ON WIEN'S DISPLACEMENT LAW.

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IN a previous note we have shown how readily the general principles of thermodynamics lend themselves to the treatment of the problem of a moving hollow space which is the seat of black radiation. In the present note we shall show that the same method can be used to establish Wien's displacement law.

If we consider a hollow space which is the seat of black radiation and if we denote by ε, η, v, t, p the energy, entropy, volume, temperature and pressure of the system, we know that we may write the fundamental thermodynamic equation

$$d\varepsilon = t d\eta - p dv.$$

If, however, we suppose that the radiation within the hollow space instead of containing waves of all possible frequencies, contains only waves whose frequencies lie between zero and a certain upper limit ν, then the above equation no longer holds; we must complete it by adding a term which shall correspond to the change in the energy of the system due to a change in the upper limit of frequencies. We are thus led to assume the equation

$$d\varepsilon = t d\eta - p dv + \nu dr, \tag{1}$$

where νdr represents the change in the energy of the system when, at constant entropy and constant volume, the upper limit of frequencies is varied. The new variable r which we have thus introduced may be called the radiation of the system.

If now we assume that the energy of the system is a homogeneous function of the first degree of the entropy η, the volume v and the radiation r, we may write

$$\varepsilon = \eta \frac{\partial \varepsilon}{\partial \eta} + v \frac{\partial \varepsilon}{\partial v} + r \frac{\partial \varepsilon}{\partial r}. \tag{2}$$

From equation 1, however, it follows that the partial derivatives in equation 2 are respectively equal to t, $-p$ and ν. Accordingly, equation 2 becomes

$$\varepsilon = t\eta - pv + \nu r. \tag{3}$$

If we differentiate equation 3 and make use of equation 1 we obtain at once the equation

$$0 = \eta dt - vdp + rd\nu. \tag{4}$$

From this equation it is easy to show that p must be a function of t and ν. Accordingly, we may write

$$p = f(t, \nu). \tag{5}$$

Moreover, since equation 4 can be written in the form

$$dp = \frac{\eta}{v}dt + \frac{r}{v}d\nu, \tag{6}$$

it follows that

$$\frac{\eta}{v} = \frac{\partial f}{\partial t}, \tag{7}$$

$$\frac{r}{v} = \frac{\partial f}{\partial \nu}. \tag{8}$$

Finally, from equations 3, 5, 7 and 8 we get

$$\frac{\varepsilon}{v} = t\frac{\partial f}{\partial t} + \nu\frac{\partial f}{\partial \nu} - f. \tag{9}$$

Let us now introduce the fact that the radiation pressure is equal to one third of the specific energy. Then from the equation

$$p = \frac{1}{3}\frac{\varepsilon}{v} \tag{10}$$

and the equations 5 and 9 we get at once the equation

$$t\frac{\partial f}{\partial t} + \nu\frac{\partial f}{\partial \nu} = 4f. \tag{11}$$

Thus f must be a homogeneous function of the fourth degree in t

and ν. Accordingly, we may change our notation and replace $f(\nu, t)$ by the expression $\nu^4 f(\nu/t)$. Equations 5, 7, 8 and 10 then become

$$p = \nu^4 f\left(\frac{\nu}{t}\right), \tag{12}$$

$$\frac{\eta}{\nu} = \frac{\partial}{\partial t}\left[\nu^4 f\left(\frac{\nu}{t}\right)\right], \tag{13}$$

$$\frac{r}{v} = \frac{\partial}{\partial \nu}\left[\nu^4 f\left(\frac{\nu}{t}\right)\right], \tag{14}$$

$$\frac{\varepsilon}{v} = 3\nu^4 f\left(\frac{\nu}{t}\right). \tag{15}$$

Wien's displacement law now follows at once. From equation 15 we get

$$\frac{\partial}{\partial \nu}\left(\frac{\varepsilon}{v}\right) = \frac{\partial}{\partial \nu}\left[3\nu^4 f\left(\frac{\nu}{t}\right)\right], \tag{16}$$

and if we carry out the indicated differentiation we shall find without difficulty that the right-hand side of the equation is equal to the product of ν^3 by a function of ν/t. Thus we may write

$$\frac{\partial}{\partial \nu}\left(\frac{\varepsilon}{v}\right) = \nu^3 F\left(\frac{\nu}{t}\right). \tag{17}$$

This equation is one of the forms of Wien's law.

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