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XLVII. The development of arbitrary functions

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the margins come in contact with one another and thus cause a diminution of the total resistance of the grating. Especially the fact that the spacing of the strips should have so large an influence as to reduce the effect almost to nothing seems to me to favour the above supposition. Also the fact that the change of resistance depends upon the orientation of the strips seems to point in the same direction. Still another fact which is equally important from the above point of view is that the resistance of the tinfoil grating once diminished by an exposure to the action of electric waves can be restored nearly to the original value by tapping the grating. It may be pointed out that Prof. Minchin's experiment * on the action of electromagnetic radiation on films containing metallic powders may stand in close connexion with the present subject.

XLVII. *The Development of Arbitrary Functions.*
By Prof. JOHN PERRY and Mr. H. F. HUNT†.

STUDENTS at the Finsbury Technical College have for some years carried out the Fourier development idea of the late Professor Clifford by practical Geometrical methods. The curve which graphically represents the function is supposed to be wrapped round a circular cylinder, and the areas of its projections upon two diametral planes enable the coefficients to be found. In 'The Electrician' of June 28th, 1895, one of us gave detailed instructions for carrying out the process. We do not know of any method of development in a Fourier series which can compare with this in either rapidity or accuracy.

In the discussion of Prof. Henrici's paper, April 13th, 1894, one of us described a graphical method of developing any arbitrary function in a series of other normal forms than sines and cosines, such as Bessels or Zonal Spherical Harmonics. It consisted in wrapping the curve which represents the function round a specially shaped cylinder, not circular, and projecting it upon a certain plane. One of us has wasted many months in finding with great exactness a sufficient number of coordinates of the trace of the cylinder suitable for a Zeroth Bessel development. The labour was unnecessary because the coordinate most troublesome to calculate is not really needed: that is, the actual shape of the cylinder is

* Phil. Mag. [5] xxxvii. p. 90 (1894).

† Communicated by the Physical Society: read October 25, 1895.

not needed, and indeed it is better to put aside altogether the idea of a curve being wrapped round a cylinder.

To develop any arbitrary function of x (let us call it y) in normal forms, the real difficulty consists in finding the value of an integral such as $\int_0^a y \cdot Q(x) \cdot dx$, where $Q(x)$ is some tabulated function. If now z is another tabulated function which is the integral of $Q(x)$ the required integral is $\int y \cdot dz$.

Suppose the values of y for 25 equidistant values of x to be known, from $x=0$ to $x=a$ (that is, a is divided into 24 equal parts). Let the corresponding values of z be also tabulated, and let a curve be drawn with the values of y as ordinates and the values of z as abscissæ; the area between this curve and the axis of z gives the value of the integral required.

To illustrate the method we shall show how it is applied in a well-known kind of problem. An arbitrary function $y=f(r)$ of r from $r=0$ to $r=a$ is to be developed in Zeroth Bessels; that is, we must determine the constants $A_1, A_2, \&c.$, in

$$f(r) = A_1 J_0(\mu_1 r) + A_2 J_0(\mu_2 r) + \&c. + A_s J_0(\mu_s r) + \&c., \quad (1)$$

where $\mu_1, \mu_2, \&c.$, are the successive roots of

$$J_0(\mu a) = 0. \quad (2)$$

Now it is well known that

$$A_s = \frac{2}{a^2 [J_1(\mu_s a)]^2} \int_0^a r f(r) J_0(\mu_s r) \cdot dr; \quad (3)$$

or, letting $\mu_s r = x$, (3) becomes

$$A_s = \frac{2}{a^2 [J_1(\mu_s a)]^2} \frac{1}{\mu_s^2} \int_0^{\mu_s a} x f(x) J_0(x) \cdot dx. \quad (4)$$

Now

$$\int x \cdot J_0(x) \cdot dx = x \cdot J_1(x),$$

which is easily found for any value of x .

To evaluate the integral in (4), take the 25 equidistant values of our function from 0 to a and use them as the ordinates of points in a curve. The values of the corresponding abscissæ are obtained in the following way. The values of $\mu_s a$ &c. are the successive roots of $J_0(\mu a) = 0$, and are

$$\begin{aligned} \mu_1 a &= 2.40483 \\ \mu_2 a &= 5.52008 \\ \mu_3 a &= 8.65373 \\ \mu_4 a &= 11.79153 \\ &\text{and so on.} \end{aligned}$$

Take 25 equidistant values of x beginning with 0 and ending with the value $\mu_s a$: the corresponding values of $x J_1(x)$ are the abscissæ of points in the curve required to find A_s .

The area of the curve is easily obtained by means of the planimeter. Having found the 25 abscissæ once for all, they may be used for the development of any arbitrary function of r of which 25 equidistant values are given. We now publish four tables giving these abscissæ, which we have carefully calculated for the first four terms of any such development.

In the column headed P we give the number of the point in the curve of which $xJ_1(x)$ is the abscissa, there being 25 such points altogether. In Table I. the point 24 corresponds to $x=2.40483$. In Table II. the point named 24 corresponds to $x=5.52008$, and so on.

