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LIX. *On some Cases of Propagation of Electric Oscillations along a Number of Parallel Wires.* By W. B. MORTON, M.A., Professor of Natural Philosophy, Queen's College, Belfast*.

§ 1. *Introduction.*

WHEN simple harmonic electromagnetic waves are guided along conducting wires of finite resistance, the effect of the leads is to produce, first, a retardation of the speed of propagation as compared with that in free space, and, second, an attenuation of the amplitude of the waves as they proceed. Both these effects are, in general, functions of the frequency of the oscillations and of the constants of the conducting system. Their determination is what is mainly wanted in the solution of the problem. The complete solution, of course, involves in addition a knowledge of the distribution of the electric and magnetic vectors inside and outside the conductors.

Let λ be the wave-length, κ the attenuation-factor, $\frac{p}{2\pi}$ the frequency. Then, for waves travelling along the axis of z in the positive direction, the different vectors contain the factor

$$e^{-\kappa z} \sin \left(\frac{2\pi z}{\lambda} - pt \right).$$

Replacing this by

$$e^{i(mz-pt)}, \text{ where } m = \frac{2\pi}{\lambda} - i\kappa,$$

we see that a knowledge of the complex quantity m gives us at once the wave-length (and therefore the speed of propagation) and the attenuation factor. The case of a single wire surrounded by a sheath was worked out by Prof. J. J. Thomson†; that of a single wire isolated in space has been solved by Sommerfeld‡. In the latter case the return currents are carried by the dielectric; lines of force which start from a positive section of the surface of the wire bend round and end on an adjoining negative section, forming in general very long loops.

In experimental investigations on the subject the common arrangement consists of two similar parallel wires. For slow oscillations we have then Heaviside's well-known formula

$$m^2 = -(R + ipL)(S + ipC)$$

* Communicated, in abstract, to the British Association at Bradford

† J. J. Thomson, *Recent Researches*, p. 262.

‡ Sommerfeld, *Wied. Ann.* lxxvii. p. 233 (1899).

where $R L S C$ are respectively the resistance, inductance, leakage-conductance, and capacity, per unit-length of the leads. When the oscillations are not slow we may still employ this formula, but $R L S C$ will now be functions of p .

In the *Annalen der Physik* for June of the present year, G. Mie* has given a very complete solution of this problem of propagation along two similar parallel wires. He finds an expression for the wave-length and attenuation involving a series of ascending powers of the ratio of radius of wire to distance of the wires apart, which is usually a small quantity. He also determines completely the distribution of the electric and magnetic vectors.

§ 2. Scope of the present Note.

Before this paper appeared I had been working at the problem by a method of successive approximations. The bulk of this work has been rendered nugatory by the publication of Mie's much more satisfactory analysis, which gives a single formula from which the solution can be got to any desired degree of accuracy. The object of the present note is to point out how the *first approximation* to the complete solution may be very simply deduced from the known single-wire solution. This approximation, as Mie has shown, means that the *square* of the small quantity, radius of wire divided by distance apart, is neglected. The advantage of this way of arriving at the approximate solution lies in the fact that the method admits of application to more complicated cases where an exact solution is not possible. In fact we can obtain an approximate equation for m in the general case of any number of parallel wires of any sizes and materials, and any arrangement in space, provided that all the radii are small compared with all the relative distances. The different roots of this equation correspond to the different ways of grouping the wires into two opposite sets, such as might perhaps be realized by connecting them in different ways, each wire to a separate secondary plate in a Lecher arrangement. If we take corresponding points on the wires we shall have positive charges and currents on all the wires of one set, negative on those of the other set. We shall use "similar" and "opposite" to indicate wires belonging to the same or different groups.

§ 3. List of Cases worked out.

The equation in its general form is intractable. The following special cases are treated :—

* Mie, *Ann. d. Phys.* ii. p. 202 (1900).

- (1) Two wires of slightly different diameters, the wires being (a) opposite, as in the usual experimental arrangement, or (b) similar.
- (2) Three wires in an isosceles triangle, the base wires being opposite.
- (3) Four wires in rectangular arrangement, any grouping.
- (4) $2n$ wires in regular polygon, consecutive wires being opposite.
- (5) n wires in regular polygon, all the wires being similar.

§ 4. Statement of Results for Single-wire case.

