be very much greater than that of the train passing over. But it will be shown that the mass of the former does not, for heavy loads, exceed that of the latter so greatly as to perceptibly diminish the deflexion. It has sometimes been found useful to add to the inertia of the girder by laying on it heavy ballast, and by this means the structure is rendered *steadier*,—that is, the slight lateral oscillations and other irregularities of motion are reduced. But it is only these smaller or subsidiary movements that can be diminished by adding to the weight of the girder. Its mass, and that of the permanent load upon it, is not in general so large as to materially influence the main, or vertical, deflexion, when produced by nearly as heavy a load as it will safely bear.

When the train passes over the girder, the centre of gravity of the whole system sinks, the impressed moving force downwards being the weight of the train, and the motion of the centre of gravity being retarded by the elastic force of the girder.

To take a case every way unfavorable to the conclusion which we wish to establish, let the greatest deflexion be 3 inches, and the velocity of the transit so great that the weight passes over the girder in a second of time. This would be the time of transit over a girder 88 feet long, at the rate of a mile a minute. Now, the extreme deflexion may be supposed to be accomplished in half the time of transit, or the centre of the girder sinks

3 inches in half a second. The centre of gravity of the whole system at no time sinks so much as the centre of the beam sinks, for its two ends do not sink at all. On the whole, it seems an ample allowance to suppose the maximum vertical descent of the centre of gravity of the beam $1\frac{1}{2}$ inch. Now, to find the work which would *alone* prodnce this velocity, we must have an equation of *vis viva*, excluding the retarding force.

By the ordinary rules for calculating the rectilineal motion of bodies, if a given mass M originally at rest be acted upon at its centre of gravity by one uniform force f moving through a space x in the time t,

$$2\,Mx = f\,t^2.$$

Suppose the force to be that of a small weight. The mass of this weight will be found (on substituting numerical values and putting gravity $= 32$) to be only the 32d part of M, if the latter move through $1\frac{1}{2}$ inch in half a second.

The beam is usually constructed to bear a pressure considerably exceeding its own weight. In that case *less* than one 32d of the work actually exerted by the traveling weight would suffice for the mere acceleration of the beam: and we come to the conclusion, that even at the highest practicable velocity, the power required to set the beam in motion subtracts very little of the power producing deflexion. In other words, when the mass of the load is not small compared with that of the beam, the deflexion is never materially influenced by the inertia of the beam.

Influence of the Velocity of Transit on the Deflexion in the case of a Single Weight.

Having arrived at the important conclusion that when the traveling weight is large, the inertia of the beam is an immaterial consideration, or that the effective moving forces are inconsiderable compared with the impressed forces, we might suppose the mutual pressure between the beam and the weight statically equal to the force which the former by its elasticity exerts in an upward direction to resist deflexion.

But, in fact, the mutual pressure between the beam and the weight is an unknown force, not generally susceptible of exact determination. During the first part of the motion, the weight does not, so to speak, exert its full pressure on the beam, for the surface yields and recedes before it. During the latter part of the descent, on the contrary, the pressure in question exerts a superior power, to destroy the momentum previously acquired by the descending weight. The weight then moves downwards, first with an accelerated, and subsequently with a retarded, velocity: or the pressure on its under side is in the former stage of motion less, and in the latter stage greater, than the effect of gravity.

The path of the weight is likewise unknown, for the motion is made up of two parts—the motion along the beam, and the motion of the beam itself. If, indeed, it be assumed that the motion is always along the beam, or that at every instant the curvature of the beam has, at the point of mutual contact, the same tangent as the path of the weight, the problem would be capable of solution. The investigations of Professor Mosely and M. Navier have determined the curvature of the beam sufficiently to afford means of tracing the curve described by the moving weight; and therefore its pressure, which is equal to its centrifugal force + the effect of gravity, might be ascertained.

The hypothesis which would lead to these results is, however, arbitrary and unsafe: and besides, the curvature of the beam as mathematically determined, is not exactly that which occurs in actual practice, where the elasticity is always more or less imperfect. The difficulty is however of no great importance, because, as will be presently shown, it does not occur where the moving body is not a single weight, but a long train. And the subject is here referred to, merely to show the almost insuperable difficulties of determining the motion of a single weight along an elastic beam.

(TO BE CONTINUED.)

Railway Accidents in Great Britain and Ireland.

By an Analysis of the Returns made to the Railway Department, it appears that of the 90 persons killed and 99 injured, on all the Railways open for public traffic in Great Britain and Ireland, during the half-year ending the 30th June, 1848, there were—

6 Passengers killed, and	60 injured from causes beyond their own control.
5 Passengers killed, and	2 injured, owing to their own misconduct or want of caution.
7 Servants of Companies or of Contractors killed, and	14 injured from causes beyond their own control.
52 Servants of Companies or of Contractors killed, and	18 injured, owing to their own misconduct or want of caution.
18 Trespassers and other persons, neither Passengers nor Servants killed, and	5 injured by improperly crossing or standing on the railway.
1 person run over and killed, at a crossing, through misconduct of an engine-driver.	
1 suicide.	
—	—
90 killed.	99 injured.

And for the same period the number of passengers amounted to 26,330,492.

Application of Air to the Brakes of Railway Carriages.

Mr. Samuel Cunliffe Lister, of Manningham Hall, Bradford, has taken out a patent for so applying air, either by compression or by exhaustion, as to act upon the wheels of railway carriages simultaneously as a powerful break. To each carriage is to be attached an air-reservoir, or tank, of sufficient strength to sustain a pressure of 50 to 60 lbs. on the square inch. This reservoir, or tank, will be charged with compressed air, or exhausted of air, by means of an air-pump, actuated in any convenient manner. In connexion with each tank a pipe communicates with a main