In Table II. an additional abscissa is given corresponding to some unknown point between numbers 10 and 11; this is the extreme abscissa of the curve, and the corresponding ordinate would be a tangent to the curve. In Table III. there are two additional or bounding abscissæ and in Table IV. there are three of them.

TABLE I.

P.	$x \cdot J_1(x)$.
0	0.0000
1	.0050
2	.0200
3	.0450
4	.0787
5	.1216
6	.1727
7	.2312
8	.2932
9	.3667
10	.4416
11	.5197
12	.5999
13	.6807
14	.7609
15	.8392
16	.9142
17	.9845
18	1.0490
19	1.1062
20	1.1552
21	1.1949
22	1.2242
23	1.2423
24	1.2485

TABLE II.

P.	$x \cdot J_1(x)$.
0	0.0000
1	+ .0267
2	.1010
3	.2242
4	.3800
5	.5578
6	.7458
7	.9191
8	1.0706
9	1.1825
10	1.2417
11	1.2160
12	1.1141
13	.9418
14	.7052
15	.4132
16	+ .0791
17	- .2801
18	.6455
19	.9963
20	1.3118
21	1.5723
22	1.7605
23	1.8626
24	1.8783
10 A	1.2485

TABLE III.

P.	$x \cdot J_1(x)$.
0	0.0000
1	+ .0640
2	.2434
3	.5086
4	.7921
5	.9485
6	1.2135
7	1.2395
8	1.0986
9	.8366
10	+ .3353
11	- .2107
12	.7807
13	1.2957
14	1.6788
15	1.8667
16	1.8195
17	1.5279
18	1.0163
19	- .3817
20	+ .4154
21	1.1561
22	1.7807
23	2.2002
24	2.3490
6 A	+1.2485
15 A	-1.8783

TABLE IV.

P.	$x \cdot J_1(x)$.
0	0.0000
1	+ .1171
2	.4268
3	.8168
4	1.1373
5	1.2468
6	1.0555
7	+ .5501
8	- .1542
9	.9246
10	1.5554
11	1.8658
12	1.7443
13	1.1825
14	- .2851
15	+ .7485
16	1.6705
17	2.2544
18	2.3065
19	1.8114
20	+ .8497
21	- .3674
22	1.5548
23	2.4214
24	2.7410
4 A	+1.2485
10 A	-1.8783
17 A	+2.3490

As an example we have taken a function which has the value 1 at the centre of a circle and the value 0 at the circumference, and which at any place is a linear function of the radius. The outer radius a of the circle is 1.

In fig. 1 the function is shown graphically by the straight line MDD, and the numbered points in this line correspond to 25 equidistant ordinates. In our actual drawing, of which fig. 1 is a reduction, the scale is such that one unit of value of the function corresponds to 10 inches.

The distances from 0 to any number in the axis of abscissæ ON represented in inches ten times the corresponding values of $xJ_1(x)$ of Table I. Projecting vertically from any point in ON, and horizontally from the corresponding point in DD we obtained points in the curve MBB. We use inches because British planimeters usually measure areas in square inches. We found the area OMBBNO to be 61.17 square inches, and as both coordinates were magnified ten times, the

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Fig. 1.

Fig. 2.

value of the integral in (4) or

$$\int_0^{2.40483} x f(x) J_0(x) . dx = 0.6117,$$

and hence by (4), as $J_1(2.40483) = 0.519$,

$$A_1 = \frac{2 \times 0.6117}{(0.519)^2 (2.405)^2} = 0.7852.$$

To find A_2 . The 25 equidistant ordinates of the original curve (straight line) are the ordinates of points in the curve MBLBBP (fig. 2), the abscissæ being taken from Table II. We found it convenient to multiply the abscissæ by 5 and represent in inches. Our ordinates were multiplied by 10 and represented in inches. The actual area was 2.36 square inches and one-fiftieth of this, or 0.0472, is the value of the integral in (4).

Hence, as $J_1(5.5201) = .3403$,

$$A_2 = \frac{2 \times 0.0472}{(0.3403)^2 \times (5.5201)^2} = 0.0268.$$

We need not show the curves used in finding A_3 and A_4 . The area of fig. 2 is the positive area MLNOM minus the area LQPNL, but one need not think about whether an area is positive or negative. It is only necessary to start the planimeter-tracer from the point numbered 0 in every case, and go from 0 to M and along the curve in the direction of the increasing numbers to 24, then along the axes of abscissæ, ending at the point 0 from which we started.

XLVIII. *The Relation between the Atom and the Charge of Electricity carried by it.* By J. J. THOMSON, M.A., F.R.S.,
Professor of Experimental Physics, Cambridge*.

IN the electrolysis of solutions, the persistency of the sign of the electric charge carried by an ion is almost as marked a feature as the constancy of the magnitude of the charge. Thus the hydrogen ion always carries a positive charge, the chlorine ion a negative one. In the electrolysis of gases, however, the sign of the charges carried by the atoms of the different elements is much more variable: here an atom of hydrogen does not always carry a positive charge, nor an atom of chlorine always a negative one; each of these

* Communicated by the Author.