We shall use the following quantities in addition to those already defined :—

$$k_1 = \frac{p}{V} = \frac{2\pi}{\lambda_0}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where V is the velocity of radiation, λ_0 the wave-length in free space.

$$k_2 = (1-i)\sqrt{\frac{2\mu\rho}{p}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

μ, ρ being the permeability and resistivity of the wire.

$$c^2 = k_1^2 - m^2 = \left(\frac{2\pi}{\lambda_0}\right)^2 - \left(\frac{2\pi}{\lambda} - i\kappa\right)^2. \quad . \quad . \quad . \quad . \quad (3)$$

c^2 is the function of the quantities λ, κ which will appear in the final equations.

Take now the case of the single isolated wire as worked out by Sommerfeld. We have three vectors to deal with, viz., lengthwise electric force, radial electric force, and magnetic force in circles concentric with the wire. The values of these which satisfy the differential equations, inside and outside the wire, are :—

	Inside.	Outside.	
Lengthwise electric force...	$dJ_0(k_2r)$	$DK_0(cr)$	}
Radial electric force	$d \cdot \frac{im}{k_2} J_1(k_2r)$	$D \frac{im}{c} K_1(cr)$	
Magnetic force	$d \cdot \frac{ik_2}{p} J_1(k_2r)$	$D \frac{ik_1}{pc} K_1(cr)$	

(4)

r is the distance from the axis of the wire. The J 's and K 's are the cylinder functions; d, D are constants. The magnitude m has been neglected in comparison with k_2 (*v. Thomson or Sommerfeld, loc. cit.*). Expressing that the tangential components are continuous we have, if a is the radius of the wire,

$$\frac{d}{D} = \frac{K_0(ca)}{J_0(k_2a)} = \frac{k_1^2 K_1(ca)}{k_2 c J_1(k_2a)} \quad . \quad . \quad . \quad . \quad (5)$$

∴ the equation for c is

$$\frac{cK_0(ca)}{K_1(ca)} = \frac{k_1^2 J_0(k_2 a)}{k_2 J_1(k_2 a)} \dots \dots \dots (6)$$

Now ca is always, as Sommerfeld has shown, a very small quantity (order 10^{-4} or 10^{-3}); K_0, K_1 therefore approximate to the values

$$\left. \begin{aligned} K_0 &= \log \frac{2i}{\gamma ca} \\ K_1 &= \frac{1}{ca} \end{aligned} \right\} \dots \dots \dots (7)$$

where γ is the constant 1.781 . . .

The equation then reduces to

$$c^2 \log \frac{2i}{\gamma ca} = \frac{k_1^2 J_0(k_2 a)}{k_2 a J_1(k_2 a)} = f, \text{ say.} \dots \dots \dots (8)$$

If we write x for $\frac{-\gamma^2 c^2 a^2}{4}$ this assumes the form

$$x \log x = \text{constant} = y, \text{ say.} \dots \dots \dots (9)$$

Sommerfeld has shown how a numerical solution of this transcendental equation may be found by an ingenious method of successive approximations, and has worked out a number of typical cases. He has proved that we may write the result in the form $x = -\Theta y$, where Θ is a quantity which varies very slowly with y . Sommerfeld's typical examples cover the whole range of practically interesting cases, and for them Θ comes out a magnitude with a negligible imaginary part, and lying between $\frac{1}{20}$ and $\frac{1}{30}$. We shall have occasion to employ this approximate solution, using Θ to mean a real fraction of this order of magnitude. It is usual and convenient for purposes of discussion to distinguish two extreme cases, viz., those in which $k_2 a$, the argument of the J functions, is very large, and those in which it is very small. The former condition is secured by low resistance and permeability, high frequency and large radius; reversal of any of these circumstances tends to bring a case under the second head. It will be seen that large $(k_2 a)$ favours the development of the "skin effect." The corresponding values of the constant f are

$$\left. \begin{aligned} \text{large } (k_2 a) \quad \quad \quad f &= \frac{ik_1^2}{k_2 a} \\ \text{small } (k_2 a) \quad \quad \quad f &= \frac{2k_1^2}{k_2^2 a^2} \end{aligned} \right\} \dots \dots \dots (10)$$

§ 5. Application to the Ordinary Two-wire Case.

