VIII.—*Note as to the Dynamical Equivalent of Temperature in Liquid Water, and the Specific Heat of Atmospheric Air and Steam*, being a Supplement to a Paper *On the Mechanical Action of Heat*. By WILLIAM JOHN MACQUORN RANKINE, Civil Engineer, F.R.S.E., F.R.S.S.A., &c.

(Read 2d December 1850.)

(33*.) In my paper on the Mechanical Action of Heat, published in the 1st Part of the 20th Volume of the *Transactions of the Royal Society of Edinburgh*, some of the numerical results depend upon the dynamical equivalent of a degree of temperature in liquid water. The value of that quantity which I then used, was calculated from the experiments of DE LA ROCHE and BÉRARD on the apparent specific heat of atmospheric air under constant pressure, as compared with liquid water.

The experiments of Mr JOULE on the production of heat by friction, give, for the specific heat of liquid water, an equivalent about one-ninth part greater than that which is determined from those of DE LA ROCHE and BÉRARD. I was formerly disposed to ascribe this discrepancy in a great measure to the smallness of the differences of temperature measured by Mr JOULE, and to unknown causes of loss of power in his apparatus, such as the production of sound and of electricity; but, subsequently to the publication of my paper, I have seen the detailed account of Mr JOULE's last experiments in the *Philosophical Transactions* for 1850, which has convinced me, that the uncertainty arising from the smallness of the elevations of temperature, is removed by the multitude of experiments (being forty on water, fifty on mercury, and twenty on cast iron); that the agreement amongst the results from substances so different, shews that the error by unknown losses of power is insensible, or nearly so; and that the necessary conclusion is, that the dynamical value assigned by Mr JOULE to the specific heat of liquid water, viz. :—772 feet per degree of Fahrenheit, does not err by more than two, or, at the utmost, three feet; and therefore, that the discrepancy originates chiefly in the experiments of DE LA ROCHE and BÉRARD.

I therefore take the earliest opportunity of correcting such of my calculations as require it, so as to correspond with Mr JOULE's equivalent. They relate to the specific heat of atmospheric air as compared with liquid water, and to that of steam, and are contained in the second and third Sections of my paper, Articles 14 and 20; Equations 28, 34, and 36.

SPECIFIC HEAT OF ATMOSPHERIC AIR AS COMPARED WITH LIQUID WATER.—(Section II
Article 14.)

The dynamical values of the specific heat of atmospheric air are calculated independently from the velocity of sound, without reference to the specific heat of liquid water; and from the closeness of the agreement of the experiments of M.M. BRAVAIS and MARTINS, MOLL and VAN BEEK, STAMPFER and MYRBACH, WERTHEIM and others, it is clear that the limits of error are about $\frac{1}{300}$ for the velocity of sound, $\frac{1}{150}$ for the ratio, and from $\frac{1}{40}$ to $\frac{1}{50}$ for the dynamical values of the specific heat of air, at constant volume and constant pressure. Those values, as given by Equation 27, are—

Real specific heat,—

$$\begin{aligned} k &= 238.66 \text{ feet} = 72.74 \text{ metres per centigrade degree.} \\ &= 132.6 \text{ feet per degree of Fahrenheit.} \end{aligned}$$

Apparent specific heat under constant pressure,—

$$\begin{aligned} K_p &= 334 \text{ feet} = 101.8 \text{ metres per centigrade degree.} \\ &= 185.6 \text{ feet per degree of Fahrenheit.} \end{aligned}$$

The ratio of these two quantities being taken as

$$\frac{K_p}{k} = 1 + N = 1.4$$

The dynamical equivalent of the specific heat of liquid water, as determined by Mr JOULE, is

$$\begin{aligned} K_w &= 1389.6 \text{ feet} = 423.54 \text{ metres per centigrade degree.} \\ &= 772 \text{ feet per degree of Fahrenheit.} \end{aligned}$$

The specific heat of air, that of liquid water being taken as unity, has therefore the following values :—

Real specific heat,—

$$\frac{k}{K_w} = \frac{132.6}{772} = 0.1717.$$

Apparent specific heat under constant pressure,—

$$\frac{K_p}{K_w} = \frac{185.6}{772} = 0.2404$$

This last quantity, according to DE LA ROCHE and BÉRARD, is

$$0.2669$$

The discrepancy being,

$$0.0265$$

or *one-ninth* of the value according to Mr JOULE's equivalent.

SPECIFIC HEAT OF STEAM. (Section III. Art. 20.)

The apparent specific heat of steam (Eq. 34 and 36) as a gas under constant

pressure is equal to that of liquid water $\times 0.305$. Its dynamical value is therefore

$$\begin{aligned} K_p &= k + \frac{1}{C_n M} = 1389.6 \times 0.305 \\ &= 422.83 \text{ feet} = 129.18 \text{ metres per centigr. degree.} \end{aligned}$$

$$\text{But} \quad \frac{1}{C_n M} = 153.48 \text{ feet} = 46.78 \text{ metres per centigr. degree.}$$

Therefore the real specific heat is

$$k = 269.35 \text{ feet} = 82.40 \text{ metres per centigr. degree.}$$

Or, that of liquid water being taken as unity,

$$\frac{k}{K_w} = \frac{269.35}{1389.6} = 0.194$$

The ratio of these two values of the specific heat of steam is

$$1 + N = 1.57.$$

Their dynamical equivalents for FAHRENHEIT'S scale are

$$k = 149.64 \text{ feet} \quad . \quad . \quad . \quad K_p = 235.46 \text{ feet.}$$

Neither the formulæ in the fourth Section, respecting the working of the steam-engine, nor the tables at the end of the paper, require any alteration; for the action of steam at full pressure being calculated from data independent of its specific heat, is not at all affected by the discrepancy I have mentioned; and the expansive action is not affected to an extent appreciable in practice.