Let us now take the case of two similar parallel wires each of radius a , at a distance apart b which is large compared with a . Suppose we try, as a first attempt at a solution, to superpose two single-wire solutions of the type given in the last paragraph. The compounded vectors will, of course, satisfy the connecting equations for the dielectric. The difficulty arises when we come to the surface-conditions at the wires. But here we notice that of the three components specified in (4), there is one, the longitudinal electric force, which varies with $\log r$; the other two contain the factor $\frac{1}{r}$. The falling off in the values of the two $\left(\frac{1}{r}\right)$ components is much more rapid than for the $(\log r)$ component. Take for example $\frac{a}{b} = \frac{1}{100}$. The quantity $\log \frac{2i}{\gamma ca}$ is, in actual cases, roughly of order 10. At the surface of the second wire this is reduced by $\log_e 100 = 4.6$. So that at the second wire the longitudinal electric force due to the field of the first wire is about half as great as the force due to the field of the second wire itself; whereas the radial electric force and the magnetic force only amount to one per cent. of the corresponding values. In fact, to neglect at the surface of the second wire the radial electric force due to the first, involves the same degree of error as if we had the two wires statically charged with equal and opposite charges, and neglected the small variations from uniform distribution on the wires, due to their mutual influence.

If now we agree to retain at the surface of the second wire only the lengthwise component of the field of the first, and further, if we neglect the variation of this component from one point to another over the surface of the wire, we can arrange the ratio of the internal and external constants (d, D above) so as to satisfy the surface condition. We have then, instead of (4),

	Inside.	Outside.
Lengthwise electric force...	$dJ_0(k_2a)$	$D[K_0(ca) - K_0(cb)]$.
Magnetic forces as before.		

$$\therefore \frac{d}{D} = \frac{K_0(ca) - K_0(cb)}{J_0(k_2a)} = \frac{k_1^2 K_1(ca)}{k_2 c J_1(k_2a)}. \quad (11)$$

The smallness of ca enables us to use the small argument value for $K_0(cb)$ also, in all cases except where $\frac{b}{a}$ is so great that the problem ceases to resemble experimental conditions.

The equation then reduces to

$$c^2 \log \frac{b}{a} = \frac{k_1^2 J_0(k_2 a)}{k_2 a J_1(k_2 a)} = f. \quad (12)$$

This agrees with Mie's result, neglecting $\frac{a^2}{b^2}$ and higher powers. It will be seen that a simple equation for c^2 replaces the transcendental form treated by Sommerfeld.

§ 6. Equation for the most General Case.

It is clear that we can apply the same method to the most general case, on the understanding that the mutual distances of the wires are all large in comparison with their radii. We shall have a different pair of constants (d, D) for each wire. The continuity of magnetic force for each wire will give

$$\frac{d}{D} = \frac{k_1^2 K_1(ca)}{k_2 c J_1(k_2 a)}, \quad (13)$$

the constants k_1, k_2, a having now different values for the different wires.

The lengthwise electric force inside the wire is as before,

$$\begin{aligned} dJ_0(k_2 a) &= D \frac{k_1^2 J_0(k_2 a)}{k_2 c J_1(k_2 a)} K_1(ca) \text{ by (13)} \\ &= D \frac{k_1^2 J_0(k_2 a)}{c^2 k_2 a J_1(k_2 a)} \\ &= D \frac{f}{c^2}. \end{aligned}$$

Distinguishing the wires by subscripts, we get for the first wire

$$D_1 \frac{f_1}{c^2} = D_1 K_0(ca_1) + D_2 K_0(cb_{12}) + D_3 K_0(cb_{13}) + \dots \quad (14)$$

$b_{12}, b_{13}, \&c.$ being the mutual distances.

For shortness write

$$\left. \begin{aligned} K_0(ca_1) &= \log \frac{2i}{\gamma ca_1} = A_1 \\ K_0(cb_{12}) &= B_{12}, \text{ and so on.} \end{aligned} \right\} \dots \quad (15)$$

The equation giving c for the system is

$$\begin{vmatrix} A_1 - \frac{f_1}{c^2} & B_{12} & B_{13} \dots \\ B_{12} & A_2 - \frac{f_2}{c^2} & B_{23} \dots \\ \dots & \dots & \dots \end{vmatrix} = 0. \quad (16)$$

§ 7. Case of Two nearly equal Wires.

Let $a_1 = a$, $a_2 = a(1 + \epsilon)$, where ϵ is a small fraction whose square may be neglected; then

$$\log a_2 = \log a + \epsilon,$$

$$A_2 = A_1 - \epsilon.$$

Taking the case where $k_2 u$ is large, $f_2 = f_1(1 - \epsilon)$.

The general equation becomes for this case

$$\left(A - \frac{f}{c^2}\right) \left[A - \epsilon - \frac{f}{c^2}(1 - \epsilon)\right] = B^2. \quad (17)$$

Regarding this as a quadratic in $\frac{f}{c^2}$ and solving, we get

$$\frac{f}{c^2} = A + \frac{1}{2}\epsilon(A - 1) \pm B(1 + \frac{1}{2}\epsilon). \quad (18)$$

(a) The lower sign gives the result for two *opposite* wires

$$\begin{aligned} \frac{f}{c^2} &= (A - B)(1 + \frac{1}{2}\epsilon) - \frac{1}{2}\epsilon \\ &= \log \frac{b}{a} + \frac{1}{2}\epsilon \left(\log \frac{b}{a} - 1 \right). \end{aligned} \quad (19)$$

It is of interest to compare the magnitudes of the currents in the two wires. The magnetic force just outside the wire is $D \frac{ik_1^2}{pc^2 a}$, the total current across the section is therefore equal to $D \frac{ik_1^2}{2pc^2}$, and the currents at corresponding points of the two wires are in the ratio $D_1 : D_2$. It is easy to prove that the ratio of the surface-charges has the same value. Referring to equation (14) we find

$$\begin{aligned} -\frac{D_2}{D_1} &= \frac{A - \frac{f}{c^2}}{B} = \frac{B - \frac{1}{2}\epsilon(A - B - 1)}{B} \\ &= 1 - \frac{1}{2}\epsilon \frac{\left(\log \frac{b}{a} - 1\right)}{\log \frac{2i}{\gamma cb}}. \end{aligned} \quad (20)$$

Examination shows that the small quantity has a positive real part, therefore the modulus of D_2 is smaller than that of D_1 , or the *larger* wire carries *less* current and surface-charge than the other.

Looking at the matter from a more physical standpoint, we

see that, while the greater number of lines of force, starting from a positive section of one wire, find their way to the surface of the other wire, there will be also, close to the points of zero surface-charge, some *re-entrant* lines, with their ends on adjoining sections of the same wire. Our analysis shows that the proportion of these latter lines is greater for the smaller wire. It will help us to understand this result if we consider that when the wires are isolated, as in Sommerfeld's case, the *scale* is smaller when the radius of the wire is smaller. The loops formed by the lines of force, now all re-entrant, are shorter; and so, when a second wire is brought up, the proportion of these looped lines which lie close enough to the wire to preserve their arrangement undisturbed, is greater.

I have worked out, numerically, the case of two copper wires of the dimensions and conductivity taken by Sommerfeld in his first case, and found the effect of making the radius of one wire 1 per cent. greater than the other, their distance apart being 100, 200, and 300 times the radius. The numerical data are:—

$$a = 0.2 \text{ cm.}, \quad \frac{p}{2\pi} = 10^9, \quad \frac{1}{\rho} = 5.83 \times 10^{-4}, \quad \epsilon = \frac{1}{100}.$$

The ratio $\frac{-D_2}{D_1}$ comes out:—

$$\begin{aligned} \text{for } \frac{b}{a} = 100, & \quad \frac{99.5 + .07i}{100}; \\ \frac{b}{a} = 200, & \quad \frac{99.3 + .11i}{100}; \\ \frac{b}{a} = 300, & \quad \frac{99.2 + 1.58i}{100}. \end{aligned}$$

So that the differences amount roughly to 5, 7, and 8 parts in a thousand. The imaginary part indicates a small phase-difference between corresponding points on the wires, the larger wire being in advance.

(b) When the two wires are similar we have, taking the positive sign in equation (18),

$$\begin{aligned} \frac{f}{c^2} &= A + B + \frac{1}{2}\epsilon(A + B - 1) \\ &= \log\left(\frac{-4}{\gamma^2 c^2 ab}\right) + \frac{1}{2}\epsilon\left[\log\left(\frac{-4}{\gamma^2 c^2 ab}\right) - 1\right]. \quad (21) \end{aligned}$$

A similar investigation shows that in this case the larger wire carries more current than the other. We obtain results of the same character when we suppose $k_2 a$ small.

§ 8. *Three Wires in Isosceles Triangle.*

Use 1 for the vertex wire, 2, 3 for the base wires. Let the base wires have the same radius and material, then $B_{12}=B_{13}$, $A_2=A_3$, $f_2=f_3$. The determinant equation becomes

$$\begin{vmatrix} A_1 - \frac{f_1}{c^2} & B_{12} & B_{12} \\ B_{12} & A_2 - \frac{f_2}{c^2} & B_{23} \\ B_{12} & B_{23} & A_2 - \frac{f_2}{c^2} \end{vmatrix} = 0. \quad (22)$$

Subtracting the third column from the second we see that $A_2 - B_{23} - \frac{f_2}{c^2}$ is a factor. Using this root of the equation,

$$\frac{f_2}{c^2} = A_2 - B_{23} = \log \frac{b}{a}, \quad (23)$$

where b is the length of the base. Putting in the value for $\frac{f_2}{c^2}$ in the equations, we get at once

$$D_1 = 0, \quad D_2 = -D_3. \quad (24)$$

Therefore we have got the case in which the base wires are carrying equal and opposite currents, and the vertex wire is quite free from current. The equation for c^2 is the same as if there were no third wire. The remaining two roots of the cubic in c^2 correspond to the cases where the base wires are similar, and either opposite or similar to the vertex wire. The equation for these arrangements is intractable.

We infer from the above that when two wires are conveying electric oscillations, a third wire equidistant from them plays a passive part. It follows at once that if we have two pairs of wires at the corners of a *rhombus* there will be no mutual influence between the pairs. For example, telephone-messages sent along the pair at the ends of one diagonal of the rhombus will not be overheard along the other pair.

When the triangle is *equilateral* we can have currents, whose algebraic sum is zero, divided among the wires in any manner.

§ 9. *Four Wires in Rectangular Arrangement.*

In this case we can split up the determinant into four factors corresponding to the four ways of grouping the wires.

We may indicate these arrangements by the notation (1234), (12) (34), (13) (24), (14) (23), numbers enclosed in one bracket indicating similar wires which are opposite to the wires in the other bracket. The wires in this and the remaining cases are identical in size and material. All the A's and f 's are equal, and $B_{12}=B_{34}$, $B_{13}=B_{24}$, $B_{14}=B_{23}$. The determinant is

$$\begin{vmatrix} A - \frac{f}{c^2} & B_{12} & B_{13} & B_{14} \\ B_{12} & A - \frac{f}{c^2} & B_{14} & B_{13} \\ B_{13} & B_{14} & A - \frac{f}{c^2} & B_{12} \\ B_{14} & B_{13} & B_{12} & A - \frac{f}{c^2} \end{vmatrix} \dots \dots \dots (25)$$

$$= \left(A - \frac{f}{c^2} + B_{12} + B_{13} + B_{14} \right) \left(A - \frac{f}{c^2} + B_{12} - B_{13} - B_{14} \right)$$

$$\times \left(A - \frac{f}{c^2} - B_{12} + B_{13} - B_{14} \right) \left(A - \frac{f}{c^2} - B_{12} - B_{13} + B_{14} \right).$$

Taking the factors in turn, and finding the corresponding ratios for the D's, we find:—

$$\begin{aligned} D_1 = D_2 = D_3 = D_4 & \text{ grouping (1234),} \\ D_1 = D_2 = -D_3 = -D_4 & \text{,, (12) (34)} \\ D_1 = -D_2 = D_3 = -D_4 & \text{,, (13) (24),} \\ D_1 = -D_2 = -D_3 = D_4 & \text{,, (14) (23).} \end{aligned}$$

For the first case, where the wires are similar, we get an equation which can be reduced to Sommerfeld's $x \log x$ type, viz.:—

$$\frac{f}{c^2} = \log \frac{16}{\gamma^4 a b_{12} b_{13} b_{14} c^4} \dots \dots \dots (26)$$

The other cases give simple equations of the two-wire type, e. g. for (12) (34):—

$$\frac{f}{c^2} = \log \frac{b_{13} b_{14}}{a b_{12}} \dots \dots \dots (27)$$

Thus when the oscillations are propagated along *four* wires in rectangular arrangement, two wires serving as returns to the other two, *the speed and the attenuation are the same as if we had two wires at distance $\frac{b_{13} b_{14}}{b_{12}}$; i. e., the rectangle between*

the two distances of opposite wires divided by the distance of the similar wires.

§ 10. $2n$ Wires at corners of Regular Polygon,
consecutive wires being opposite.

It is clear from symmetry that $D_1 = -D_2 = D_3 = -D_4 = \&c.$; therefore equation (14) gives at once

$$\begin{aligned} \frac{f}{c^2} &= A - B_{12} + B_{13} - \dots - B_{1(2n-2)} + B_{1(2n-1)} - B_{12n} \\ &= \log \frac{b_{12} b_{14} \dots b_{12n}}{a b_{13} b_{15} \dots b_{1(2n-1)}}. \quad \dots \quad (28) \end{aligned}$$

Let r be the radius of the circumscribing circle, then

$$\begin{aligned} b_{12} &= 2r \sin \frac{\pi}{2n}, \quad b_{13} = 2r \sin \frac{2\pi}{2n}, \quad \&c.; \\ \therefore \frac{f}{c^2} &= \log \frac{2r}{a} + \log \frac{\sin \frac{\pi}{2n} \sin \frac{3\pi}{2n} \dots \sin \frac{(2n-1)\pi}{2n}}{\sin \frac{2\pi}{2n} \sin \frac{4\pi}{2n} \dots \sin \frac{(2n-2)\pi}{2n}} \\ &= \log \frac{2r}{a} + \log \frac{1}{n} \\ &= \log \frac{2r}{na}. \quad \dots \quad (29) \end{aligned}$$

Therefore the *speed and attenuation with the given arrangement* are the same as for a pair of leads at distance $\frac{2r}{n}$.

§ 11. n Wires at corners of Regular Polygon,
all carrying similar currents.

Here $D_1 = D_2 = D_3, \&c.$, and we have

$$\begin{aligned} \frac{f}{c^2} &= A + B_{12} + B_{13} + \dots B_{1n} \\ &= n \log \frac{2r}{\gamma c} - \log a - (n-1) \log 2r - \log \sin \frac{\pi}{n} \sin \frac{2\pi}{n} \dots \sin \frac{(n-1)\pi}{n} \quad (30) \end{aligned}$$

Write g for $\sin \frac{\pi}{n} \sin \frac{2\pi}{n} \dots \sin \frac{(n-1)\pi}{n}$,

$$\begin{aligned} \frac{f}{c^2} &= -\log \left(\frac{\gamma^n c^n r^{n-1} g}{2i^n} \right) \\ &= -\frac{n}{2} \log \left(-\frac{\gamma^2 r^{\frac{2(n-1)}{n}} g^n}{2a^{\frac{2}{n}}} c^2 \right) \quad \dots \quad (31) \end{aligned}$$

We may write this

$$-c^2 \log(-hc^2) = \frac{2f}{n}$$

(h being the constant inside the bracket);

$$\therefore -hc^2 \log(-hc^2) = \frac{2hf}{n}.$$

Therefore, using Sommerfeld's approximate solution as stated in § 4 above,

$$\begin{aligned} -hc^2 &= -\frac{2hf}{n} \Theta, \\ c^2 &= \Theta \frac{2f}{n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32) \end{aligned}$$

So that, if we neglect the slow variation of Θ , the solution is independent of h .

To find a measure of the effect of the n leads under discussion, we may compare them with a single wire of the same material. For this we have by (8), a' being the radius, and f' the corresponding constant,

$$\begin{aligned} -c^2 \log\left(-\frac{\gamma^2 a'^2}{4} c^2\right) &= 2f' \\ c^2 &= \Theta \cdot 2f'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33) \end{aligned}$$

Therefore if the single wire is to produce the same effect as the n wires, we must have

$$nf' = f.$$

Referring to (10), we see that when $k_2 a$ is large this gives

$$a' = na;$$

when $k_2 a$ is small,

$$a'^2 = na^2.$$

In other words, when the skin-effect is marked the n wires produce the same effect on the waves as a single wire of n times the *perimeter*; but when the currents are not concentrated on the surface, but occupy the whole section, the n wires can be replaced by a single wire of n times the *area*. This result was of course to be expected from general considerations. The same approximate method of treatment might be applied to equation (26) above.

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19th October, 1900.